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PRACTICAL ARITHMETIC

BOOK II

BY

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PREFACE

BOOK TWO is a complete arithmetic furnishing material for a review and a more comprehensive treatment of the subjects included in Book One, together with the advanced work required for the higher grades.

The book is designed for pupils in the last three years of the elementary grades as well as for normal school classes reviewing the subject of arithmetic. The drill exercises and problems aim to give such training in arithmetical operations and relations as will enable pupils to cope with the problems they may have to deal with in the ordinary affairs of business life. It has not been the aim to furnish every kind and form of problem that anybody might have occasion to solve. Many so-called industrial and practical problems present conditions and data entirely outside of the range of experience of the majority of pupils, and are as unreal and improper for them as any that could be devised.

A number of topics that are omitted from the body of this book as of minor importance to most pupils are included in an Appendix designed for reference or for such use as the teacher may deem wise. Among these subjects are several lists of problems of special interest to the country boy, dealing as they do with the practical concerns of farm life. Some of these problems, while requiring arithmetical operations of little difficulty, have their chief value in the interest they may awaken in matters vital to the farmer.

Special attention is called to the chapter on "Use of Symbols in Arithmetical Analysis." The object of such problems

as are given in this chapter is not to give practice in operations, but to test the pupils' power of reasoning. If the pupils could give instantly each result in the several operations involved without performing the indicated operations, they could proceed with the necessary reasoning without delay, and could analyze several problems in the time usually taken to perform the operations. This saving of time is made possible by merely *indicating* the operations and using letters as symbols to represent the various results. The pupils thus get, in a given time, very much more practice in arithmetical reasoning, and in the statement of such reasoning, than is possible if they are required actually to perform all the indicated operations before they begin the analysis. A half hour devoted to work of this kind will enable the pupils to review rapidly the problems of several days preceding, or to analyze in advance many problems to be worked out completely, if thought desirable, on succeeding days.

While the number of problems in the body of the book is not so great as in some arithmetics, it is believed that it is sufficient for ample practice in the important operations. The thorough drills for accuracy and rapidity in the fundamental processes given both in Book One and in the first part of this book make it possible, in the subsequent applications of these processes, to reduce the number of problems, many of which are included in other books for no other purpose than to afford practice in such operations. The aim has been to provide material adequate for the purposes for which arithmetic is taught, and at the same time to reduce the amount of time usually given to this subject in the schools.

For the mastery of any topic in arithmetic there are certain things that the pupils must *know* and *do*. As a rule, very few of these things are entirely new, yet the

knowledge and skill previously acquired by pupils are not always available by them at the time when needed. It is evident, therefore, that at every stage the choice and arrangement of matter should be made with reference to the proper preparation of pupils for the next stage of work. While the things that must be known and done for the mastery of each new topic are not stated in the book, they have been carefully considered in its preparation. The subject-matter, accordingly, is so arranged as to insure the pupils' power to attack what is new with the greatest success.

The introduction to a new topic or to a difficult phase of a topic by means of the *Study Recitation*, in which the pupils work under the guidance and inspiration of the teacher in their preparation for the mastery of the new, is a distinctive feature of this book as well as of Book One. The Study Recitation shows the teacher the present state of the pupils' knowledge of what is necessary for the next subject and the degree of their skill in using that knowledge. It also enables the teacher to determine what, if any, review work is necessary and what special difficulties are likely to be encountered. Then, by careful guidance of the children's activities in the proper direction, the instructor may become a real teacher instead of a mere testing machine.

The *Notes to the Teacher* have been prepared with care, and are introduced at those points where experience has shown that they will be helpful for the pupils' advancement.

The *Review Questions* at the close of each important topic are designed not only to test the pupils' knowledge and power of clear statement, but also to test their power of reasoning and their ability to put that reasoning into proper form. Many of the questions are framed for the purpose of leading the pupils to arrive at and formulate the reasons for arith-

metical operations not previously explained. Thus they require the pupils *to reason*, instead of confining their work to an attempt to understand, state, and apply the reasoning given in the book. Both lines of work are regarded as valuable and both are provided for.

The books of this series are the result of many years of practical experience in teaching arithmetic and in supervising the teaching of others in all classes of public schools and in professional schools for the training of teachers. It has not been the aim to make a series of books whose chief merit is striking novelty in the character or arrangement of matter, but rather to make a series of workable arithmetics adapted to the needs of pupils and teachers in the schools of to-day.

Acknowledgment is made to M. A. Bussewitz of the Milwaukee State Normal School for valuable assistance rendered in the preparation of material for this book; and to George A. Works of the Menomonie High School for assistance in reading the proofs.

L. D. HARVEY.

SUGGESTIONS TO TEACHERS

PART I of this book should be used in sixth grade classes, Part II in seventh grade classes, and Part III in eighth grade classes. Where the work in Book One and in Part I of Book Two has been properly done, Parts II and III can be satisfactorily completed in about one half the time usually given to arithmetic in the seventh and eighth grades. It is probably better to distribute this time over the two years than to attempt to complete the work in arithmetic in the seventh grade.

Upon the first introduction of this book, it will be well for the teachers of seventh and eighth grade classes to familiarize themselves with the drill exercises in Book One and in the first part of Book Two, and to give such of these drill exercises to their classes as may be found necessary to secure accuracy and rapidity in the handling of numbers.

While the Notes to the Teacher are not intended to interfere with the teacher's use of those methods which his experience has justified, it is believed that if careful consideration is given to them they will prove valuable aids. They are the outcome of wide experience not only in teaching arithmetic but in the training of teachers.

Under the heading "Study Recitation" matter is presented which should be worked out by pupils in the class under the direction and guidance of the teacher. In order to do this properly it will be necessary for teachers to make themselves thoroughly familiar with the order and plan of work in the study recitation. This work must in no sense be considered

a waste of time. If properly done, it will prepare the class to enter with intelligence and interest upon the work of preparation of the lesson that is assigned, and will make possible a surer and more rapid advance.

In the assignment of lessons the teacher should be sure that pupils understand the meaning of terms used and that they know exactly what is expected of them, both in the way of solution of examples and problems and in the requirements for oral and written recitations. It will be well to devote a considerable portion of the time of a number of recitation periods to such study of definitions and rules as may be necessary to enable the pupils of the class to recite readily and accurately. This exercise gives good training in concentration of attention upon the subject-matter of a lesson, and presents proper standards for a recitation. With such training pupils will understand better what is required in the assignment of a lesson.

At the close of the most important subjects, a set of review questions is given. As the pupils proceed in their work upon a topic it is an excellent plan for the teacher to assign as a part of the lesson such of these questions as have been covered by the work already done. In this way pupils will be required to prepare upon not more than one or two questions a day, and will need but little time, when the subject is finished, to prepare for a complete summary of it.

New and unusual words are underscored as in Book I. The teacher should make sure that pupils understand the meaning of the underscored words.

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BOOK TWO

PART I

DEFINITIONS — NOTATION AND NUMERATION

1. A **unit** is one.

A **number** is a unit or a collection of units of the same kind.

The **unit of a number** is one of the collection that makes the number.

The unit of 10 mills is 1 mill; of 10 cents is 1 cent; of 10 dimes is 1 dime; of 10 dollars is 1 dollar; of 10 eagles is 1 eagle; of 10 dozen is 1 dozen; of 10 pounds is 1 pound; of 10 boys is 1 boy; of 10 cords is 1 cord; of 10 halves is 1 half; of 10 tenths is 1 tenth; of 10 million is 1 million; of 10 is 1.

A unit is **abstract** when it is not applied to any particular kind of thing; as, 1, 1 ten, 1 hundred, 1 half, 1 eighth.

A unit is **concrete** when it is applied to some particular kind of thing; as, 1 dollar, 1 dime, 1 egg, 1 half dollar, 1 quarter dollar, 1 regiment, 1 company, 1 cord, 1 pound, 1 ton, 1 mile, 1 rod, 1 foot, 1 inch.

An **abstract number** is an abstract unit or a collection of abstract units of the same kind.

A **concrete number** is a concrete unit or a collection of concrete units of the same kind.

Numbers like 1, 2, 3, 4, 5, 6, 20, 35, 70, 75, 76, etc., are called **whole numbers, integers,** or **integral numbers.**

Numeration is the art of reading numbers expressed by symbols.

Notation is the art of writing numbers by means of symbols.

In arithmetic, the **Arabic** or **decimal system of notation** is used. In this system ten figures are used in writing numbers. These are:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

one, two, three, four, five, six, seven, eight, nine, zero.

The first nine of these figures are called **significant figures** because they signify or express value. They are also called **digits.** The last figure is called **zero, cipher,** or **naught,** and expresses no value. It is used with the other figures in expressing some numbers.

Another system of notation, called the **Roman system,** is sometimes used in numbering chapters of books, headings, and in a few other cases. It is explained on page 17.

NUMERATION

2. **Study Recitation**

1
10
100
1000
10000
100000
1000000
1111111

1. Read the numbers written at the left.
2. Read their sum.
3. Beginning with 1, what is the value of each number as compared with the number following it?
4. Beginning with 1,000,000, what is the value of each number as compared with the one before it?
5. In the sum of the several numbers, where is the significant figure in each of the numbers added to be found?
6. In the number 1,111,111, give the number represented by each figure.

In any whole number, the places occupied by the figures are called **orders**; the place at the right is of the first order, the second place is of the second order, the third place is of the third order, and so on to the left.

7. In 1,111,111, what is the value of the unit of the first order? Of each succeeding order? In 2,222,222, what is the value of the figure in each order?

8. The value of a unit in any order bears what relation to its value in the next higher order? To its value in the next lower order?

9. In whole numbers the first three orders constitute a period, and each three succeeding orders to the left constitute another period.

Names of periods.	Trillions			Billions			Millions			Thousands			Units		
Numbers of orders.	15th order	14th order	13th order	12th order	11th order	10th order	9th order	8th order	7th order	6th order	5th order	4th order	3d order	2d order	1st order
Names of orders.	hundred-trillions	ten-trillions	trillions	hundred-billions	ten-billions	billions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	units

10. Beginning with the units, give the names of the orders in each period; as, units, tens, hundreds, thousands, etc.

11. Beginning at the right, give the names of the periods; as, units, thousands, millions, billions, trillions.

12. What is the name of the 1st period? The 3d? The 2d? The 5th? The 4th?

13. Give the names of the periods, in succession, beginning at the left.

14. What is the name of the first period? Give the number and name of each order in this period, thus:

The name of the first period is units.
The name of the first order is units.
The name of the second order is tens.
The name of the third order is hundreds.

15. What is the name of the second period? Give the number and name of each order in this period.

16. What is the name of the third period? Give the number and name of each order in this period.

17. What is the name of the fourth period? Give the number and name of each order in this period.

Notice that the name of the first order in each period is the same as the name of the period; that the name of the second order in each period after the first is *ten*, with the name of the period added; that the name of the third order in each period after the first is *hundred*, with the name of the period added.

RULE FOR NUMERATION: *Beginning at the units' order, point off the number into periods of three orders each.*

Beginning at the left, read each period as if it stood alone and add the name of the period.

Read the following numbers from the book:

18. 203,417,256

The third period, standing alone, is read, two hundred three; on adding the name of the period, it is read, two hundred three million. The second period, standing alone, is read, four hundred seventeen; on adding the name of the period, it is read four hundred seventeen thousand. The first period, standing alone, is read, two hundred fifty-six. In reading a number the name of the units' period is omitted.

	trillions	billions	millions	thousands	units
19.	56,	400,	321,	256	
20.	127,	370,	209,	127	
21.	21,	257,	048,	563,	047
22.	478,	000,	397,	256,	007
23.	421,	057,	400,	320,	000
24.	20,	003,	900,	708,	516
25.	106,	100,	400,	300,	260
26.	207,	000,	123,	405,	000

Before attempting to read the entire numbers, read the numbers in each period without adding the name of the period. Begin at the top and read downwards; as, two hundred fifty-six, one hundred twenty-seven, forty-seven, seven, naught, five hundred sixteen, two hundred sixty, naught. Whenever, in reading a number, you reach a period of ciphers, omit it and read the next period.

To THE TEACHER. Put other numbers on the board and require pupils to read them at sight, until they can read any number to one billion readily and accurately.

ARABIC NOTATION

3. To write units of any period requires one figure in that period; to write tens or tens and units of any period requires two figures in that period; to write hundreds, hundreds and tens, hundreds and units, or hundreds, tens, and units, of any period requires three figures in that period. If there are vacant places, they should be filled with ciphers.

Write the names of the first three periods in order; under these names write the following numbers in their proper places:

1. Five.
2. Fifty.
3. Five hundred.
4. Five thousand.
5. Fifty thousand.
6. Five hundred thousand.
7. Five million.
8. Fifty million.
9. Five hundred million.
10. Four hundred six.
11. Two thousand four hundred six.

12. Twenty-five thousand, four hundred six.

13. Two hundred thousand, four hundred six.

14. Two million, twenty-two thousand, four hundred six.

15. Twenty-two million, two thousand, four hundred six.

16. Two hundred twenty million, twenty thousand, four hundred six.

17. Fifteen thousand, nine hundred forty-eight.

18. Two hundred thousand.

19. Four million, four thousand, four hundred.

20. Four hundred thousand six.

21. Nine hundred eight million, fourteen thousand, five hundred.

22. Thirteen thousand eighteen.

23. Eighteen thousand two.

24. Five hundred seventeen million, three hundred sixty-eight thousand, seven hundred ninety-nine.

25. Three hundred million, eight hundred seventeen thousand, eight hundred seven.

26. Nine hundred ninety-nine million, nine hundred ninety-nine thousand, ninety-nine.

To THE TEACHER. Read the following numbers to pupils, requiring the numbers to be written as read. When written, require pupils to read them.

27.	40,062	785,020	2,469,304	5,860,000
28.	27,009	634,009	15,000,270	7,200,000
29.	146,350	780,350	27,039,009	2,794,352
30.	107,009	609,095	65,078,027	4,000,000

RULE FOR NOTATION: *Begin at the left hand and write the hundreds, tens, and units of each period in succession, filling vacant orders with ciphers.*

ROMAN SYSTEM OF NOTATION

4. The seven letters used in the Roman system are:

Letters: I, V, X, L, C, D, M.
Values: 1, 5, 10, 50, 100, 500, 1000.

The following principles govern the use of these letters in expressing numbers:

1. *Repeating a letter repeats its value.* XX = 20. II = 2.

2. *When a letter is placed after one of greater value, the two express the sum of their values.* XV = 15. CL = 150.

3. *When a letter is placed before one of greater value, the two express the difference of their values.* IX = 9. IV = 4.

4. *When a letter is placed between two letters of greater value, its value is to be taken from the sum of the other two.* XIX = 19. XIV = 14.

5. *Placing a dash over a letter multiplies its value by one thousand.* $\overline{X}$ = 10,000. $\overline{M}$ = 1,000,000.

The following table further illustrates the system:

I, 1	VIII, 8	XVI, 16	LXXX, 80	DCC, 700
II, 2	IX, 9	XX, 20	XC, 90	DCCC, 800
III, 3	X, 10	XXX, 30	C, 100	CM, 900
IV, 4	XI, 11	XL, 40	CCC, 300	M, 1000
V, 5	XII, 12	L, 50	CD, 400	MCM, 1900
VI, 6	XIV, 14	LX, 60	D, 500	$\overline{V}$, 5000
VII, 7	XV, 15	LXX, 70	DC, 600	$\overline{M}$, 1,000,000

6. Express the following numbers by the Arabic notation: XIX, XXVIII, XXIX, XXXIX, CIX, XCI, CC, DCL, $\overline{M}$, MC, $\overline{C}$, MM, $\overline{CMII}$, MDCCCCV.

7. Express by Roman characters all numbers from 1 to 50; from 51 to 100.

8. Express in the Roman notation: 106, 254, 372, 409, 698, 785, 843, 967, 1909, 10,000.

ADDITION

5. **Addition** is the process of uniting two or more numbers into one number.

The numbers added are called **addends.**

The **sum** is the number obtained by addition.

The **sign** of addition is +, read plus.

$$\begin{array}{r} 2 \\ 1 \\ 5 \\ \hline 8 \end{array}$$ Addends; Sum

Only **like numbers,** that is, numbers having the same unit, can be added.

The Forty-five Combinations

6. In rapid addition of numbers, a knowledge of the sum of any two single-figure numbers is a necessity. There are forty-five combinations. Thirty-six of these combinations are given in the addition table below, and should be memorized. The other nine, combinations of 1 and each of the digits, are not given.

Addition Table

Name the sums rapidly.

	1.	2.	3.	4.	5.	6.	7.	8.
9.	2 + 2	3 + 3	4 + 4	5 + 5	6 + 6	7 + 7	8 + 8	9 + 9
10.	2 + 3	3 + 4	4 + 5	5 + 6	6 + 7	7 + 8	8 + 9	
11.	2 + 4	3 + 5	4 + 6	5 + 7	6 + 8	7 + 9		
12.	2 + 5	3 + 6	4 + 7	5 + 8	6 + 9			
13.	2 + 6	3 + 7	4 + 8	5 + 9				
14.	2 + 7	3 + 8	4 + 9					
15.	2 + 8	3 + 9						
16.	2 + 9							

To the Teacher. For rapid drill, place the table on the board; point to different combinations, requiring individual pupils to give sum of each combination as it is pointed out. Give the combinations orally and have pupils give sums. Use the following drill table in the same manner.

Drill in finding Differences

7. **To the Teacher.** Require pupils to give the difference between 6 and each of the numbers at its right, as indicated by the sign of subtraction, minus (−); between 7 and each of the numbers at its right, and so on with all the numbers to 18.

Pupils may use their books. The teacher may give the number from which the subtraction is to be made, and then name the numbers to be subtracted from it, pupils giving differences; the exercises may be placed on the board and pupils required to give differences orally, or to write them on the board or on paper.

6 − 3, 1, 5, 2, 4 *7* − 4, 2, 5, 6, 1, 3 *8* − 2, 5, 7, 3, 6, 1, 4

9 − 4, 1, 3, 8, 5, 7, 6, 2 *10* − 9, 3, 1, 8, 4, 7, 5, 2, 6

11 − 9, 2, 6, 5, 8, 4, 3, 7 *12* − 3, 6, 9, 4, 7, 5, 8

13 − 4, 7, 5, 9, 6, 8 *14* − 6, 8, 5, 7, 9 *15* − 9, 7, 8, 6

16 − 7, 9, 8 *17* − 9, 8 *18* − 9

Drill in giving Units' Figure in the Sum of Any Two Single-figure Numbers

8. In the following, state first the units' figure in the sum and then the sum:

1.	6 + 8	5 + 9	6 + 4	2 + 9	3 + 4	6 + 6	4 + 4
2.	7 + 9	8 + 2	4 + 7	3 + 8	2 + 3	7 + 7	6 + 3
3.	8 + 7	7 + 6	3 + 9	5 + 3	5 + 2	9 + 9	3 + 3
4.	6 + 9	5 + 8	8 + 4	7 + 3	9 + 4	8 + 8	2 + 7
5.	7 + 5	5 + 6	5 + 4	6 + 2	5 + 5	9 + 8	2 + 4

Addition of a single-figure number to a two figure number.

Study Recitation

9. 1. Add: 35, 8

What is the units' figure in the sum of 8 and 5? It is 3. What is the first number larger than 35, having 3 for its units' figure? It is 43. That number is the sum of 35 and 8.

$$\begin{array}{r} 35 \\ 8 \\ \hline 43 \end{array}$$

2. Add: 48, 7

What is the units' figure in the sum of 7 and 8? It is 5. What is the first number larger than 48, having 5 for its units' figure? It is 55. That number is the sum of 48 and 7.

$$\begin{array}{r} 48 \\ 7 \\ \hline 55 \end{array}$$

3. Add: 39, 6

What is the units' figure in the sum of 9 and 6? It is 5. What is the first number larger than 39 having 5 for its units' figure? It is 45. That number is the sum of 39 and 6.

$$\begin{array}{r} 39 \\ 6 \\ \hline 45 \end{array}$$

4. Add: 26, 8

The units' figure obtained as in 1, 2, and 3 above is 4; the first number larger than 26, having 4 for its units' figure, is 34. This is the answer.

When 6 and 8 are added, the units' figure of the sum is always 4. When 36 and 8 are added, the units' figure is 4, and the first number larger than 36 having 4 for its units' figure is 44. When 46 and 8 are added, the units' figure is 4 and the sum 54, etc.

To the Teacher. Give further illustrations similar to examples 1, 2, 3, and 4, until pupils see clearly that to add a 1-figure number to a 2-figure number, three things should be known:

1. The sum of the 1-figure number and the units' figure of the 2-figure number.
2. The units' figure of this sum.
3. The first number larger than the 2-figure number that has this units' figure.

If further drill is needed, use drill exercises in Addition in Book I of this series.

10. To the Teacher. Put the following table on the board or on a large sheet of manila or wrapping paper, so that it may be ready for daily use:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
1	2	3	4	5	6	7	8	9
71	72	73	74	75	76	77	78	79
51	52	53	54	55	56	57	58	59
91	92	93	94	95	96	97	98	99
21	22	23	24	25	26	27	28	29
41	42	43	44	45	46	47	48	49
81	82	83	84	85	86	87	88	89
61	62	63	64	65	66	67	68	69
31	32	33	34	35	36	37	38	39
11	12	13	14	15	16	17	18	19

1. Require pupils to give the sum of 5 and each of the numbers under *A*; as, 6, 76, 56, 96, etc. Continue the drill until 5 has been added to every number in the table.

2. Substitute each of the other digits for 5, and require pupils to give the sum of each digit and each number in the table.

3. Name a digit and point to numbers in the table, requiring pupils to give the sum of the digit named and each number as you point to it. Do not point to the numbers in order from right to left, nor from left to right.

4. Require pupils to subtract 6 from each larger number under *A*, beginning with 11; as, 5, 25, 55, 75, etc. Continue the drill until 6 has been subtracted from every larger number in the table. In each column, begin by subtracting from the number at the foot of the column. Substitute each of the other digits for 6, and continue the drill.

5. Vary Drill 3 by requiring differences instead of sums. The drills called for above should be conducted in such a manner as to hold the attention of every member of the class and to require prompt and vigorous work, and should be continued for a few minutes daily long after pupils have passed on to further work in the book. If pupils in advanced classes do not add and subtract accurately and rapidly, they should be given a few minutes' drill on this table each day.

Written

11. Add:

1.	2.	3.	4.	5.	6.
246	366	476	759	9786	74076
36	427	244	346	7976	27345

7.	8.	9.	10.	11.
986	358	497	273	497
437	449	359	598	235
592	575	648	467	643
785	892	223	523	857
649	261	578	709	923
427	135	421	854	237

12.	13.	14.	15.	16.
2475	9468	6047	6438	3649
8434	7275	2793	4729	5784
9568	4683	8729	5643	2709
4793	9748	4649	7285	4658
5784	6572	5782	4697	2734
7286	5648	2341	4789	3251

17.	18.	19.	20.	21.
6184	6347	2784	56409	72003
2321	8459	1347	7348	15564
4598	6572	9106	28407	7005
8727	8137	3490	10456	758
2635	1549	7029	20017	26405
5248	1672	8462	90008	1509

Write and add the following numbers:

22. Two hundred six, four hundred fifty-seven, eight hundred twenty-eight, five hundred nine, seven hundred sixty, four hundred forty-eight.

23. Three hundred twenty-seven, four hundred ninety-four, eight hundred thirty-nine, two hundred twenty, five hundred eight, six hundred forty-nine.

24. Four hundred thirty-nine, one hundred forty-eight, twenty-seven, thirty-four, three hundred forty-eight, sixteen, nine hundred six, forty-three, seven.

25. Seventy-five, three hundred forty-six, seven hundred ninety-five, nine, eight hundred fifty-six, thirty-eight, four hundred fifty-six.

26. Eight hundred seventy-five, ninety-eight, six hundred seven, forty-two, three hundred nine, eight, four hundred forty-seven.

27. One thousand four hundred forty-seven, two thousand three hundred forty-four, nine thousand seven hundred ninety-eight, eight hundred, seven hundred forty-four.

28. Two thousand forty-six, five thousand three hundred fifty-seven, nine hundred one, eight thousand five, seven thousand eight hundred forty-seven, eighteen, nine.

To THE TEACHER. For further practice, dictate numbers given in examples 17–21 on page 22, and have pupils write and add them.

Drill thoroughly on the work in addition until pupils can add accurately and rapidly. As pupils advance, it will be well to come back to these pages for drill work. It is an excellent plan to time pupils on a given amount of this work. A thorough mastery of addition will save much time in subsequent work and is an absolute necessity in business transactions involving addition. Addition probably constitutes three fourths of all work in computation required in business.

12. Addition of numbers representing dollars and cents.

Prefix the dollar sign, $, to the first number in each example. It need not be prefixed to the other numbers.

Do not insert "and" when reading numbers representing dollars and cents.

Express the following in words:

ILLUSTRATION: Example 4 is read, twenty-seven dollars four cents; fifteen dollars twelve cents; nine dollars nine cents; seventy-five cents; one dollar twenty-eight cents; seven cents; five dollars seventy cents.

In each case place the decimal point before beginning to add.

1.	2.	3.	4.	5.
$ 27.50	$13.42	$18.04	$27.04	$35.06
15.13	12.15	27.00	15.12	27.98
26.25	9.22	10.10	9.09	19.01
42.13	10.20	20.08	.75	27.50
85.75	13.45	13.75	1.28	46.90
23.25	25.42	12.82	.07	82.07
42.51	18.95	26.98	5.70	19.36
$262.52				

6. Add $24.76, $766.07, $.79, $6.25, $987.60.

7. Add $17, $.08, $.96, $2.83, $.75, $1.99, $3.25, $.16, $2.40, $1.65.

8. Find the sum of $3676.09, $47.55, $5707.07, $7.85, $3206.75.

9. Add twenty-nine dollars seventeen cents ; two cents; three thousand seven hundred sixty dollars forty cents ; one dollar seventy-eight cents; eighty-three cents; six hundred six dollars one cent.

10. Write six numbers representing dollars and cents, placing units of the same kind in the same column. Add them.

Oral Drill

13. Give the sums of the following numbers:

1.	10	10	45	35	75	80	30	40	50	70
	5	30	10	20	5	20	60	20	50	40
2.	85	65	45	55	25	95	55	60	30	10
	20	30	40	30	40	20	40	25	55	15
3.	15	20	25	25	75	125	150	175	75	65
	5	15	15	50	25	25	50	25	75	35

4.

17	28	34	7	4	5	26	37	54	73
3	2	6	33	56	45	4	3	6	7

In adding two two-figure numbers, proceed as follows:

5. Add: 17, 13. Think of 17 as 1 ten and 7 units; add the 1 ten to 13, making 23; add the 7 units to 23, making 30.

6. Add: 48, 57. Think of 48 as 4 tens and 8 units; add the 4 tens to 57, making 97; add the 8 units to 97, making 105.

7. Add: 36, 45. Operation shortened: add 3 to 4 in 45, making 75; add 6 to 75, making 81.

8.

19	28	34	13	14	15	26	37	54	73	69
13	12	16	33	56	45	14	13	16	17	28

9.

15	14	18	26	37	45	56	75	86	52	48
12	17	13	12	14	12	14	16	16	14	14

PROBLEMS

Oral

14. To the Teacher. In solving the following problems, several plans may be followed. The pupil *may* read the problem and give the answer without explanation or analysis; he *should* also read the problem and analyze. The teacher should read the problem and require pupils to repeat from memory and analyze. The teacher *may* read each problem, give pupils sufficient time to solve it mentally, and at a signal have each pupil write the answer on a piece of paper. Require rapid work. The class exercises conducted as indicated above should train attention and memory and give practice in the correct use of language.

1. John has 6 marbles, Henry has 9, and James has 5. How many have all?

All have as many marbles as the sum of 6, 9, and 5 marbles.

2. A boy spends 8 cents for apples, 5 cents for candy, and 10 cents for a pencil. How many cents does he spend? Analyze.

3. Two men start from the same place and travel in opposite directions; one travels 9 miles and the other 15 miles. How far apart are they?

Since they travel in opposite directions, one 9 miles and the other 15, they are as far apart as the sum of 9 miles and 15 miles, or —— miles.

4. How many days are there in April and June? In September and November? In February and April? Make similar questions, combining any two months not named, and answer them.

April has —— days; June has —— days; both months have as many days as the sum of —— days and —— days.

5. A dealer buys corn at 76 ¢ a bushel and sells it so as to gain 9 ¢ a bushel. At what price does he sell?

If he sells the corn so as to gain 9 ¢ a bushel, he sells it for 9 ¢ more than 76 ¢, or —— ¢.

6. In this class there are —— pupils; in my reading class there are —— pupils. How many pupils are there in both classes?

7. Mary buys ribbon for 20 ¢, tape for 15 ¢, and pins for 10 ¢. How much does she pay for all?

She pays for all the sum of ——, ——, ——, or —— ¢.

8. John pays a half dollar for a knife, a quarter for a spelling book, and a dime for paper. How much does he pay for all?

He pays for all the sum of ——, ——, ——, or —— ¢.

9. A farmer has two cows; one gives 10 quarts of milk a day and the other 9 quarts. How many quarts will they both give in two days? Analyze.

10. If you add 8 to 16 and then 9 and 4 to this sum, what will the result be?

11. At present prices, what is the value of 1 dozen eggs, 1 pound of sugar, and 2 pounds of tea?

At present prices 1 dozen eggs cost ——; 1 pound of sugar costs ——; 2 pounds of tea cost ——. All are worth the sum of ——, ——, ——.

12. The top of my desk is —— inches long and —— inches wide. What is the distance from one corner to the corner farthest from it, measuring around the desk? What is the entire distance around it?

13. What is the length of the room in which you now are? What is its width? What is the distance halfway around it? The whole of the way around? Analyze.

14. John is 9 years old, and is 27 years younger than his father. How old is his father?

If John is 9 years old, and is 27 years younger than his father, his father's age is 27 years + 9 years, or —— years.

15. On a fishing trip Alfred caught 7 fish and James caught 8 more than Alfred. How many did both catch? Analyze.

16. One year ago a boy weighed 195 pounds. During the year he gained 15 pounds. How much does he now weigh? Analyze.

Written

17. Mrs. Brown purchases in a grocery store the following articles: sugar, \$1.50; coffee, \$.29; flour, \$1.05; pepper, 8¢; apples, 45¢; lettuce, 15¢; canned goods, \$2.50. What is the amount of the bill?

The amount of the bill is the sum of ——, ——, ——, ——, ——, ——, ——.

18. Mr. Smith paid $34 for a cow, $145 for a horse, $87.50 for a carriage, and $35.75 for a harness. Find the entire amount paid.

Indicate the operation; as, $34 + $145 + $87.50 + $35.75 = amount paid. Solve.

19. A farmer has 125 bushels of wheat in one bin, 184 bushels in another, and 85 bushels in each of two other bins. How many bushels has he in all?

Indicate the operation by writing the numbers in columns, and solve.

20. A merchant had $87.75 in his cash drawer at the opening of business. He took in during the day $32.25; $2.87; $39.70; $9.85; $.75; $2.40; $27.09; $9.87; $25; $.50; $3.20. How much cash had he on hand at the close of business?

Write the amounts in columns for addition and explain as in example 17.

21. July has 31 days, August 31, September 30, October 31, November 30, December 31. How many days are there in the second half of the year? In the first half?

22. I bought a farm for $6775. I spent $1024.50 for building a barn, $187.25 for fencing, $67.75 for other improvements, and then sold the farm at a profit of $596. How much did the farm cost with improvements? What was the selling price?

23. Find the number of strokes made in a day by a town clock that strikes the hours.

24. John earns $5.60; James earns $3.75 more than John; and Charles earns as much as James and John together. What is the entire amount earned? Make and answer three questions in solving this problem.

25. The amount of money in circulation in the United States on September 1, 1908, was as follows:

Gold coin and bullion	$619,990,263
Gold certificates	806,653,109
Standard silver dollars	74,891,095
Silver certificates	475,083,723
Subsidiary silver	124,005,574
Treasury notes of 1890	4,837,418
United States notes	341,311,926
National Bank notes	630,633,800

Find the total money in circulation.

REVIEW

15. 1. Define addition; addends; sum.

2. How should numbers be written for addition? Why?

```
 467
 985
 386
1838
```

What is the sum of the numbers in the first column? What is the denomination of the left-hand figure in this sum? Why is the 1 added to the next column? What is the sum of 1 and the numbers in the second column? What is the denomination of the right-hand figure in this sum? Of the left-hand figure? Why is the 2 added to the third column?

SUBTRACTION

16. Subtraction is the process of finding the difference between two numbers.

The **minuend** is the number from which we subtract.

The **subtrahend** is the number subtracted.

The **difference**, or **remainder**, is the result obtained by the subtraction.

The **sign** of subtraction is $-$, and is read *minus*.

Minuend		Subtrahend		Difference or Remainder.
27	−	9	=	18

The minuend equals the sum of the subtrahend and the difference.

17. Subtraction of any number of more than one figure from any larger number.

To the Teacher. Before proceeding with this exercise, review Exercise 7, p. 19, and Drills 4 and 5, Exercise 10, p. 21.

Adding a number to the *minuend* and an equal number to the *subtrahend* does not change the *difference*.

$14 + 10 = 24$
$\underline{9 + 10 = 19}$
$5 \qquad\quad 5$

1. From 46 take 29.

$$\begin{array}{r} 46 \\ \underline{29} \\ 17 \end{array}$$

10 must be added to 6 before 9 can be subtracted. 9 from 16 leaves 7. 1 added to the next figure, 2, in the subtrahend, gives 3; 3 from 4 leaves 1.

Explain why adding 10 to 6 in the minuend, and 1 to the tens' figure, 2, in the subtrahend, does not make the answer wrong.

2. From 507 take 398.

507
398
109

10 added to 7 gives 17. 8 from 17 leaves 9. 1 added to 9 gives 10; 10 added to 0 gives 10; 10 from 10 leaves 0. 1 added to 3 gives 4; 4 from 5 leaves 1.

3. From 46 take 29.

46
29
17

If a number is added to the minuend, and an equal number is subtracted from it, the difference is not changed.

Instead of adding 1 to the next figure of the subtrahend, after adding 10 to any figure in the minuend, 1 may be subtracted from the next figure of the minuend.

10 must be added to 6 before 9 can be subtracted. 9 from 16 leaves 7. 1 taken from 4, the next figure in the minuend, leaves 3. 2 from 3 leaves 1.

Explain why adding 10 to 6, and taking 1 from 4, does not make the result wrong.

Find the differences:

4.	**5.**	**6.**	**7.**	**8.**
2457	3856	5947	6984	5697
1529	1938	2953	1897	1499

9.	**10.**	**11.**	**12.**	**13.**
35609	52168	27431	59430	56423
26437	18539	16927	29726	17317

14. From 2767 take 398.

15. From 8646 take 3926.

16. Subtract 4763 from 967,608.

17. Subtract 9761 from 24,630.

18. Diminish 30,406 by 5904.

19. Diminish 46,076 by 3987.

20. From seventy-seven thousand take seven thousand seventy-seven.

18. Subtraction when the numbers represent dollars and cents.

Arrange the numbers for subtraction, as in former work, by placing units of the same kind in the same column. Place the decimal point in the difference below the decimal points above. Subtract, and then prefix the dollar sign to the difference.

Find the remainders:

1.	2.	3.	4.	5.
\$563.15	\$529.24	\$206.25	\$495.85	\$385.93
128.79	139.87	178.25	294.87	197.98
\$434.36				

6. Subtract \$7.50 from \$15.95.

7. Subtract \$256.85 from \$9867.24.

8. Diminish \$39,076.56 by \$2879.78.

9. What is the difference between \$4976.55 and \$3487.86?

10. \$8760 − \$398.95 = ?

11. From six thousand three hundred six dollars and sixty-seven cents take three thousand eight hundred forty-seven dollars and eighty-nine cents.

PROBLEMS

ORAL

19. TO THE TEACHER. See the suggestions to the teacher on page 25.

1. If a boy earns 65¢ on Monday and 50¢ on Tuesday, how much more does he earn on Monday than on Tuesday?

The amount the boy earns on Monday is more than he earns on Tuesday by the difference between 65¢ and 50¢, or ——¢.

2. If I buy goods for $18 and sell them for $21, how much do I gain?

I gain the difference between $—— and $——, or $——.

3. Make a problem giving the cost and selling price of goods, showing a case in which the loss is required. Give the loss.

4. A man travels at the rate of 7 miles an hour; another starts with him and travels, in the same direction, at the rate of 5 miles an hour. How far apart are they at the end of one hour?

They are as many miles apart as the difference between 5 and 7.

5. John travels —— miles an hour; Henry, in the same direction, at the rate of —— miles an hour. How far apart are they at the end of 2 hours if they start together?

Fill the blanks and solve.

6. Mary buys candy for 10¢, a pear for 5¢, an apple for 3¢. How much should she pay for all? She gives the storekeeper a quarter. How much change should she get?

7. A man sells a stock buyer a cow for $25, a sheep for $7, and hogs for $8. The stock buyer gives him a $50 bill. How much has he overpaid him?

Notice that two questions are to be answered in solving the seventh problem. The answer to the first forms a part of the second.

First question: A man sells a stock buyer a cow for $25, sheep for $7, and hogs for $8. How much should he receive for all? *Ans.*, $40.

Second question: If a stock buyer owes a man $40 for a cow, sheep, and hogs, and gives the man a $50 bill, how much has he overpaid him? *Ans.*, $10.

8. A man has 125 bushels of corn, wheat, and oats. He has 25 bushels of wheat and 25 bushels of corn. How many bushels of oats has he?

What is the first question to be answered? Answer it. State the second question and answer it.

9. Make and solve a problem like the eighth, using different numbers.

10. If I have 4 recitations daily when in school and am absent Monday and Friday, how many recitations do I have during the week? Make and answer three questions in solving this problem.

11. A boy goes to a store with one dollar in his pocket. He buys a knife for a half dollar, a book for a quarter of a dollar, a comb for a dime, and five pencils for a nickel. How much money has he left after paying for his purchases?

12. James learns to spell 12 new words each school day. Mary learns to spell 8 new words each school day. How many more new words will James learn in 4 days than Mary learns?

Make and answer two questions in solving this problem.

Written

If problems can be solved mentally, express the work, using signs to indicate the operations, and give the result orally.

13. A man had 145 cords of wood and sold 97 cords. How many cords had he left?

He had left —— cords minus —— cords.

14. A man having 145 cords of wood sold a number of cords, and then had 48 cords left. How many cords did he sell?

15. A man sold 97 cords of wood and then had 48 cords left. How many cords had he at first?

Notice how this problem differs from problems 13 and 14.

16. Mr. Jones bought a farm for $8765 and sold it at a loss of $297. How much did he receive for it?

He received for it ——. (Indicate the operation.)

Had he sold it so as to gain $297, how much would he have received?

17. A boy borrows $.75 from a playmate. How much does he owe after paying back $.60?

18. A man borrows $5657.75. How much does he still owe after returning $2950.25?

How does this differ from problem 17?

19. Henry sold James a pair of skates for $1.25 and gained 30¢. How much did they cost Henry?

20. A man sold a house and lot for $2250 and lost $150 by so doing. How much did the house and lot cost him?

21. Make a problem showing a gain of $.15 on something bought and sold. Solve it. Indicate the solution, using signs.

22. Make a problem giving selling price and loss, or gain, to find the cost. Solve it.

23. A boy earns $5 in one week and $7 the second week. He buys a suit of clothes for $10. How many dollars has he left?

Indicate the operation.

24. In one year a man earns $350. The next year he earns $375. His expenses for the two years are $460. How many dollars does he save in two years?

25. A man bought a horse for $125, a carriage for $95, and a harness for $23. If he sold the horse, carriage, and harness for $225, did he gain or lose? How much? For how much should he sell them to gain $45? To lose $30?

26. A man bought a lot for $775 and built a house on it costing $2375. He sold both for $5000. Find his profit.

27. A family earns $900 a year and spends $140 for rent, $105.50 for meat, $210.75 for groceries, and $240 for clothing and other expenses. How much does this family save during the year?

This problem may be explained as follows:

The family spends for rent, meat, groceries, and clothing in one year

$$\begin{array}{r} \$140.00 \\ +\ 105.50 \\ +\ 210.75 \\ +\ 240.00 \\ \hline \$696.25 \end{array}$$

If a family earns $900 a year and spends $696.25, it saves

$$\begin{array}{r} \$900.00 \\ -\ 696.25 \\ \hline \$203.75 \end{array}$$

Make and solve a similar problem.

28. A man sold a grocer eggs for $1.50, cherries for $2.25, butter for $7.80. He bought from the grocer sugar for $3.50, vinegar for $.40, coffee for $1.20, oil for $.50, and tea for $1.12.

In settlement which owed the other, and how much?

Separate into three problems and solve.

29. A newsboy buys 25 papers of one kind for 25¢, 15 papers of another kind for 30¢, and 10 papers of a third kind for 30¢. How much does he pay for all?

30. He sells all of the first kind for 50¢, 12 of the second kind for 36¢, and 5 of the third kind for 25¢. How much does he receive?

31. A newsboy buys 50 papers for ——¢; he sells all of the papers for ——¢. How many cents does he gain?

Fill the blanks with the proper numbers and solve.

32. A man divides $2650 among his wife and two children. The wife receives $1500, the son $900 less than the mother. How much does the daughter receive?

REVIEW

20. 1. Define difference or remainder; subtrahend; minuend.

2. Show that adding 10 to the number in any order of the minuend and 1 to the number of the next higher order in the subtrahend does not affect the difference.

3. Show that adding 10 to the number in any order of the minuend and subtracting 1 from the number of the next higher order of the minuend does not affect the difference.

4. How can you prove the correctness of the result of subtraction?

MULTIPLICATION

21. Multiplication is the process of taking one number as many times as there are units in another.

It is a short process of finding the sum of several equal numbers.

The sign of multiplication is $\times$, and is read *times* or *multiplied by*. 15×8 is read 15 times 8, or 15 multiplied by 8. 6×5 is the same as $5 + 5 + 5 + 5 + 5 + 5$, or 5 taken 6 times; or it may be regarded as $6 + 6 + 6 + 6 + 6$, or 6 taken 5 times.

The **multiplicand** is the number multiplied.

The **multiplier** is the number by which the multiplicand is multiplied.

The **product** is the result of the multiplication.

The multiplier and multiplicand are **factors** of the product.

The product divided by either the multiplicand or the multiplier gives the other factor.

62 Multiplicand
3 Multiplier } Factors
186 Product

$186 \div 3 = 62$
$186 \div 62 = 3$

A **power** of a number is the product arising from using the number two or more times as a factor.

ILLUSTRATION: $10 \times 10 = 100$. 100 is the second power, or the **square** of 10. 10×10 is also written 10^2 and is read 10 square, or the second power of 10. $10 \times 10 \times 10 = 1000$, or 10^3. 10^3 is the third power, or the **cube** of 10.

The multiplicand may represent a number of abstract or concrete units; the multiplier is always an abstract number. In the actual process of multiplication both factors are used as abstract numbers.

The product always represents a number of units of the same kind as are represented by the multiplicand. 3 times \$10 = \$30. 5 times 8 horses = 40 horses.

The product is not changed by changing the order of its factors in multiplying. If a cord of wood costs \$8, how much will 24 cords cost? *Ans.*, 24 times \$8. In finding the product it is more convenient to use the smaller number as the multiplier.

Multiplication Table

22. To the Teacher. The table gives all the products found by the multiplication of any two numbers, to and including 12×12. Pupils should learn all these products of three figures and their factors; then the products between 90 and 100 and their factors; then between 80 and 90 and their factors; and so on.

In Book I the multiplication table is presented in another order.

$144 = 12 \times 12$
$132 = 12 \times 11$
$121 = 11 \times 11$
$120 = 12 \times 10$
$110 = 11 \times 10$
$108 = 12 \times 9$
$100 = 10 \times 10$

$99 = 11 \times 9$
$96 = 12 \times 8$
$90 = 10 \times 9$

$88 = 11 \times 8$
$84 = 12 \times 7$
$81 = 9 \times 9$
$80 = 10 \times 8$

$77 = 11 \times 7$
$72 = \begin{cases} 12 \times 6 \\ 9 \times 8 \end{cases}$
$70 = 10 \times 7$

$66 = 11 \times 6$
$64 = 8 \times 8$
$63 = 9 \times 7$
$60 = \begin{cases} 10 \times 6 \\ 12 \times 5 \end{cases}$

$56 = 8 \times 7$
$55 = 11 \times 5$
$54 = 9 \times 6$
$50 = 10 \times 5$

$$49 = 7 \times 7$$
$$48 = \begin{cases} 8 \times 6 \\ 12 \times 4 \end{cases}$$
$$45 = 9 \times 5$$
$$44 = 11 \times 4$$
$$42 = 7 \times 6$$
$$40 = \begin{cases} 10 \times 4 \\ 8 \times 5 \end{cases}$$

$$36 = \begin{cases} 12 \times 3 \\ 9 \times 4 \\ 6 \times 6 \end{cases}$$
$$35 = 7 \times 5$$
$$33 = 11 \times 3$$
$$32 = 8 \times 4$$
$$30 = \begin{cases} 10 \times 3 \\ 6 \times 5 \end{cases}$$

$$28 = 7 \times 4$$
$$25 = 5 \times 5$$
$$24 = \begin{cases} 12 \times 2 \\ 8 \times 3 \\ 6 \times 4 \end{cases}$$
$$22 = 11 \times 2$$
$$20 = \begin{cases} 10 \times 2 \\ 5 \times 4 \end{cases}$$

$$18 = \begin{cases} 9 \times 2 \\ 6 \times 3 \end{cases}$$
$$16 = \begin{cases} 8 \times 2 \\ 4 \times 4 \end{cases}$$
$$15 = 5 \times 3$$
$$14 = 7 \times 2$$
$$12 = \begin{cases} 6 \times 2 \\ 4 \times 3 \end{cases}$$
$$10 = 5 \times 2$$

$$9 = 3 \times 3$$
$$8 = 4 \times 2$$
$$6 = 3 \times 2$$
$$4 = 2 \times 2$$

Drill

23. 1. Give the multiples of 2 to 24; the multiples of 3 to 36.

To the Teacher. Continue the exercise until pupils can give the multiples of numbers from 1 to 12, up to 12 times each of them.

2. How many times 8 is each of the following numbers:

16, 48, 96, 32, 88, 56, 72, 24, 40, 64, 80?

3. How many times 7 is each of the following numbers:

21, 77, 14, 49, 84, 35, 63, 28, 42, 70, 56?

TO THE TEACHER. Continue the exercise by writing on the board in irregular order the multiples of the numbers from 1 to 12, inclusive; give one factor common to each set of multiples and require pupils to give the other factor.

Vary this exercise by asking for $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of each number in the first set; $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, etc., of each number in the second set.

4. 12 is the product of what two numbers? Of what two other numbers? Give the sets of two factors which multiplied together equal the following numbers, giving first the set in which the numbers are most nearly equal:

16, 30, 28, 36, 25, 32, 21, 18; thus,

$$24 = \begin{cases} 4 \times 6 \\ 3 \times 8 \\ 2 \times 12 \end{cases}$$

The factors 4 and 6 are more nearly equal than either 3 and 8, or 2 and 12.

TO THE TEACHER. Continue the exercise by writing on the board all the products given in the multiplication table and requiring pupils to name the sets of factors given in the table, whose product equals these multiples respectively.

5. Divide both terms of each of the following fractions by 9 and give the resulting fractions:

$$\frac{18}{27}, \frac{36}{108}, \frac{54}{72}, \frac{27}{99}, \frac{45}{63}, \frac{36}{54}, \frac{54}{63}.$$

6. Divide both terms of each of the following fractions by 7 and give the resulting fractions:

$$\frac{14}{21}, \frac{28}{84}, \frac{42}{56}, \frac{35}{77}, \frac{42}{63}, \frac{21}{70}, \frac{49}{56}.$$

TO THE TEACHER. Give additional exercises, using multiples of other numbers less than 13 as terms of the fractions.

24. Multiplication of any number by another of several digits.

The product of one number by another number may be found by multiplying the first number by any part of the

second number, and then by the other part or parts of the second number and finding the sum of the products.

1. Multiply 15 by 9.

$$\begin{array}{rrrrl} 15 & 15 & & 15 & \\ 9 & 4 & & 5 & 9 = 4 + 5 \\ \hline 135 & = 60 & + & 75 & \end{array}$$

Any number of two or more figures equals the sum of the units of the first order represented by each figure in the number. Thus, $123 = 3 + 20 + 100$.

2. Multiply 256 by 123.

	A	*B*	
Multiplicand	256	256	
Multiplier	123	123 =	3 + 20 + 100
1st partial product	768	768 =	3 × 256
2d partial product	512	5120 =	20 × 256
3d partial product	256	25600 =	100 × 256
Complete product	31488	31488	123 × 256

In *A* the 0's at the right of the second and third partial products are omitted.

In *B* each partial product is written out in full.

Multiply as under *A* above:

3.	5.	7.	9.	11.	13.
13	907	308	24301	897	493
21	23	29	21	86	587

4.	6.	8.	10.	12.	14.
578	2341	599	258	956	246
42	12	68	495	89	35

15.	16.	17.	18.	19.	20.
234	485	426	678	14232	856
222	987	728	496	122	629

25. Multiplication of numbers when the multiplier contains zeros.

The product of a number by 10 or any power of ten is found by annexing the zeros of the multiplier to the multiplicand.

Multiply 426 by 10; by 100; by 1000.

The product of one number by another number may be found by multiplying the first number by one of the two factors of the other number, and the product by the other factor.

1. Multiply 21 by 15.

$15 = 3 \times 5$

$15 \times 21 = 21 \times 3 \times 5$

$3 \times 21 = 63$

$5 \times 63 = 315$

$$\begin{array}{r} 15 \\ \hline 105 \\ 21 \\ \hline 315 \end{array}$$

2. Multiply 26 by 40.

$40 = 4 \times 10$

$26 \times 40 = 26 \times 4 \times 10$

$26 \times 4 = 104$

$104 \times 10 = 1040$

$$\begin{array}{r} 40 \\ \hline 1040 \end{array}$$

3.	5.	7.	9.	11.	13.
567	786	494	853	2468	120
20	20	30	50	150	30

4.	6.	8.	10.	12.	14.
426	378	2070	540	3600	205
10	100	130	200	1000	400

26. Multiplication of two numbers when one of them represents dollars and cents.

In the complete product a decimal point is placed under the decimal point in the number representing dollars and cents, or two places from the right.

A dollar sign should be prefixed to the product. Find products.

1.	2.	3.	4.	5.
$4.75	$.85	$.07	$16.07	$3006
12	48	16	325	800

6. Multiply $8.07 by 13.
7. Multiply $4300.25 by 91.
8. Multiply $7.83 by 1000.
9. Multiply $.75 by 1000.

ORAL DRILL IN MULTIPLICATION

27.

Multiply by 2:		Multiply by 3:	
15	17	15	16
18	19	18	25
14	20	22	30
16	21	27	24
22	36	34	36
23	27	56	50
24	28	35	44
25	39	28	29
32	45	32	33
44	50	37	55

TO THE TEACHER. In conducting this drill exercise, name the number to be used as a multiplier and then give the number to be multiplied, and call on pupils for the product.

Answers may be placed on paper, in order, at the call of the word "time" by the teacher. Any pupil not having the product should make a dash in place of it. The work must be entirely mental, the only work with the pencil being to write the result. The result should always be written, beginning with the left-hand figure of the product. If too much time is given pupils to secure the required results, most of the value of the exercise is lost.

Prepare other lists of numbers, and in a similar way continue the drill exercises from time to time, to secure facility in the mental operations involved in multiplication.

PROBLEMS

Oral

28. **1.** A farmer sells 5 loads of wheat, of 60 bushels each, at 80 cents a bushel. How much does he receive?

2. How many feet are there in 1 yard? In 3 yards? How many inches are there in 1 foot? In 2 feet? In 1 yard? In 3 yards? In 2 feet? In one half of a yard?

3. If 1 inch of lace is worth ten cents, how much is one half of a yard worth? 3 yards? $3\frac{1}{2}$ yards?

4. What will be the cost of $\frac{1}{2}$ of a ton of coal at \$8 a ton, 3 cords of wood at \$4.80 a cord, and $\frac{1}{4}$ of a barrel of flour at \$4 a barrel?

5. How many quarts are there in a peck? In a bushel? How many pints equal a peck? A bushel? If a bushel of peanuts costs \$1.90, and is sold at the rate of 5 cents a pint, what is the profit?

6. John has 113 marbles, William has twice as many and 20 more, and Frank has 4 more than both. How many marbles have the three?

Written

7. Find the cost of 5 cords of wood at \$5.75 a cord.

8. Find the cost of 32 yards of silk at \$2.25 a yard.

9. How many minutes equal an hour? A day? A week?

10. Define *average*. If a railway train averages 35 miles an hour, how far will a passenger travel in four days?

11. A fruit dealer bought 12 bushels of cranberries at $2.25 a bushel and sold them at 10 cents a quart. How much did he gain?

12. From the product of 2987 and 78 take their sum.

13. I paid $36 an acre for 50 acres of woodland. I sold the wood for $1576 and the land for $17 an acre. Did I gain or lose, and how much? Solve in two ways.

14. 15 bushels of apples were purchased at $1.25 per bushel. If they were sold at 45 cents a peck, was there a gain or a loss by the transaction, and how much? Solve in two ways.

15. I sell to a dealer 14 cords of hard maple at $5.75 per cord and buy of him 8 tons of hard coal at $6.50 a ton. What sum is due me?

16. What change should be received from $50 in payment for 16 dozen buttons at 36¢ a dozen, 6 yards of velvet at $4.75 a yard, and 15 yards of lining at 18¢ a yard?

17. If a mill weaves on the average 76,080 yards of cloth in a day, how many yards will it weave in a year of 313 working days?

18. Two express trains leave St. Louis at the same time. One travels east at the rate of 50 miles an hour, the other west at the rate of 45 miles an hour. How far apart will they be in 12 hours?

19. If the daily pay of a railway conductor is $3.45, what is his salary for a year of 340 working days? If he spends on an average $12.75 a week, how much can he save in a year of 52 weeks?

BILLS

SAMPLE BILL

29. 1.

MILWAUKEE, WIS., Jan. 9, 1908.

MR. CHARLES R. BROWN,

Bought of R. H. HOLD & COMPANY,

DEALERS IN STAPLE AND FANCY GROCERIES, 34 GRAND AVE.

Jan. 2	30 lb. Granulated Sugar	@ $ .06	$ 1.80
Jan. 2	12 lb. California Prunes	@ .14	1.68
Jan. 7	15 lb. Carolina Rice	@ .08	1.20
Jan. 9	5 doz. Eggs	@ .17	.85
			$ 5.53

Received payment,
R. H. HOLD & Co.
Per H. Hanson.

The amount of this bill was paid to H. Hanson, who was authorized to collect the bill for the firm.

A **bill** is a written statement of services performed or of goods sold, with the price of each item.

A **debtor** (Dr.) is one who owes a debt.

A **creditor** (Cr.) is one to whom a debt is owed.

An **account** is a written statement of the business transactions between two parties for a given time.

A **receipt** is a written statement that a sum of money or some goods have been received.

A bill is receipted when the creditor or his agent writes "Paid" or "Received payment" and adds his signature. This is usually done at the bottom of the bill.

When was the above bill made out? When were the goods bought? Who is the debtor? Who is the creditor?

Who bought the goods? Who sold the goods? Who received payment for the goods? Who made out the bill? Who receipted it? After the payment is made who should have the bill? Of what use will it be to him? What does "Per H. Hanson" mean?

2.

CHICAGO, ILL., Feb. 18, 1909.

BOARD OF EDUCATION, MAYVILLE, WIS.,

To AMERICAN BOOK COMPANY,

EDUCATIONAL PUBLISHERS, 521–531 WABASH AVE., DR.

20 Fyffe's Greek History Primer	@	$.35
25 Bryant & Stratton's Bookkeeping (Com. Sch.)	@	.80
16 Overton's Applied Physiologies — Adv.	@	.80
80 Metcalf & Bright's Lang. Les. Part I.	@	.35
12 doz. Curtiss's Semi Vert. Copy Books, No. 1	@	.96
8 doz. Curtiss's Semi Vert. Copy Books, No. 2	@	.96
6 doz. Curtiss's Semi Vert. Copy Books, No. 3	@	.96

Finish and receipt the bill.

When were these books bought? Who bought them? When were they paid for? Who sold them?

3. W. L. Spring bought of Green & Harris, Grocers, S. Toledo, Ohio, March 12, 1900, 4 tubs of butter (170 lb.), at $.31; 20 lb. Rio coffee, at $.35; 4 lb. Japan tea, at $.85. March 18, 8 doz. eggs, at 14¢; 25 lb. prunes, at 12¢; 8 bu. potatoes, at 55¢; 50 lb. granulated sugar, at 6¢. April 2, 30 lb. dried apples, at 11¢; 8 lb. cheese, at 14¢.

Make out a receipted bill.

4. J. W. Stevens bought of R. M. Sell & Company, Dry Goods and Notions, April 6, 1905, 25 yd. muslin, at 8¢; 33 yd. calico, at 6¢; 25 pr. men's boots, at $3.25. April 30,

15 yd. alpaca, at 70¢. May 8, 1 Clay worsted suit, $16.75; 2 pr. rubbers, at $.75; 2 neckties, at $.60; 6 collars, at 12¢.

Stevens paid this account June 3, 1899. Make out a receipted bill.

5. Make out a receipted bill of your own.

REVIEW

30. 1. Define multiplication; multiplicand; multiplier; product; power.

2. What are partial products?

3. In the process of multiplication, where is the first figure in each partial product written?

4. Multiply 24 by 32. What is the first partial product? What is the first partial multiplier? What is the second partial product? What is the second partial multiplier? What is the complete product? Show that the sum of the two partial multipliers equals the complete multiplier. Show that the second partial product, if written in full, equals $24 \times 3 \times 10$. The complete product, 768, is the sum of what two products?

Why is this sum the complete product? See example 2, p. 42.

5. Having given the product and multiplier, how can you find the multiplicand?

6. Having given the product and multiplicand, how can you find the multiplier? Illustrate each case.

7. Having given the number of tons of any article and the cost per ton, how can you find the entire cost? Which number is the multiplicand? What is the denomination of the product? Illustrate.

8. Having given the cost of a quantity of coal and the number of tons, how can you find the cost of one ton? Illustrate.

9. Having given the cost of a quantity of coal, and the cost of one ton, how can you find the number of tons? Illustrate.

10. Knowing the rate a vessel sails per hour and the number of hours it takes to go from one port to another, how can you find the distance between the two ports? Illustrate. Which number is the multiplicand? What is the denomination of the product?

11. Knowing the distance between two ports and the rate a vessel sails per hour, how can you find the number of hours required to sail from one port to the other? Illustrate.

12. Knowing the number of hours it requires for a vessel to sail from one port to another and the distance between the ports, how can you find the rate the vessel sails per hour? Illustrate.

DIVISION

31. Division is the process of finding how many times one number is contained in another, or the process of separating a number into equal parts.

The **dividend** is the number divided.

The **divisor** is the number by which the dividend is to be divided.

The **quotient** is the result obtained by the process of division.

The **remainder** is the undivided part of the dividend, when the division is not exact.

Divisor 8)26 Dividend
Quotient 3, 2 Remainder

The sign of division is ÷, and is read *divided by*. 18 ÷ 9 is read, 18 divided by 9.

Another way of expressing division is by writing the dividend over the divisor; thus, $\frac{18}{9}$ is read, 18 divided by 9, or eighteen ninths.

1. Multiply 34 by 6.

$$\begin{array}{r} 34 \text{ Multiplicand} \\ 6 \text{ Multiplier} \\ \hline 204 \text{ Product.} \end{array} \Big\} \text{Factors}$$

The inverse process is given here.

Divisor 6)204 Dividend
34 Quotient

Divisor 34)204 Dividend
6 Quotient

The foregoing makes it clear that in division the dividend is a product, of which the divisor and quotient are factors.

The process of division gives the number of times the divisor is contained in the dividend. It also divides the dividend into as many equal parts as there are units in the divisor, and gives the value of one of the parts.

2. $6\overline{)204}$ 34

The division gives 34, the number of times 6 is contained in 204. By the division, 204 is divided into 6 equal parts; the value of each part is shown to be 34.

Short division is the process of division when the work is performed mentally, the quotient only being written; thus,

3. $2\overline{)4268}$ 2134.

Long division is the process of division in which the entire work is set down; thus,

4.
$$\begin{array}{r} 189 \\ 2\overline{)378} \\ \underline{2} \\ 17 \\ \underline{16} \\ 18 \\ \underline{18} \end{array}$$

Each figure in the quotient is called **a partial quotient**; thus, $3\overline{)639}$ 213. 2 is a partial quotient; also 1 and 3.

A **partial dividend** is the portion of the dividend used in obtaining each figure in the quotient; thus, 8 hundreds is the partial dividend from which 1 hundred in the quotient is obtained: 17 tens is the partial dividend from which 8 tens in the quotient is obtained; 18 units is the partial dividend from which 9 units in the quotient is obtained.

Division Tables

32. Review multiplication and division tables, with special emphasis on the latter.

DRILL

33. 1. Give the multiples of 4, to 48.

2. Give the multiple of 4 next below 17. It is 16. $16 \div 4 = ?$

4 is contained in 17 how many times and how many remain?

3. Give the multiple of 4 next below 43. It is 40. $40 \div 4 = ?$

4 is contained in 43 how many times and how many remain?

4. Give the multiple of 4 next below 38. $36 \div 4 = ?$ $38 \div 4 = ?$

5. Give the multiple of 4 next below each of the following numbers, and the number of times 4 is contained in each multiple: 27, 31, 14, 22, 39, 46, 26, 18, 47.

6. Give the quotient and remainder resulting from dividing each of the above numbers by 4.

7. Divide each of the following numbers by 4. Prove results by multiplying the quotient in each case by the divisor, adding the remainder, if any. If the result equals the dividend, the division is correct.

8. 256. 10. 3844. 12. 2796. 14. 4804. 16. 7964.

9. 562. 11. 2794. 13. 1480. 15. 2998. 17. 1987.

18. Give the multiples of 6, to 72.

19. Give the multiples of 6 next below each of the following numbers, and the quotient of each multiple divided by 6: 15, 70, 34, 22, 65, 28, 39, 58, 40, 53.

20. Give the quotient and remainder resulting from dividing each of the above numbers by 6.

Divide each of the following numbers by 6 and prove results:

21. 438. **23.** 4632. **25.** 3984. **27.** 13,242. **29.** 7948.

22. 2772. **24.** 5796. **26.** 17,418. **28.** 2785. **30.** 4725.

To the Teacher. Give further exercises similar to 5 and 6, using other numbers from 5 to 12 inclusive as divisors. Thorough drill here will shorten the subsequent work in division.

SHORT DIVISION — DIVISORS 2 TO 12 INCLUSIVE

Oral Drill

34. Name the quotients and remainders:

1. $9 \div 2$; $11 \div 2$; $23 \div 3$; $5 \div 2$; $7 \div 2$; $7 \div 3$.

2. $30 \div 4$; $48 \div 9$; $23 \div 6$; $16 \div 9$; $12 \div 5$; $37 \div 8$.

3. $\frac{45}{10}$; $\frac{90}{11}$; $\frac{87}{12}$; $\frac{82}{9}$; $\frac{40}{6}$; $\frac{53}{11}$; $\frac{35}{8}$; $\frac{46}{6}$.

4. $\frac{78}{9}$; $\frac{81}{10}$; $\frac{49}{8}$; $\frac{42}{12}$; $\frac{13}{4}$; $\frac{18}{7}$; $\frac{59}{8}$; $\frac{64}{12}$.

5. $\frac{99}{8}$; $\frac{41}{12}$; $\frac{83}{12}$; $\frac{79}{10}$; $\frac{12}{9}$; $\frac{97}{9}$; $\frac{111}{12}$; $\frac{143}{12}$.

6. $92 \div 11$; $80 \div 9$; $27 \div 8$; $35 \div 12$; $98 \div 10$; $19 \div 8$; $24 \div 9$.

7. $103 \div 12$; $47 \div 8$; $39 \div 4$; $79 \div 9$; $83 \div 9$; $48 \div 7$; $31 \div 8$.

8. $29 \div 4$; $80 \div 11$; $27 \div 5$; $86 \div 9$; $45 \div 12$; $62 \div 9$; $73 \div 8$.

9. $12 \div 11$; $105 \div 11$; $29 \div 6$; $58 \div 6$; $70 \div 9$; $84 \div 11$; $63 \div 10$.

10. $140 \div 12$; $28 \div 6$; $77 \div 9$; $89 \div 12$; $124 \div 12$; $38 \div 7$; $56 \div 9$.

11. What number nearest 54 is a multiple of 9? 7? 12? 6? 11?

12. What one nearest 83 is a multiple of 6? 9? 8? 4? 12?

13. What one nearest 36 is a multiple of 3? 4? 8? 5? 7?

14. What one nearest 47 is a multiple of 11? 10? 7? 5? 9?

15. What one nearest 103 is a multiple of 12? 9? 11? 7? 10?

Divide orally:

ILLUSTRATION: 459 ÷ 8. Say: 8 in 45, 5 times; in 59, 7 times with 3 remainder; quotient, 57⅜.

16. 33 ÷ 2; 82 ÷ 3; 46 ÷ 3; 74 ÷ 4; 83 ÷ 5; 66 ÷ 5.

17. 44 ÷ 3; 37 ÷ 2; 92 ÷ 5; 83 ÷ 3; 72 ÷ 5; 81 ÷ 4.

18. 56 ÷ 4; 64 ÷ 6; 89 ÷ 4; 148 ÷ 5; 97 ÷ 6; 147 ÷ 6.

19. 96 ÷ 5; 346 ÷ 4; 144 ÷ 3; 94 ÷ 7; 270 ÷ 3; 249 ÷ 3.

20. 123 ÷ 7; 245 ÷ 6; 839 ÷ 7; 914 ÷ 10; 464 ÷ 5; 836 ÷ 9.

WRITTEN DRILL

35. Solve the following and prove results:

1.	4.	7.	10.
2)476	2)8376	3)40890	3)89067
2.	**5.**	**8.**	**11.**
4)1806	4)71084	5)280076	5)72430
3.	**6.**	**9.**	**12.**
6)34062	7)13064	8)176432	9)834064

13. Divide 3240 by 9.

14. Divide 80,765 by 8.

15. Divide 4,076,436 by 11.

16. Divide 9,700,872 by 12.

LONG DIVISION

36. To divide, when the value of the second figure from the left in the divisor is small.

Study Recitation

When the value of the second figure from the left in the divisor is small, its first figure is considered a trial divisor, and a partial quotient is obtained as in short division. If the resulting product exceeds the partial dividend, the quotient figure is diminished by one.

The first partial dividend has the same number of figures as the divisor, unless the number represented by these figures is less than the divisor; in that case it has one more figure than the divisor.

The first quotient figure is written over the last figure in the partial dividend; the remaining quotient figures are placed in order over succeeding figures in the dividend.

1. Divide 579,063 by 937.

$$\begin{array}{r} 617\frac{334}{937} \\ 937\overline{)579063} \\ 5622 \\ \hline 1686 \\ 937 \\ \hline 7493 \\ 6559 \\ \hline 934 \end{array}$$

579 is less than 937; therefore there are four figures in the first partial dividend.

The second figure from the left in the divisor is 3, a small number; 9 is used as a trial divisor. Divide as in short division. 9 in 57, 6 times with a remainder. Write 6 over 0, the last figure in the partial dividend, and multiply the divisor 937 by 6. Subtract the product, 5622, from the partial dividend, 5790. The remainder is 168. To 168 annex 6, the next figure to the right of those already used in the dividend. 1686 is the second partial dividend. Proceed as before.

2. $568475 \div 615 = ?$

$$
\begin{array}{r}
924\tfrac{215}{615} \\
615\overline{)568475} \\
5535 \\
\hline
1497 \\
1230 \\
\hline
2675 \\
2460 \\
\hline
215
\end{array}
$$

How many figures are there in the first partial dividend? Why? What is the trial divisor? What is the first quotient figure? How is it found? Where is it written? What is the next step? What is the result? What next? What is done to the result? What is 1497?

State each remaining step in order. Notice that in long division, after the first partial dividend and trial divisor are determined, the following steps occur in order:

1. Divide.
2. Multiply.
3. Subtract.
4. Annex next figure of dividend.

The four steps are then repeated as often as necessary.

Examples with 0's in the quotient.

3. Divide 8652 by 42.

$$
\begin{array}{r}
206 \\
42\overline{)8652} \\
84 \\
\hline
252 \\
252 \\
\hline
\end{array}
$$

4. Divide 8197 by 63.

$$
\begin{array}{r}
130\tfrac{7}{63} \\
63\overline{)8197} \\
63 \\
\hline
189 \\
189 \\
\hline
7
\end{array}
$$

Application of Study Recitation

37. Find the first figure of the quotient in the following examples. After finding the first figure in each quotient, solve each example, finding the entire quotient in each case.

1.	**3.**	**5.**	**7.**
$21\overline{)4662}$	$41\overline{)27,798}$	$61\overline{)530,763}$	$203\overline{)19,683}$
2.	**4.**	**6.**	**8.**
$22\overline{)9350}$	$42\overline{)390,876}$	$72\overline{)57,024}$	$318\overline{)119,087}$

9.	11.	13.	15.
$31\overline{)1271}$	$51\overline{)46,076}$	$81\overline{)240,570}$	$426\overline{)196,606}$
10.	**12.**	**14.**	**16.**
$32\overline{)89,856}$	$52\overline{)867,643}$	$91\overline{)380,764}$	$824\overline{)546,312}$

38. To divide, when the value of the second figure from the left in the divisor is large.

Study Recitation

When the value of the second figure from the left in the divisor is 8 or 9, its first figure, increased by one, is to be used as a trial divisor ; if the remainder is equal to or greater than the divisor, the quotient figure is increased by one.

Finding the first quotient figure.

1.
$$289\overline{)198076}\quad 6$$
$$1734$$
$$246$$

8, the second figure from the left in the divisor, is large; divide 19 by 3.
$19 \div 3 = 6$, with a remainder.
$6 \times 289 = 1734$.
$1980 - 1734 = 246$.

2.
$$582\overline{)45863764}\quad 7$$
$$4074$$
$$512$$

$45 \div 6 = 7$, with a remainder.
$7 \times 582 = 4074$.
$4586 - 4074 = 512$.

When the second figure from the left in the divisor is large, the reason for dividing by the first figure increased by 1, is to allow for the number to be carried when the divisor is multiplied by the quotient figure.

3. $482\overline{)27,642}$

The second figure from the left in the divisor is 8. If we divide 27, the first partial dividend, by 4, we have 6 for the first quotient figure; multiplying by 6, we have six times 8 is 48, six times 4 is 24, and with 4 to carry, we have 28, which is larger than the partial dividend 27;

hence the quotient figure is too large. Calling the first figure at the left in the divisor 5, and dividing 27 by it, we have 5 for the quotient figure, which is correct.

In practice, when the trial quotient figure is found, the pupil should multiply the first two figures at the left by it, as above, to see whether it is too large or too small. Usually it is not necessary to multiply more than the two figures.

Application of the Study Recitation

39. Find the first figure in the quotients. After finding the first figure in each quotient, solve each example, finding the entire quotient in each case.

1.	4.	7.	10.
$19\overline{)4674}$	$28\overline{)73,440}$	$29\overline{)86,420}$	$37\overline{)279,744}$
2.	**5.**	**8.**	**11.**
$48\overline{)110,640}$	$55\overline{)176,715}$	$69\overline{)476,376}$	$76\overline{)2,761,384}$
3.	**6.**	**9.**	**12.**
$98\overline{)2,764,286}$	$87\overline{)442,325}$	$189\overline{)146,286}$	$194\overline{)83,420}$

40. Examples in which the division by the trial divisor does not give the correct quotient figure.

Study Recitation

$$\begin{array}{rr} & 81 \\ 1.\ 35\overline{)} & 2835 \\ & \underline{280} \\ & 35 \\ & \underline{35} \end{array}$$

The trial divisor is 3. Dividing 28 by 3, we obtain 9 for a trial quotient figure. Multiply the divisor mentally by 9; 9 times 35 is 315; 315 is larger than the partial dividend 283; therefore the quotient 9 is too large. Try 8.

Multiply the divisor mentally by 8. 8 times 35 is 280, which is less than 283. 8 is the correct quotient figure, and is written above 3. The next figure, 5, in the dividend is annexed to 3. The next quotient figure is 1.

$$\begin{array}{r} 8\tfrac{10}{46} \\ 46\overline{)378} \\ \underline{368} \\ 10 \end{array}$$

2.

The trial divisor is 4. 4 in 37, 9 times with a remainder. When the divisor is multiplied, it is clear that if there is more than 1 to carry to the product of 9×4, the quotient figure is too large. $9 \times 6 = 54$. As there is a 5 to carry to the next product, it is seen that the quotient figure is less than 9. Try 8. Multiply and subtract.

$$\begin{array}{r} 8\tfrac{63}{84} \\ 84\overline{)735} \\ \underline{672} \\ 63 \end{array}$$

3.

8 in 73 how many times and what remainder? What is the largest number which can be carried to 9×8 when the divisor is multiplied by 9? *Ans.* 1.

$9 \times 4 = 36$. As 3 is the number carried to the next product, 9 is too large. Try 8. Multiply and subtract.

$$\begin{array}{r} 7\tfrac{31}{76} \\ 76\overline{)563} \end{array}$$

4.

7 in 56, 8 times, no remainder. Nothing can be carried when 7 is multiplied by 8. 8×6 gives 4 to carry; therefore, 8 is too large. Try 7. Multiply and subtract.

5. $7684\overline{)56{,}379}$

Find the quotient figure the same way as in example 4, giving no attention to any of the figures in the divisor except the two at the left, until the quotient figure is found; then multiply the entire divisor by the quotient figure and subtract.

In a very few cases it is necessary to consider the third figure in the divisor. The next example is one of the few.

6. $459\overline{)3212}$

It will be seen that the third figure, 9, is large. 4 in 32, 8 times, no remainder; but as 8×5 gives 4 to carry to the next product, 8 is too large. Try 7. As 7×5 is 35, giving 3 to carry, it would seem that 7 is the correct quotient figure. On multiplying 459 by 7, however, the product is found to be 3213, or 1 too large. 6 is therefore the correct quotient figure.

TO THE TEACHER. If pupils need further practice in multiplication, require them to prove the correctness of their work in each of the following examples.

Find the quotients:

7.	75)647	13.	469)1785	19.	129)734
8.	43)365	14.	839)4056	20.	215)825
9.	93)462	15.	752)6435	21.	427)2563
10.	756)6478	16.	987)8435	22.	196)408
11.	436)3658	17.	105)639	23.	497)2563
12.	934)4629	18.	308)2712	24.	798)6347

41. Divisor 10 or some power of 10.

Review operation of multiplying by 10 or some power of 10.

1. Multiply 215 by 10; by 100; by 1000.

2. Divide the first quotient by 10; the second quotient by 100; the third by 1000.

What is the shortest process of making the division in each case?

To divide any number by 10 or some power of 10, cut off as many figures from the right of the dividend as there are zeros in the divisor. The remaining figures, with the figures cut off written over the divisor, are the quotient.

ILLUSTRATION: $275 \div 100 = 2\frac{75}{100} = 2.75$

TO THE TEACHER. Have the remainders written as common fractions in the quotient; also as decimals if division of decimals has been mastered.

Divide the following numbers by 10; by 100; by 1000:

3.	278	4.	386	5.	4972	6.	27,856

Divide the following numbers by 100; by 1000:

7.	2785	9.	3946	11.	27,856	13.	4610
8.	2730	10.	4000	12.	50,230	14.	2805

42. Divisor a multiple but not a power of 10.

1. Divide 38,613 by 1500.

$$15\,00\overline{)386\,13}\quad 25\tfrac{11}{1500}$$

30

86

75

11

Cut off the zeros at the right in the divisor, and the same number of figures at the right in the dividend. Divide 386, the number remaining in the dividend, by 15, the number remaining in the divisor; to the remainder, 11, annex 13, the figures cut off, and write this number over the divisor, thus, $\frac{11}{1500}$, as a part of the quotient. State the process when there are three zeros at the right of significant figures in the divisor.

2. 92,678 ÷ 90.

3. 876,783 ÷ 5400.

4. 83,076 ÷ 420.

5. 468,846 ÷ 8300.

43. Division of a number representing dollars and cents by an abstract number.

Place the decimal point in the quotient directly above the decimal point in the dividend. Then proceed as before.

1. \$287.75 ÷ 15.

$$15\overline{)287.75}\quad \$19.18\tfrac{5}{15}$$

15

137

135

27

15

125

120

5

Solve the following:

Where the fraction of 1 cent is $\frac{1}{2}$ or more than $\frac{1}{2}$, increase the last quotient figure by 1 and omit the fraction.

2. \$78.46 ÷ 8.

3. \$4.55 ÷ 5.

4. \$246.08 ÷ 28.

5. \$8240.76 ÷ 88.

6. \$1806.40 ÷ 225.

7. \$676.65 ÷ 9.

PROBLEMS

ORAL

44. 1. Divide $24.64 equally among 7 men. How much does each receive?

Each man receives $\frac{1}{7}$ of $24.64, which = .

2. How many pecks are there in 424 quarts?

There are as many pecks in 424 quarts as 8 quarts are contained times in 424 quarts, or as 8 is contained times in 424.

3. How many feet are there in 1728 inches?

4. How many dozen are there in 1728 units?

5. How many hours are there in 21,600 minutes? How many days?

6. A man bought nine cows for $378 and sold them for $4 more for each cow than he gave. How much did he pay for each cow, and how much did he get for all of them?

7. If 25 pencils cost $.75, what is the cost of 60 pencils?

8. If my horses eat 45 bushels of oats in 9 days, how long will 225 bushels last them?

What is the first question to be answered? State the second question.

9. If I buy milk at 4 cents a quart and sell it at 3 cents a pint, how much do I gain in selling 4 cans of $12\frac{1}{2}$ gallons each?

Work the problem in two ways. Which is the shorter way?

10. A boy had 30¢; he bought as many melons as he could at 8¢ each, and spent the remainder for apples at 2¢ each. How many melons and how many apples did he buy?

At 8¢ each, for 30¢ he could buy 3 melons and have 6¢ left. For 6¢ he could buy as many apples at 2¢ each as 2 is contained times in 6, or ——. He bought —— melons and —— apples.

Make and solve a similar problem.

11. A boy pays 15 cents for 3 oranges. How much should he pay for $3\frac{1}{2}$ dozen oranges at the same rate? Work in two ways.

12. I bought tea at 60 cents a pound and received 25 cents change out of $1. How many pounds did I get?

13. If 36 yards of velvet can be bought for $54, how much will one half of a yard cost? One eighth of a yard? One quarter of a yard?

WRITTEN

14. How many cows at $27 a head can be purchased for $4725?

15. A bushel of corn weighs 56 pounds. How many bushels are there in 244,160 pounds of corn?

16. Charles Williamson earns $71.50 a month. How much does he earn per day, counting 26 working days per month?

17. Rhode Island contains 1085 square miles; Wisconsin 56,040 square miles. How many states the size of Rhode Island could be made from Wisconsin?

18. The dividend is 203,984, the divisor, 836. Find the quotient.

19. A drover with $2144 bought as many horses as possible for $165 each, and spent the remainder for sheep at $4 a head. How many of each did he buy?

20. How many feet are there in a mile? How many inches? A bicycle wheel is 88 inches in circumference. How many revolutions does it make in going 5 miles?

21. Find the wages of a laborer who has worked 568 hours at $1.60 a day of 8 hours each. What is the first question to be answered?

22. 128 cubic feet make a cord. What is the cost of 1920 cubic feet of wood at $4.50 per cord?

What is the first question to be answered? What is the second question?

23. If 9 pears are sold for 27¢, how much are 6 pears worth at the same rate?

24. 1800 bushels of wheat are sold for $1612. How much are 1240 bushels worth at the same rate?

25. A grocer paid $67.20 for 240 pounds of coffee, and sold it at 30 cents a pound. Find his gain. Work the problem in two ways.

26. A man earned in 7 weeks the following amounts: $10.75; $14.25; $9.80; $15.60; $16.05; $7.85; $17.65. What was the average earned per week?

27. The sum of the ages of 15 pupils is 180 years. What is the average age?

28. In a certain school 8 pupils are 14 years of age; 6 are 13; 7 are 12; 2 are 9; 12 are 8; 14 are 7. What is the sum of their ages? What is their average age?

29. Of what use is a thermometer? During a day in June the thermometer at 6 in the morning stood at 69°; at 7 it showed a temperature of 74°; at 8, 76°; at 9, 80°. Find the average temperature for the four hours.

30. Two men started on their wheels from the same place, at the same time, and traveled in opposite directions, one at the rate of 65 miles a day, the other at the rate of 83 miles a day. At their journey's end they were separated by a distance of 3208 miles. How many days did they travel? Represent the conditions mentioned by means of a diagram. What are the two questions that must be answered?

31. A clerk earns $950 a year. His board is $25 per month and his other expenses are $11.50 per month. In how many years can he save enough to pay $1096 for a lot and build on it a $3000 house?

32. The divisor is 4, the quotient is 8, and the remainder 3. What is the dividend?

33. The divisor is 246, the quotient 384, and the remainder 157. What is the dividend?

When the divisor, quotient, and remainder are given, how can you find the dividend?

34. Construct a practical problem, requiring multiplication and division in its solution. Solve it.

REVIEW

45. 1. Define division; dividend; quotient; remainder.

2. What is short division? When is it used?

3. What is long division? When is it used?

4. How can you find the dividend when the divisor and quotient are given? Illustrate.

5. Multiplying or dividing both dividend and divisor by the same number has what effect upon the quotient? Show the truth of your answer by examples.

6. Multiplying the dividend has what effect upon the quotient? Show by examples what effect multiplying or dividing the dividend has upon the quotient.

7. Show by examples what effect multiplying or dividing the divisor has upon the quotient.

COMBINED OPERATIONS

Study Recitation

46. The parenthesis indicates that the numbers inclosed are to be considered as a single number.

ILLUSTRATION: $3 \times (10 + 6)$.
$(10 + 6) = 16$.
$3 \times 16 = 48$. *Ans.*

In examples involving several of the fundamental operations, the operations of division and multiplication must be performed before the operations of addition and subtraction.

ILLUSTRATION: $18 + 6 \div 2 - 3 \times 4 = ?$
$6 \div 2 = 3$; $3 \times 4 = 12$.
$18 + 3 - 12 = 9$. *Ans.*

If a parenthesis is used, the operations called for by signs within the parenthesis must be performed before the operation indicated by the sign that precedes it is performed.

ILLUSTRATION: $9 + 12 \div 4 - (10 - 6) = ?$
$12 \div 4 = 3$; $(10 - 6) = 4$.
$9 + 3 - 4 = 8$. *Ans.*

Find the value of the following expressions:

1. $7 \times 2 - 8 \div 4$.
2. $3 - (8 - 6) + 10 \times 3$.
3. $5 + 6 \times 4 - 12 \div 6$.
4. $(8 + 3 - 7) \div 2 + 16$.
5. $4 + 5 \times 6 + 10$.
6. $(4 + 5) \times 6 + 10$.
7. $4 + 5 \times (6 + 10)$.
8. $(12 + 18 \div 3) \div 9 + 2$.
9. $48 - 3 \times 3 + 5 \times 2 - (30 - 6) \div 6$.
10. $(6 + 10 - 8) \div 4 + (28 - 5 \times 2) \div 9 + 19 - 6$.

REVIEW

Written

47. Add the following. Time yourself. Can you get the correct results of the first five examples in ten minutes?

1.	2.	3.	4.	5.
26	246	365	4641	3041
38	374	274	2738	2758
47	895	381	4254	2943
29	408	623	2134	9137
43	639	472	6403	4251
54	421	857	2790	6327
87	375	321	4023	8124
56	946	978	4256	2756
93	258	462	3007	3423
72	724	357	9842	5245
14	397	825	7345	6327
23	486	634	6213	8142
98	923	275	7856	2739
87	847	149	9427	4231
46	282	206	2341	6145
23	197	372	7205	2007
75	218	425	8006	3009
89	643	986	2037	2748
44	584	724	5002	4623
55	729	375	6343	2721
63	134	428	2784	9871
98	423	653	1462	6295
74	156	527	2275	8179

6. Add ten thousand four hundred fifty-two; twenty thousand eight hundred fifty-one; fifty-six thousand seven

hundred; five hundred sixty-six; ninety-eight thousand seven hundred nine; two hundred six; eighty-five.

7. Add seventy-five thousand six hundred fifty-seven; nineteen thousand four; six thousand fifty-six; seven hundred forty-nine; eighty-four thousand five hundred eight; seven hundred nine; four hundred thirty-six; fifty-seven.

8. Mr. Smith's bank book at the end of March shows the following debits and credits:

1900	*Cr.*			*Dr.*	
March	1, Deposit,	$967.60	March	2, Checks,	$346.78
	5, Deposit,	25.74		4, Checks,	25.96
	9, Note,	875.00		5, Checks,	327.30
	11, Deposit,	146.75		12, Checks,	24.93
	16, Deposit,	85.36		30, Note and Int.,	879.40
	26, Deposit,	128.98		31, Checks,	32.48
	30, Deposit,	475.50			
	31, Deposit,	326.84			

Find the balance. What is meant by balance? State how it is found.

9. March 1, 1900, the Goodyear Shoe Factory of Milwaukee sold to S. M. Jones of Chillicothe, Ohio, 36 pairs of calfskin boots at $4.25; 94 pairs of gaiters at $1.25; 26 pairs of slippers at $.98; 136 pairs of arctics at $1.05; 250 pairs of rubbers at $.62.

Make out a bill in due form and properly receipt it.

10. A bought a block of land for $75,350, paid $25 for having it divided into 325 lots, spent $675 in building sidewalks, and then sold the entire block at a profit of $50 per lot. What was the selling price of a lot?

State the several questions which must be answered to solve the problem.

11. Mr. Bain has 1560 sheep; Mr. Smith has 6 times as many as Mr. Bain less 240; Mr. Mason has one half as many as Mr. Bain. How many sheep have the three?

12. What is the cost of 30 oranges at 20 cents a dozen, 132 eggs at 14 cents a dozen, and 2 pecks of apples at 80 cents a bushel?

13. Find the difference between the sum and product of 756 and 894.

14. A clerk has a salary of $42 a month, and his expenses are $27 a month. How many years will it take him to save $900?

15. A grocer bought 2250 pounds of sugar at $5 a hundredweight; he sold 250 pounds at 6 cents, and the remainder at $5\frac{1}{4}$ cents a pound. How much did he gain?

16. A merchant finds that during the day he has taken in 208 half-dollars, 73 quarters, 110 dimes, 42 nickels, and 13 cents. How much has he received in all?

17. A boy buys from a dealer in canceled stamps 24 German stamps, at 4 for a cent; 39 English stamps, at 3 for a cent; 76 Mexican stamps, at 2 for a cent; 13 French stamps, at 2 cents apiece; 16 Cuban stamps, at $3\frac{1}{2}$ cents each. If he hands the dealer two dollars, what change should the boy receive?

18. How much will my milk bill be for this year if I take a quart in the morning and a pint at night, the price being 6 cents a quart? How much will it be during the first 6 months? During the winter months? During the spring months? During the summer months? During the autumn months?

19. A man has an income of $1825 a year. He spends $\frac{1}{5}$ of it for groceries, $200 for rent, $437 for clothing, and $579 for sundries. How much will he have left at the end of the year? What is his average net income per month? What is the meaning of "net income"? Of "sundries"?

20. If a boy spends 25 minutes each day in reading, how many hours will he read in a year? What more do you need to know to find how many books he could read in a year?

21. Mr. Roberts bought a horse for $80, paid $2.50 a week for keeping it, and received $3.75 a week for its work. He kept it a year (52 weeks) and then sold it for $70. How much did he gain?

22. Find the sum of all the numbers between 50 and 150 that are divisible by 12.

23. Lord and Taylor advertised a sale of 8500 yd. of Scotch flannel reduced from 48¢ a yard to 28¢ a yard. What was their loss on the entire lot?

24. Mrs. Ames bought 14 yd. of the flannel before it was reduced. How many more yards could she have bought for the same price at the sale?

25. Construct five problems each involving the four fundamental processes. Solve them.

FACTORING

48. A **factor** is one of the makers of the product. 3 and 5 are factors of 15. The multiplicand and multiplier are factors of the product.

Factoring is the process of separating a product into its factors. Thus, $15 = 3 \times 5$.

A **prime number** is a number that is not exactly divisible by any whole number except itself and 1. 17 is a prime number, because 17 and 1 are its only integral factors.

A **composite number** is one that has other integral factors besides itself and one. 21 is a composite number because it has the factors 3 and 7.

An **even number** is one that is exactly divisible by two. All even numbers end in 0, 2, 4, 6, or 8.

An **odd number** is one that is not exactly divisible by 2. Odd numbers end in 1, 3, 5, 7, or 9.

Are the following numbers even or odd? Prime or composite? 4, 9, 7, 12, 35, 120, 8, 11, 39, 40, 25, 19, 36?

49. Tests of divisibility.

Any even number is exactly divisible by 2.

A number is exactly divisible by 4 if the number represented by its last two figures is exactly divisible by 4, or if the last two figures are zeros; as, 324, 948, 500.

A number is exactly divisible by 8 if the number represented by its last three figures is exactly divisible by 8, or if the last three figures are zeros; as, 3456, 7000.

A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3; as, 147, 5142.

A number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9; as, 342.

A number is exactly divisible by 5 if its last digit is 0 or 5.

A number is exactly divisible by 25 if the number represented by its last two figures is divisible by 25, or if the last two figures are zeros; as, 575, 550, 6500.

By what numbers are the following exactly divisible? Give test.

1. 24, 72, 45, 30, 125
2. 145, 152, 144, 196, 920
3. 60, 96, 100, 231, 625
4. 320, 1728, 2250, 3600, 8376

50. Resolving a number into its prime factors.

Study Recitation

A factor of a number is an *exact divisor* of that number.

Two or more equal factors may be written as a power of the number.

a. $4 = 2 \times 2 = 2^2$ is read, 2 to the second power, or 2 square.

b. $8 = 2 \times 2 \times 2 = 2^3$ is read, 2 to the third power, or 2 cube.

c. $16 = 2 \times 2 \times 2 \times 2 = 2^4$ is read, 2 to the fourth power.

d. $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$ is read, 3 to the fifth power.

A **power** of a number is the product arising from using the number two or more times as a factor.

The numbers 2, 3, 4, 5, written to the right and a little above the numbers, 2, 3, in *a*, *b*, *c*, and *d*, are **exponents.** An exponent shows how many times the number to which it is

attached is used as a factor. Thus, 3^5 shows that 3 is used 5 times as a factor, in $3 \times 3 \times 3 \times 3 \times 3$; 2^3 shows that 2 is used 3 times as a factor in $2 \times 2 \times 2$.

A **prime factor** is a factor that cannot be separated into other integral factors.

To the Teacher. Have pupils learn thoroughly the prime factors of numbers to 13, and the prime numbers to 20 as given below. In the table of prime factors, the numbers under *A* are the products found in the multiplication table; the numbers under *B* at the right of each number under *A* are the factors of that number; the numbers under *C* at the right of each set of numbers under *B* are the prime factors of those numbers and also of the numbers at the left under *A*.

If the multiplication table has been learned, the factors as given under *B* will be known. If the prime factors of numbers to 13 have been learned, the prime factors under *C* will be known and, therefore, any product found in the multiplication table can be readily separated into its prime factors.

After the composition of the table has been thoroughly studied, place the numbers under *A* on the board and require pupils to give orally, or to write the factors of these numbers as given in the multiplication table (or those under *B*), and then the prime factors as given under *C*.

The teacher may also give the numbers orally and require pupils to give the two sets of factors orally, and finally to think of the first set of factors and give orally the prime factors.

Prime factors of 4, 6, 8, 9, 10, and 12.

	Learn:
$4 = 2^2$	The prime factors of 4 are two 2's.
$6 = 2 \times 3$	The prime factors of 6 are 2 and 3.
$8 = 2^3$	The prime factors of 8 are three 2's.
$9 = 3^2$	The prime factors of 9 are two 3's.
$10 = 2 \times 5$	The prime factors of 10 are 2 and 5.
$12 = 2^2 \times 3$	The prime factors of 12 are two 2's and 3.

The prime numbers from 1 to 20 are: 1, 2, 3, 5, 7, 11, 13, 17, 19.

The remaining prime numbers to 100 are: 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Table of Prime Factors

A	*B*	*C*
4	2×2	2^2
6	2×3	2×3
8	2×4	2^3
9	3×3	3^2
10	2×5	2×5
12	3×4	$2^2 \times 3$
14	2×7	2×7
15	3×5	3×5
16	4×4	2^4
18	3×6	2×3^2
20	4×5	$2^2 \times 5$
21	3×7	3×7
22	2×11	2×11
24	3×8	$2^3 \times 3$
25	5×5	5^2
27	3×9	3^3
28	4×7	$2^2 \times 7$
30	3×10	$2 \times 3 \times 5$
32	4×8	2^5
33	3×11	3×11
35	5×7	5×7
36	4×9	$2^2 \times 3^2$
40	5×8	$2^3 \times 5$
42	6×7	$2 \times 3 \times 7$
44	4×11	$2^2 \times 11$
45	5×9	$3^2 \times 5$
48	6×8	$2^4 \times 3$
49	7×7	7^2
50	5×10	2×5^2
54	6×9	2×3^3
55	5×11	5×11
56	7×8	$2^3 \times 7$
60	5×12	$2^2 \times 3 \times 5$
63	7×9	$3^2 \times 7$
64	8×8	2^6
66	6×11	$2 \times 3 \times 11$
70	7×10	$2 \times 5 \times 7$
72	8×9	$2^3 \times 3^2$
77	7×11	7×11
80	8×10	$2^4 \times 5$
81	9×9	3^4
84	7×12	$2^2 \times 3 \times 7$
88	8×11	$2^3 \times 11$
90	9×10	$2 \times 3^2 \times 5$
96	8×12	$2^5 \times 3$
99	9×11	$3^2 \times 11$
100	10×10	$2^2 \times 5^2$
108	9×12	$2^2 \times 3^3$
110	10×11	$2 \times 5 \times 11$
120	10×12	$2^3 \times 3 \times 5$
121	11×11	11^2
132	11×12	$2^2 \times 3 \times 11$
144	12×12	$2^4 \times 3^2$

Drill in Factoring

51. Separate the following numbers into the two factors most nearly equal, when such factors are readily seen, writing the smaller of the two first; then write the prime factors in the order of their size.

1. Find the prime factors of 72.

To the Teacher. — From his knowledge of the multiplication table the pupil should know that $72 = 8 \times 9$, and from his knowledge of prime factors of numbers to 13 he should know that the prime factors of 8 are three 2's, and of 9 two 3's. He should be able to say from memory $72 = 8 \times 9 = 2^3 \times 3^2$.

The numbers with stars are products not found in the multiplication table. Apply the tests of divisibility in factoring the starred numbers. Give the two factors of the other numbers from memory and their prime factors from memory.

2. Find the prime factors of 94.

94 is not one of the products found in the multiplication table. Apply tests of divisibility.

94 is an even number, therefore divisible by 2.

$2)\overline{94}$ 47 is a prime number. The prime factors of 94 are 2 and 47.
$\quad 47$

Find the prime factors of :

3.	28	54	81	100	11.	44	70	91 *	125 *
4.	30	57 *	82 *	108	12.	45	72	92 *	105 *
5.	33	60	84	121	13.	46 *	74 *	93 *	118 *
6.	35	64	85 *	132	14.	48	75 *	94 *	225 *
7.	36	65 *	86 *	144	15.	49	76 *	95 *	250 *
8.	38 *	66	87 *	128 *	16.	50	77	96	168 *
9.	40	68 *	88	130 *	17.	51 *	78 *	98 *	170 *
10.	42	69 *	90	140 *	18.	52 *	80	99	150 *

ORAL

State the value of:

19. 2^2, 3^2, 6^2

20. 2^3, 3^3, 7^2

21. 2^4, 3^4, 8^2

22. 2^5, 4^3, 9^2

23. 2^6, 5^3, 11^2

24. 4^2, 3^5, 3^6

Express as powers:

Thus, $16 = 2^4$, $25 = 5^2$.

25. 4, 8, 9, 16, 36, 49, 64

26. 32, 27, 81, 100, 125, 144

27. 100, 1000, 10,000, 100,000

28. $10^5 = ?$ $10^2 = ?$ $10^1 = ?$

29. $10^3 = ?$ $10^4 = ?$ $10^6 = ?$

WRITTEN

Which of the following numbers are prime:

30. 16, 317, 319, 389?

31. 11, 121, 131, 287?

The tests of divisibility show that 317 is not a multiple of 2, 3, or 5. We next try 7, 11, 13, 17, and 19 as divisors, and find that 317 is not exactly divisible by any of them. There is no need of trying higher factors since 361, the square of 19, exceeds 317. Hence, 317 is a prime number.

GREATEST COMMON DIVISOR

Study Recitation

52. A **common divisor** or a **common factor** of two or more numbers is a divisor or a factor common to all of them. 6, 2, and 3 are common divisors or common factors of 30, 18, 24.

The **greatest common divisor** or the **greatest common factor** of two or more numbers is the greatest factor common to all of them. Its abbreviation is G. C. D. or G. C. F.

53. To find the G. C. D. of several numbers.

RULE: *Find the prime factors of the given numbers; the product of such of the prime factors as are common to all the numbers is the G. C. D. of the numbers.*

1. Find the G. C. D. of 48, 72, 96.

$48 = 2^4 \cdot 3$
$72 = 2^3 \cdot 3^2$
$96 = 2^5 \cdot 3$
$2^3 \times 3 = 24$, G.C.D.

In 48, 2 is found 4 times as a factor; in 96, 5 times; in 72, it is found but 3 times; hence, 2 must be used as a factor 3 times in the G. C. D.

In the same way, it is found that 3 must be used but once as a factor in the G. C. D.

The factors common to these numbers are 2^3 and 3; and their product, 24, is the G. C. D. of 48, 72, and 96.

Find the G. C. D. of:

2. 12, 18	**7.** 4, 12, 16	**12.** 60, 105, 120
3. 16, 24	**8.** 8, 10, 18	**13.** 42, 72, 180
4. 27, 36	**9.** 25, 75, 100	**14.** 448, 875, 567
5. 22, 33	**10.** 60, 45, 120	**15.** 144, 1296, 1728
6. 40, 48	**11.** 9, 27, 30, 93	**16.** 108, 162, 188, 204

LEAST COMMON MULTIPLE

Study Recitation

54. A **multiple** of a number is a number exactly divisible by the number.

A **common multiple** of two or more numbers is a multiple of each of them. 36 is a common multiple of 2, 3, 4, and 6, because each of the numbers is an exact divisor of 36.

24 is a common multiple of 2, 3, 4, 6, for the same reason.

12 is a common multiple of 2, 3, 4, 6, for the same reason.

The **least common multiple** of two or more numbers is the least number that is a multiple of each of them. 36, 24, and 12 are common multiples of 2, 3, 4, and 6. 12 is the smallest of these multiples, and as it is the smallest number exactly divisible by 2, 3, 4, and 6, it is the least common multiple.

The abbreviation is L. C. M.

55. To find the L. C. M. of two or more numbers.

RULE: *Find the prime factors of the numbers. Find the product of all the different prime factors of the given numbers, using each as a factor the greatest number of times it is found in any of the numbers.*

1. Find the L. C. M. of 16, 24, 30, and 36.

$16 = 2^4$
$24 = 2^3 \times 3$
$30 = 2 \times 3 \times 5$
$36 = 2^2 \times 3^2$
$2^4 \times 3^2 \times 5 = 720$, L. C. M.

The different prime factors are 2, 3, and 5. In 16, 2 is used 4 times as a factor, the greatest number of times it is found in any of the numbers. In 36, 3 is used 2 times as a factor, the greatest number of times it is found in any of the numbers. 5 is found but once as a factor in any of the numbers. On bringing these factors together, the L. C. M. is found to be $2^4 \times 3^2 \times 5 = 720$.

2. Find the L. C. M. of 12, 25, 48, and 50.

$48 = 2^4 \times 3$
$50 = 2 \times 5^2$
$2^4 \times 3 \times 5^2 = 1200$, L. C. M.

Since 48 is a multiple of 12, *any* multiple of 48 is also a multiple of 12; hence 12 may be rejected from the work. Likewise, since 50 is a multiple of 25, and *any* multiple of 50 is also a multiple of 25, 25 may be rejected. The L. C. M. of 12, 25, 48, and 50 is therefore the same as the L. C. M. of 48 and 50, which is found as in example 1.

3. Find the L. C. M. of 3, 5, and 7.

These numbers are prime to each other, hence their product, 105, is their L. C. M.

Oral

Find by inspection the L. C. M. of each set of numbers:

4. 2, 3, 5	**12.** 15, 10	**20.** 6, 9
5. 3, 7, 2	**13.** 2, 3, 4	**21.** 4, 10
6. 5, 7, 3	**14.** 3, 4, 5	**22.** 5, 9
7. 16, 24, 2	**15.** 7, 9, 3	**23.** 9, 12
8. 16, 8	**16.** 11, 12, 6	**24.** 10, 24
9. 24, 16	**17.** 21, 14, 7	**25.** 12, 20
10. 7, 21	**18.** 12, 36	**26.** 16, 48
11. 18, 27	**19.** 9, 15	**27.** 4, 18

The chief use of the L. C. M. is in finding the least common denominator of several fractions. Thorough drill here will save time in addition and subtraction of fractions.

Written

Find the L. C. M. of:

28. 9, 12, 21	**36.** 11, 33, 44, 55
29. 15, 24, 30	**37.** 15, 25, 45, 50
30. 20, 24, 72	**38.** 2, 3, 4, 5, 6, 7, 8
31. 75, 90, 150	**39.** 7, 8, 16, 42, 48
32. 25, 60, 300	**40.** 5, 8, 12, 24, 120
33. 75, 63	**41.** 20, 30, 40, 50, 60
34. 240, 360	**42.** 2, 5, 10, 100, 4, 8, 1000
35. 144, 1728	**43.** 1, 2, 3, 4, 5, 6, 7

44. A tank is filled with the least number of gallons of water that can be exactly measured both by a vessel holding 12 gallons and by another holding 14 gallons. How many gallons does the tank contain? How many times could the smaller vessel be filled from the tank? The larger vessel?

REVIEW

56. 1. Define and give an example of: factor; prime factor; prime number; composite number; divisor; common divisor; greatest common divisor; multiple; common multiple; least common multiple; power; exponent.

2. Give the rule for finding the G. C. D. of several numbers.

3. Give the rule for finding the L. C. M. of several numbers.

4. How can you determine whether a number is even or odd?

5. A divisor of any number is a divisor of any number of times that number. Illustrate by an example.

6. A divisor of any two numbers is a divisor of their sum.

4)12(3	4)200(50	8)32000(4000
4)16(4	4) 28(7	8) 928(116
4)28(7	4)228(57	8)32928(4116

7. Any number of more than two figures is the sum of the number of units represented by the first two figures + the number of hundreds represented by the remaining figures; as, $2756 = 2700 + 56$. 4 is an exact divisor of one hundred, and therefore of any number of hundreds. Show that if 4 is an exact divisor of the number represented by the first two figures of a number, it is an exact divisor of the entire number.

8. Any number of more than three figures is the sum of the number of units represented by the first three figures + the number of thousands represented by the remaining figures; as, $36{,}872 = 36{,}000 + 872$. 8 is an exact divisor of

one thousand and therefore of any number of thousands. Show that if 8 is an exact divisor of the number represented by the first three figures of a number, it is an exact divisor of the entire number.

9. Any number whose first figure is 0 is made up entirely of tens. Any number whose first figure is 5 is the sum of 5 and the number of tens represented by the remaining figures of the number; as, $275 = 270 + 5$. Show that a number whose first figure is 0 or 5 is divisible by 5. Show that a number whose first figure is 0 is divisible by 10.

10. 1 ten, or 10, = 1 nine + 1; 2 tens, or 20, = 2 9's + 2; 5 tens, or 50, = 5 9's + 5; 45 tens = 45 9's + 45. Any number of 10's equals the same number of 9's + the same number of units; thus, 243 10's, or 2430, = 243 9's + 243 units. 243 units equals 24 10's + 3 units; 24 10's equals 24 9's + 24 units. 24 units + 3 units = 27 units. 9 is an exact divisor of 27. Any number may be regarded as made up of 9's + the number of units represented by the sum of its digits. If 9 is an exact divisor of the sum of the digits of any number, it is an exact divisor of the number, because it is an exact divisor of the two parts of the number. Show why 576 is exactly divisible by 9.

FRACTIONS

57. An **integer** is a whole number; as, 2, 5, 12.

An **integral unit** is a single thing considered as an undivided whole. 1, the least integer, is the unit in arithmetic.

A **fractional unit** is one of the equal parts into which a unit has been separated; as, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{16}$.

A **fraction** consists of one or more fractional units; as, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{7}$, $\frac{8}{8}$, $\frac{9}{4}$. Such fractions are called simple fractions.

A **fraction** may also be defined as a quotient expressed by writing the dividend above a horizontal line and the divisor below; thus, $2 \div 7 = \frac{2}{7}$; $4 \div 9 = \frac{4}{9}$; $8 \div 2 = \frac{8}{2}$.

The **denominator**, written below the line, shows into how many equal parts the whole has been divided.

The **numerator**, written above the line, shows how many of the equal parts are taken.

The numerator and denominator are the **terms of the fraction.**

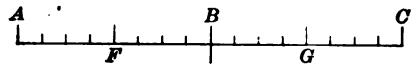
A **proper fraction** is one whose numerator is less than its denominator; as, $\frac{2}{5}$, $\frac{5}{6}$, $\frac{4}{8}$.

An **improper fraction** is one whose numerator equals or exceeds its denominator; as, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{12}{7}$.

A **mixed number** is an integer and a fraction united; as, $2\frac{1}{2}$, $12\frac{3}{4}$, $16\frac{3}{8}$.

58. Principles governing operations in fractions.

Study Recitation



The line AC is divided into 16 equal parts; by the lines at F, B, and G it is divided into 4 equal parts; by the line at B it is divided into 2 equal parts.

1. What part of the line AC is the line AB?

2. What part of the line AC is the line AF?

3. What part of the line AB is the line AF?

4. What is the fractional unit in each of the fractions:

$$\frac{3}{4},\ \frac{4}{8},\ \frac{2}{3},\ \frac{4}{5},\ \frac{5}{6}?$$

5. What is the number of fractional units in each of the above fractions?

6. What figure in the fraction $\frac{3}{4}$ shows the number of parts into which a whole has been divided? What shows the number of the parts?

7. Draw a line representing the line AC and under it draw lines to represent $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ of the line AC.

A	B
$\frac{1}{2} \times 2$	$= \frac{1}{4}$
$\frac{1}{4} \times 2$	$= \frac{1}{8}$
$\frac{1}{8} \times 2$	$= \frac{1}{16}$

8. If the denominator of each fraction under A is multiplied by 2, are the resulting fractions larger or smaller than the fractions whose denominators have been multiplied?

9. Show by means of the line AC above, the value of each fraction under B as compared with the fraction at its left under A.

10. Multiplying the denominator of a fraction produces what effect upon the fraction? Why?

11. If the denominator of each fraction under B is divided by 2, what are the resulting fractions?

12. Dividing the denominator of a fraction has what effect upon the fraction? Why?

13. What does the numerator of a fraction show?

14. If the numerator of a fraction is multiplied by any number, what is the effect upon the fraction? Why? Illustrate.

15. If the numerator of a fraction is divided by any number, what is the effect upon the fraction? Illustrate.

16. How can the fraction $\frac{8}{16}$ be obtained from the fraction $\frac{4}{8}$? What is its value as compared with $\frac{4}{8}$? Illustrate by use of line AC. What is the effect of multiplying both terms of a fraction by the same number?

17. How can $\frac{4}{8}$ be obtained from $\frac{8}{16}$? What is the effect of dividing both terms of a fraction by the same number? Illustrate by use of line AC.

Complete the following statements and then learn them. Give an example to illustrate each.

PRINCIPLE 1. Multiplying the numerator of any fraction by any number . . . . . the fraction by . . . . . .

PRINCIPLE 2. Dividing the numerator of a fraction by any number . . . . . the fraction by . . . . . .

PRINCIPLE 3. Multiplying the denominator of a fraction by any number . . . . . the fraction by . . . . . .

PRINCIPLE 4. Dividing the denominator of a fraction by any number . . . . . the fraction by . . . . . .

PRINCIPLE 5. Multiplying and dividing both terms of a fraction by the same number . . . . . the value of the fraction.

18. What is $\frac{3}{4}$ of 24?

With 2 as multiplier and divisor perform upon the fraction $\frac{3}{4}$ the operations indicated in Principles 1, 2, 3, 4, and 5, and find the parts of 24 indicated by the resulting fractions. Notice in each case the effect upon the result as compared with the result in taking $\frac{3}{4}$ of 24.

19. ILLUSTRATION applying Principle 1.

$\frac{3}{4}$ of 24 is 12.

Multiplying the numerator of $\frac{3}{4}$ by 2 gives $\frac{1}{4}$.

$\frac{1}{4}$ of 24 is 24, which is 2 times 12, or 2 times $\frac{3}{4}$ of 24.

20. Apply Principle 2; Principle 3; Principle 4; Principle 5.

REDUCTION OF FRACTIONS

59. Reducing fractions to lowest terms.

A fraction is in its lowest terms when the numerator and denominator have no common factor.

1. Reduce $\frac{280}{840}$ to lowest terms or denominations.

$$\frac{280}{840} = \frac{28}{84} = \frac{4}{12} = \frac{1}{3}$$

Inspection shows that 10 is a common factor of the terms. Dividing them by 10 gives $\frac{28}{84}$. Inspection shows also the common factor 7. Dividing both terms of $\frac{28}{84}$ by 7 gives $\frac{4}{12}$. Inspection shows that 4 is a common factor of both terms of $\frac{4}{12}$. Dividing both terms by 4, gives $\frac{1}{3}$, a fraction in its lowest terms.

Why has the value of the fraction $\frac{280}{840}$ not been changed?

To reduce a fraction to its lowest terms.

RULE. *Divide both terms of the fraction by any common divisor; divide the resulting terms by any common divisor, and so continue until the terms have no common divisor. Or,*

Divide both terms of the fraction by their G. C. D.

ORAL AND WRITTEN

Reduce to lowest terms:

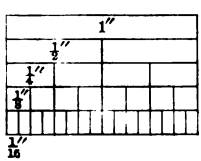
2. $\frac{2}{4}, \frac{3}{6}$
3. $\frac{4}{6}, \frac{6}{8}$
4. $\frac{5}{10}, \frac{10}{12}$
5. $\frac{3}{12}, \frac{3}{9}$
6. $\frac{2}{14}, \frac{10}{15}$
7. $\frac{6}{18}, \frac{4}{20}$
8. $\frac{12}{18}, \frac{16}{20}$
9. $\frac{6}{21}, \frac{9}{24}$
10. $\frac{18}{24}, \frac{21}{27}$
11. $\frac{20}{25}, \frac{25}{30}$
12. $\frac{14}{16}, \frac{36}{42}$
13. $\frac{15}{45}, \frac{21}{28}$
14. $\frac{5}{100}, \frac{10}{100}$
15. $\frac{20}{100}, \frac{15}{100}$
16. $\frac{25}{100}, \frac{40}{100}$
17. $\frac{60}{100}, \frac{80}{100}$
18. $\frac{90}{100}, \frac{50}{100}$
19. $\frac{35}{100}, \frac{55}{100}$
20. $\frac{75}{100}, \frac{95}{100}$
21. $\frac{45}{100}, \frac{85}{100}$
22. $\frac{105}{100}, \frac{15}{87}$
23. $\frac{17}{34}, \frac{17}{51}$
24. $\frac{26}{39}, \frac{22}{66}$
25. $\frac{49}{70}, \frac{63}{81}$
26. $\frac{12}{84}, \frac{45}{60}$
27. $\frac{19}{38}, \frac{57}{60}$
28. $\frac{24}{96}, \frac{25}{125}$
29. $\frac{125}{600}, \frac{250}{1200}$
30. $\frac{170}{100}, \frac{875}{1000}$
31. $\frac{25}{100}, \frac{60}{100}$
32. $\frac{375}{1000}, \frac{500}{1000}$
33. $\frac{625}{1000}, \frac{1125}{1000}$
34. $\frac{750}{1000}, \frac{900}{1000}$
35. $\frac{250}{400}, \frac{1500}{2000}$
36. $\frac{210}{231}, \frac{288}{864}$
37. $\frac{770}{990}, \frac{187}{328}$

60. Reduction of fractions to fractions of higher denominations.

ORAL

1. Reduce $\frac{1}{2}$ to 4ths.

SOLUTION: $1 = \frac{4}{4}$. $\frac{1}{2}$ of $1 = \frac{1}{2}$ of $\frac{4}{4} = \frac{2}{4}$.



Prove your answer by reference to the illustration.

Thus, $\frac{1}{2}$ inch equals $\frac{2}{4}$ of an inch.

Reduce and analyze. Prove your answers by reference to the illustration :

2. $\frac{1}{2}$ to 8ths.
3. $\frac{1}{2}$ to 16ths.
4. $\frac{1}{4}$ to 8ths.
5. $\frac{1}{4}$ to 16ths.
6. $\frac{3}{4}$ to 8ths.
7. $\frac{3}{8}$ to 16ths.
8. $\frac{3}{4}$ to 16ths.
9. $\frac{5}{8}$ to 16ths.

SOLUTION: $1 = \frac{8}{8}$. $\frac{1}{4}$ of $1 = \frac{2}{8}$. $\frac{2}{4}$ of $1 = 2$ times $\frac{2}{8}$, or $\frac{4}{8}$.

What principle is applied in reducing a fraction to another fraction of a higher denomination?

10. $\frac{3}{4}$ and $\frac{3}{5}$ to 20ths.
11. $\frac{5}{6}$ and $\frac{2}{3}$ to 12ths.
12. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ to 16ths.
13. $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$ to 24ths.
14. $\frac{3}{7}$, $\frac{1}{5}$, and $\frac{6}{7}$ to 35ths.
15. $\frac{1}{2}$, $\frac{1}{7}$, and $\frac{5}{6}$ to 42ds.
16. $\frac{3}{8}$, $\frac{4}{7}$, and $\frac{1}{2}$ to 56ths.
17. $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{1}{2}$ to 100ths.

When the examples have all been analyzed, learn the rule and then solve each of the examples in the above list as directed by the rule.

To reduce a fraction to an equivalent fraction having a given larger denominator.

RULE: *Divide the required denominator by the denominator of the fraction, and multiply the quotient by the numerator. Write the product over the given denominator.*

Notice that the operations performed in analyzing the example are the same as required by the rule.

61. Common and least common denominator.

Study Recitation

Fractions have a **common denominator** when they have the same denominator. The fractions $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$ have a common denominator, 8.

The fractions $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$ have not a common denominator, but may be reduced to other fractions having a common denominator. The *smallest* denominator common to these three fractions is 16. Thus, $\frac{3}{4} = \frac{12}{16}$; $\frac{5}{8} = \frac{10}{16}$; $\frac{7}{16} = \frac{7}{16}$.

The *smallest* denominator common to several fractions is called their **least common denominator.**

The least common denominator of several fractions is the L. C. M. of their denominators.

ORAL

1. Reduce $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{3}{16}$ to fractions having the least common denominator.

SOLUTION: The L. C. M. of 8, 6, and 16 is 48; hence, the fractions must be reduced to 48ths.

$$\frac{3}{8} = \frac{18}{48} \quad \frac{5}{6} = \frac{40}{48} \quad \frac{3}{16} = \frac{9}{48}.$$

To reduce fractions to equivalent fractions having the least common denominator.

RULE: *Find the L. C. M. of the denominators. Reduce each fraction to a fraction having this L. C. M. as a denominator.*

NOTE. Skill in addition and subtraction of fractions depends upon the thorough mastery of this work. When the fractions have been reduced to a common denominator, their addition or subtraction by simple addition or subtraction of the numerators involves no new difficulty.

Reduce to least common denominator, and add:

2. $\frac{1}{2}$ and $\frac{1}{3} = ?$
3. $\frac{1}{2}$ and $\frac{2}{3} = ?$
4. $\frac{3}{4}$ and $\frac{3}{8} = ?$
5. $\frac{3}{5}$ and $\frac{2}{3} = ?$
6. $\frac{4}{5}$ and $\frac{3}{4} = ?$
7. $\frac{1}{3}$ and $\frac{1}{6} = ?$
8. $\frac{5}{6}$ and $\frac{3}{5} = ?$
9. $\frac{7}{9}$ and $\frac{4}{7} = ?$
10. $\frac{5}{11}$ and $\frac{3}{8} = ?$
11. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4} = ?$
12. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6} = ?$
13. $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{4}{10} = ?$

14. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{6}$ = ?

15. $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$ = ?

16. $\frac{3}{4}$, $\frac{1}{8}$, and $\frac{7}{16}$ = ?

17. $\frac{7}{8}$, $\frac{3}{12}$, and $\frac{5}{6}$ = ?

18. $\frac{3}{5}$, $\frac{5}{7}$, and $\frac{1}{2}$ = ?

19. $\frac{5}{7}$, $\frac{9}{14}$, and $\frac{1}{2}$ = ?

WRITTEN

20. $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{5}{7}$ = ?

21. $\frac{4}{5}$, $\frac{7}{9}$, and $\frac{5}{12}$ = ?

22. $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{5}{8}$ = ?

23. $\frac{7}{12}$, $\frac{9}{16}$, and $\frac{11}{20}$ = ?

24. $\frac{3}{4}$, $\frac{5}{9}$, and $\frac{13}{14}$ = ?

25. $\frac{11}{12}$, $\frac{7}{20}$, and $\frac{19}{30}$ = ?

26. $\frac{7}{10}$, $\frac{13}{18}$, and $\frac{41}{90}$ = ?

27. $\frac{5}{12}$, $\frac{7}{15}$, and $\frac{9}{50}$ = ?

28. $\frac{3}{5}$, $\frac{7}{9}$, and $\frac{29}{30}$ = ?

29. $\frac{7}{9}$, $\frac{5}{16}$, and $\frac{7}{24}$ = ?

30. $\frac{7}{8}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{30}$ = ?

31. $\frac{3}{10}$, $\frac{4}{15}$, $\frac{13}{20}$, and $\frac{9}{50}$ = ?

32. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$, and $\frac{7}{60}$ = ?

33. $\frac{3}{8}$, $\frac{7}{9}$, $\frac{3}{5}$, and $\frac{23}{24}$ = ?

34. $\frac{7}{10}$, $\frac{4}{25}$, $\frac{9}{50}$, and $\frac{83}{200}$ = ?

35. $\frac{9}{20}$, $\frac{5}{50}$, $\frac{6}{100}$, and $\frac{9}{40}$ = ?

36. Reduce the following fractions to hundredths:

TO THE TEACHER. The following exercise should be thoroughly mastered because it prepares for work in percentage.

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{50}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{3}{20}$, $\frac{7}{20}$, $\frac{9}{20}$, $\frac{13}{20}$, $\frac{17}{20}$, $\frac{19}{20}$, $\frac{3}{50}$

62. Reduction of improper fractions to whole or to mixed numbers.

ORAL

1. Reduce $\frac{39}{12}$ to a mixed number.

$1 = \frac{12}{12}$. $\frac{39}{12} = 39 \div 12 = 3\frac{3}{12}$, or $3\frac{1}{4}$.

$\frac{39}{12}$ is an indicated division. Performing the indicated division gives $3\frac{3}{12}$: reducing the fractional part, $\frac{3}{12}$, to lowest terms gives the result $3\frac{1}{4}$.

To reduce an improper fraction to a whole or a mixed number.

RULE: *Divide the numerator by the denominator.*

Reduce to whole or mixed numbers:

2.	$\frac{72}{5}$	**4.**	$\frac{96}{6}$	**6.**	$\frac{87}{11}$	**8.**	$\frac{45}{16}$	**10.**	$\frac{79}{8}$	**12.**	$\frac{68}{10}$
3.	$\frac{98}{8}$	**5.**	$\frac{126}{9}$	**7.**	$\frac{376}{12}$	**9.**	$\frac{560}{9}$	**11.**	$\frac{185}{7}$	**13.**	$\frac{281}{8}$

WRITTEN

14.	$\frac{285}{27}$	**15.**	$\frac{786}{95}$	**16.**	$\frac{278}{19}$	**17.**	$\frac{567}{84}$	**18.**	$\frac{575}{25}$	**19.**	$\frac{584}{78}$

63. Reduction of mixed numbers to improper fractions.

1. Reduce $7\frac{2}{3}$ to an improper fraction.

$7\frac{2}{3}$

21 Multiplying 3 by 7 gives 21. Adding the numerator 2 to 21,

2 gives the sum 23. Writing 23 over 3 gives $\frac{23}{3}$.

23 $1 = \frac{3}{3}$. $7 =$ 7 times $\frac{3}{3}$ or $\frac{21}{3}$. $\frac{21}{3} + \frac{2}{3} = \frac{23}{3}$.

3

To reduce a mixed number to an improper fraction.

RULE: *Multiply the denominator of the fractional part by the whole number; add the numerator to the product, and write the sum over the denominator.*

ORAL

Reduce to improper fractions:

2.	$2\frac{1}{3}$	**4.**	$4\frac{1}{2}$	**6.**	$3\frac{3}{4}$	**8.**	$8\frac{9}{10}$	**10.**	$6\frac{2}{3}$
3.	$3\frac{1}{3}$	**5.**	$5\frac{5}{6}$	**7.**	$4\frac{2}{5}$	**9.**	$\frac{5}{12}$	**11.**	$\frac{8}{9}$

WRITTEN

12.	$3\frac{1}{13}$	**14.**	$8\frac{5}{13}$	**16.**	$275\frac{3}{8}$	**18.**	$256\frac{1}{2}$	**20.**	$75\frac{5}{8}$
13.	$8\frac{5}{11}$	**15.**	$12\frac{9}{15}$	**17.**	$125\frac{2}{3}$	**19.**	$127\frac{5}{9}$	**21.**	$80\frac{4}{5}$

Oral Drill

64. Reduce the mixed numbers under *A* to improper fractions.

Reduce the fractions under *B* to lowest terms:

A.		*B.*			
$3\frac{1}{4}$	$7\frac{4}{5}$	$\frac{58}{60} =$	$\frac{6}{15} =$	$\frac{25}{100} =$	$\frac{8\frac{1}{3}}{100} =$
$5\frac{1}{6}$	$6\frac{3}{8}$	$\frac{15}{45} =$	$\frac{10}{15} =$	$\frac{75}{100} =$	$\frac{12\frac{1}{2}}{100} =$
$2\frac{2}{3}$	$8\frac{3}{4}$	$\frac{15}{25} =$	$\frac{21}{24} =$	$\frac{40}{100} =$	$\frac{33\frac{1}{3}}{100} =$
$3\frac{5}{8}$	$10\frac{1}{2}$	$\frac{8}{12} =$	$\frac{21}{28} =$	$\frac{60}{100} =$	
$5\frac{3}{8}$	$9\frac{1}{2}$	$\frac{9}{12} =$	$\frac{28}{32} =$	$\frac{70}{100} =$	$\frac{66\frac{2}{3}}{100} =$

ADDITION AND SUBTRACTION OF FRACTIONS

65. In fractions, as in whole numbers, only units of the same denomination can be added or subtracted; hence, the fractions to be added or subtracted must be reduced to fractions having a common denominator. The indicated operation may then be performed and the result reduced to lowest terms, or to a whole number or a mixed number.

When mixed numbers are to be added, the fractions and integers may be added separately. In subtraction, they may be subtracted separately.

1. $\frac{2}{3} + \frac{7}{8} + \frac{5}{6} = ?$

$\frac{2}{3} + \frac{7}{8} + \frac{5}{6} =$

$\frac{16}{24} + \frac{21}{24} + \frac{20}{24} = \frac{57}{24} = 2\frac{9}{24}$

The L. C. D. of the fractions to be added is the L. C. M. of 3, 8, and 6, or 24. On reducing the given fractions to 24ths and adding, the result is $\frac{57}{24}$, or $2\frac{9}{24}$.

2. $\frac{7}{12} + \frac{9}{16} + \frac{5}{8} = ?$

$$\frac{28 + 27 + 30}{48} = \frac{85}{48} = 1\tfrac{37}{48}$$

$12 = 2^2 \times 3$; $16 = 2^4$; $8 = 2^3$

L. C. D. $= 2^4 \times 3 = 48$

In reducing the fractions to other fractions having a common denominator, it is convenient to write the resulting fraction in each case directly under the fraction from which it is obtained, as in example 2.

Reduce each fraction to lowest terms before finding the L. C. D.:

3. $\frac{7}{8} + \frac{9}{10} + \frac{4}{5} = ?$

4. $\frac{9}{14} + \frac{5}{8} + \frac{4}{7} = ?$

5. $\frac{10}{12} + \frac{8}{18} + \frac{6}{20} = ?$

6. $\frac{12}{18} + \frac{3}{8} + \frac{1}{4} = ?$

7. $\frac{5}{16} + \frac{7}{9} + \frac{7}{8} = ?$

8. $\frac{4}{9} + \frac{5}{12} + \frac{7}{18} = ?$

9. $\frac{15}{22} - \frac{7}{12} + \frac{3}{8} = ?$

10. $\frac{12}{18} - \frac{5}{12} = ?$

11. $\frac{13}{15} - \frac{7}{20} = ?$

12. $\frac{15}{16} - \frac{9}{24} = ?$

13. $\frac{9}{20} - \frac{3}{8} = ?$

14. Add $2\frac{3}{4}$, $6\frac{3}{5}$, and $12\frac{1}{3}$.

Whole Numbers	Fractions
2	$\frac{3}{4} =$ 45 60ths
6	$\frac{2}{5} =$ 36 60ths
12	$\frac{1}{3} =$ 20 60ths
20	101 60ths $= 1\frac{41}{60}$
$1\frac{41}{60}$	
$21\frac{41}{60}$	

The sum of the whole numbers 2, 6, and 12 is 20. The L. C. D. of the fractions $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{1}{3}$ is 60.

Reducing each fraction to 60ths and adding, gives the sum $1\frac{41}{60}$, which added to 20 $= 21\frac{41}{60}$.

15. $15\frac{5}{8} - 11\frac{2}{5}$

$15\frac{5}{8} = 15$ and 25 40ths

$11\frac{2}{5} = 11$ and 16 40ths

$\overline{4}$ and $\overline{9}$ 40ths

$\frac{2}{5}$ is less than $\frac{5}{8}$. Reduce both fractions to 40ths.

Subtract $\frac{16}{40}$ from $\frac{25}{40}$ and 11 from 15. The result is $4\frac{9}{40}$.

16. $26\frac{2}{5} - 12\frac{5}{12}$

$26\frac{2}{5} = 25 + 1 + \frac{2}{5}$ Minuend

$12\frac{5}{12} = 12 + \frac{5}{12}$ Subtrahend

$13 + \frac{7}{12} + \frac{2}{5}$ Remainder

$13 + \frac{35 + 24}{60} = 13\frac{59}{60}$

$\frac{5}{12}$ cannot be taken from $\frac{2}{5}$, but can be taken from 1. Take 1 from 26, leaving 25. From the 1 take $\frac{5}{12}$, leaving $\frac{7}{12}$. Take 12 from 25, leaving 13. The entire remainder is $13 + \frac{7}{12} + \frac{2}{5}$, which equals $13\frac{59}{60}$.

17. $18\frac{3}{8} - 12\frac{4}{7}$

18. $156\frac{5}{12} - 85\frac{1}{3}$

19. $275\frac{5}{16} - 183\frac{5}{12}$

20. $286\frac{3}{10} - 156\frac{5}{7}$

21. $37\frac{7}{8} + 9\frac{4}{5} + 12\frac{3}{4}$

22. $68\frac{1}{2} + 27\frac{5}{16}$

23. $56\frac{3}{8} + 21\frac{4}{9} - 10\frac{1}{3}$

24. $56\frac{3}{4} + 27\frac{5}{11}$

Drill Exercise

66. To the Teacher. Read the following examples, and require pupils to write results. After each example is read, the teacher should pause a moment and then give the order "Write," and at that instant each pupil should write his result. If he has not secured the result, he should make a dash in place of it. After several of the examples have been read, one pupil should be called upon to read his results in order, and errors should be corrected. The pupils who are slowest in the work should have the drill more frequently than the others.

1. $2\frac{1}{2} + 3\frac{1}{4}$	**6.** $\frac{5}{6} + \frac{2}{3}$	**11.** $\frac{7}{8} - \frac{2}{3}$	**16.** $3\frac{2}{3} - 2\frac{1}{2}$
2. $3\frac{1}{3} + 5\frac{1}{6}$	**7.** $\frac{3}{8} + \frac{3}{4}$	**12.** $\frac{5}{6} - \frac{2}{3}$	**17.** $12\frac{7}{8} - 5\frac{3}{4}$
3. $4\frac{1}{2} + 3\frac{5}{8}$	**8.** $\frac{4}{5} + \frac{2}{3}$	**13.** $\frac{3}{4} - \frac{1}{3}$	**18.** $9\frac{11}{16} - 4\frac{3}{8}$
4. $6\frac{1}{4} + 7\frac{2}{3}$	**9.** $\frac{5}{6} + \frac{1}{2}$	**14.** $\frac{4}{7} - \frac{1}{2}$	**19.** $7\frac{5}{9} + 2\frac{2}{3}$
5. $8\frac{1}{5} + 3\frac{1}{3}$	**10.** $\frac{2}{3} + \frac{5}{9}$	**15.** $\frac{7}{10} - \frac{3}{5}$	**20.** $6\frac{3}{4} - 4\frac{1}{6}$

PROBLEMS

Oral

67. Indicate operations :

1. A farmer sold $\frac{1}{3}$ of his land at one time, and $\frac{1}{4}$ of it at another time. What part of his land did he sell? What part remained?

2. Mr. Smith sold some wheat and spent $\frac{1}{2}$ of the money received for clothing, and $\frac{1}{5}$ of it for groceries. What part of the money did he spend? What part remained?

3. Anna walked $\frac{3}{4}$ of a mile in the forenoon, $\frac{3}{8}$ of a mile in the afternoon, and $\frac{1}{2}$ of a mile in the evening. How far did she walk that day?

4. A merchant sells $\frac{1}{2}$ dozen eggs to one man, $\frac{3}{4}$ dozen to another, and enough to a third to make $1\frac{2}{3}$ dozen in all. How many does the third man buy?

5. From a $2\frac{3}{4}$-acre lot were sold $\frac{1}{3}$ acre and $\frac{1}{4}$ acre. How much remained?

6. The sum of two numbers is $\frac{7}{8}$; one of the numbers is $\frac{2}{3}$; find the other.

REVIEW

WRITTEN

68.

1. Find the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{1}{6}$.
2. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$.
3. Find the sum of $\frac{3}{8}$, $\frac{7}{9}$, $\frac{9}{10}$, and $\frac{7}{12}$.
4. Find the sum of $2\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{9}$, and $\frac{8}{11}$.
5. Find the sum of $12\frac{3}{4}$, $14\frac{6}{7}$, $10\frac{3}{10}$.
6. Find the sum of $36\frac{7}{8}$, $\frac{3}{5}$, $12\frac{3}{4}$.
7. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$.
8. Subtract $\frac{5}{12}$ from $\frac{5}{7}$.
9. Subtract $\frac{8}{9}$ from $\frac{11}{12}$.
10. Subtract $\frac{3}{25}$ from $\frac{19}{65}$.
11. Subtract $\frac{3}{4}$ from $12\frac{4}{7}$.
12. Subtract $\frac{5}{6}$ from $3\frac{5}{9}$.
13. Subtract $2\frac{1}{3}$ from $7\frac{1}{4}$.

14. Subtract $10\frac{3}{5}$ from $14\frac{9}{40}$.

15. Find the sum of $30\frac{7}{15}$, $40\frac{3}{5}$, and $6\frac{7}{16}$.

16. Find the sum of $120\frac{59}{60}$ and $306\frac{1}{36}$.

17. Find the sum of $72\frac{3}{25}$ and $360\frac{9}{35}$.

18. Find the sum of $20\frac{7}{40}$, $34\frac{9}{20}$, and $64\frac{5}{50}$.

19. Find the sum of $304\frac{5}{7}$, $27\frac{7}{11}$, and $30\frac{4}{231}$.

20. Subtract $12\frac{9}{17}$ from 17.

21. Subtract $75\frac{3}{4}$ from $90\frac{5}{16}$.

22. Subtract $16\frac{1}{32}$ from $105\frac{1}{50}$.

23. Subtract $72\frac{3}{16}$ from $96\frac{1}{36}$.

24. Subtract $46\frac{2}{11}$ from $93\frac{2}{231}$.

PROBLEMS

Written

69. 1. A farmer has 5 fields containing $26\frac{3}{4}$ acres, $32\frac{2}{3}$ acres, $19\frac{5}{8}$ acres, $38\frac{1}{2}$ acres, and $40\frac{7}{10}$ acres. How many acres has he in all?

2. A man having $39\frac{1}{2}$ cords of wood sold $4\frac{3}{4}$ cords to one man, and $\frac{7}{8}$ cords to another. How much remained?

3. A field is $75\frac{1}{2}$ rods long and $33\frac{7}{8}$ rods wide. What is the distance around the field?

4. A man spends $\frac{2}{9}$ of his salary for clothing, $\frac{2}{5}$ for board, and $\frac{1}{6}$ in traveling. What part of his salary has he remaining?

5. A steamer sails $350\frac{5}{8}$ miles the first day, $312\frac{2}{3}$ miles the second day, $338\frac{4}{5}$ miles the third day, and is then $312\frac{1}{2}$ miles from port. When the port is reached, how far will the steamer have sailed on the trip?

6. A merchant sold $29\frac{3}{4}$ yards of cloth from a roll containing $60\frac{1}{3}$ yards, and $40\frac{7}{8}$ yards from another roll contain-

ing $110\frac{1}{4}$ yards. He sold what was left of both rolls to B. How many yards did B buy?

7. From a piece of cloth measuring 20 yards, $2\frac{3}{8}$ yards, $3\frac{1}{4}$ yards, and $4\frac{1}{2}$ yards were cut. How much remained?

8. What number must you take from $60\frac{2}{3}$ to leave $28\frac{3}{7}$? What number must you add to $16\frac{5}{12}$ to equal $36\frac{3}{8}$?

CANCELLATION

Study Recitation

70. Cancellation is the process of rejecting the same factors from dividend and divisor, or from numerator and denominator, by dividing both terms by such factors.

In the following statements, the word *numerator* may be substituted for *dividend*, and *denominator* for *divisor*.

1. $\frac{35}{21}$ indicates that 35 is to be divided by 21, 35 being the dividend and 21 the divisor.

$21\overline{)35}$

$1\frac{14}{21} = 1\frac{2}{3}$

If both dividend and divisor are divided by 7, the value of the resulting fraction is the same.

$$\frac{7)35}{7)21} = \frac{5}{3} = 1\tfrac{2}{3}$$

What principle is applied in canceling a factor in the numerator and denominator of a fraction?

2. Reduce $\frac{35}{21}$ to a mixed number.

By the process of cancellation, or rejecting equal factors from both dividend and divisor, the operation is shortened.

$$\frac{35}{21} = \frac{5 \times 7}{3 \times 7}$$

$$\frac{\overset{5}{\cancel{35}}}{\underset{3}{\cancel{21}}} = \frac{5}{3} = 1\tfrac{2}{3}$$

Canceling the factor 7 in both dividend and divisor, we have remaining $\frac{5}{3} = 1\frac{2}{3}$. Striking out the factor 7 is the same as dividing both dividend and divisor, or numerator and denominator, by 7. The same result may be obtained by dividing both 35 and 21 by 7.

3. Reduce $\frac{210}{78}$ to a mixed number.

$$\frac{210}{78} = \frac{2 \times 3 \times 5 \times 7}{2 \times 3 \times 13} = \frac{5 \times 7}{13} = \frac{35}{13} = 2\frac{9}{13}$$

In this case the prime factors of the numerator and denominator are found, and the common factors are canceled.

$$\frac{\overset{35}{\cancel{105}}}{\underset{78}{\cancel{210}}} = \frac{35}{13} = 2\frac{9}{13}$$

$$\overset{13}{\cancel{39}}$$

In this case both 210 and 78 are divided by 2, giving $\frac{105}{39}$; both 105 and 39 are then divided by 3, giving $\frac{35}{13}$ which equals $2\frac{9}{13}$.

When all the equal factors have been rejected from both dividend and divisor, the remaining factors in the dividend are multiplied together, and the product is divided by the product of the remaining factors in the divisor. Give this statement substituting the word *numerator* for *dividend* and *denominator* for *divisor*.

4. $$\frac{\overset{1}{\cancel{3}} \quad 8 \quad 8}{\underset{3}{\cancel{6}} \times 8 \times \underset{1}{\cancel{24}} \times \underset{1}{\cancel{40}}}{\underset{3}{\cancel{9}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{2}}} = 512$$

Here we first divide the factors 5 and 40 by 5, giving 1 in the divisor and 8 in the dividend. The factors 2 and 6 are then divided by 2, giving 1 in the divisor and 3 in the dividend. The 3 resulting from this division and the 9 in the divisor are divided by 3, giving 1 in the dividend and 3 in the divisor. The 3 in the divisor resulting from this division and the 24 in the dividend are divided by 3, giving 1 in the divisor and 8 in the dividend. Multiplying $8 \times 8 \times 1$, the remaining factors in the dividend, gives the product, 512. As there are no factors other than 1 in the divisor, and as dividing by 1 does not affect the quotient, 512 is the answer.

Solve the following by cancellation:

5. $\frac{4 \times 9 \times 16}{8 \times 3 \times 5} = ?$

6. $\frac{5 \times 18 \times 27 \times 30}{10 \times 9 \times 6 \times 2} = ?$

7. $\frac{120 \times 72 \times 48}{60 \times 9 \times 8} = ?$

8. $\frac{7}{8} \times \frac{12}{15} \times \frac{20}{24} = ?$

9. $\frac{6 \times 12 \times 14}{7 \times 3 \times 5} = ?$

10. $\frac{27 \times 30 \times 36}{15 \times 18} = ?$

11. $\frac{48 \times 30 \times 65}{18 \times 7} = ?$

12. $\frac{9}{12} \times \frac{18}{24} \times \frac{15}{25} = ?$

Oral Drill

71. In the following exercises pupils should be required to cancel factors common to numerators and denominators, and then to perform the indicated operation without any written work:

1.	$\frac{1}{2}$ of $\frac{8}{4}$	7.	$\frac{3}{8} \times \frac{1}{5}$	13.	$\frac{1}{2}$ of $2\frac{1}{3}$	19.	$\frac{1}{3} \times 3\frac{3}{4}$
2.	$\frac{3}{8}$ of $\frac{5}{6}$	8.	$\frac{2}{8} \times \frac{6}{7}$	14.	$\frac{1}{2}$ of $4\frac{3}{8}$	20.	$\frac{1}{3} \times 6\frac{1}{2}$
3.	$\frac{2}{8}$ of $\frac{6}{7}$	9.	$\frac{5}{6} \times \frac{20}{45}$	15.	$\frac{1}{2}$ of $6\frac{6}{7}$	21.	$\frac{1}{3} \times 9\frac{3}{4}$
4.	$\frac{3}{4}$ of $\frac{8}{9}$	10.	$\frac{5}{8} \times \frac{3}{75}$	16.	$\frac{1}{2}$ of $8\frac{8}{9}$	22.	$\frac{1}{3} \times 12\frac{5}{6}$
5.	$\frac{8}{9}$ of $\frac{5}{16}$	11.	$\frac{3}{12} \times \frac{10}{15}$	17.	$\frac{1}{2}$ of $10\frac{1}{3}$	23.	$\frac{1}{3} \times 18\frac{2}{3}$
6.	$2\frac{7}{24}$ of $\frac{8}{9}$	12.	$\frac{3}{11} \times \frac{33}{6}$	18.	$\frac{2}{3}$ of 7	24.	$\frac{5}{8} \times 6$

MULTIPLICATION OF FRACTIONS

Study Recitation

72. 1. In what ways may a fraction be multiplied by a whole number? Illustrate by multiplying $\frac{3}{8}$ by 2.

2. In what ways may a fraction be divided by a whole number? Illustrate by dividing $\frac{4}{5}$ by 2.

The expression $\frac{3}{4}$ equals 3 times $\frac{1}{4}$, or $\frac{1}{4}$ of 3, or $3 \div 4$.

$\frac{3}{4}$ of $8 = 3 \times \frac{1}{4}$ of 8, or $\frac{1}{4}$ of 3×8, or $3 \times 8 \div 4 = 6$

$\frac{7}{8} \times \frac{3}{4} = (\frac{7}{8} \times 3) \div 4$, or $(\frac{7}{8} \div 4) \times 3 = \frac{21}{32}$.

3. Multiply $\frac{7}{8}$ by $\frac{3}{4}$.

Multiply $\frac{7}{8}$ by 3. What is the product? Divide the product by 4. What is the result? How is the numerator of the result obtained? How is the denominator of the result obtained? Show that the result is the same if $\frac{7}{8}$ is divided by 4 and the quotient multiplied by 3.

4. Make and learn a rule for multiplying a fraction by a fraction. Apply the rule:

$$\frac{7}{8} \times \frac{4}{5} \times \frac{3}{14} = \frac{7 \times 4 \times 3}{8 \times 5 \times 14} \begin{matrix}\text{Numerator} \\ \text{Denominator}\end{matrix} = \frac{84}{560} = \frac{3}{20}.$$

Since dividing both numerator and denominator by the same number does not change the value of the fraction, the same factors in numerator and denominator may be canceled, and the operation shortened.

$$\begin{matrix}1 & & 1 \\ \frac{7 \times \cancel{4} \times 3}{\cancel{8} \times 5 \times \cancel{14}} & = & \frac{3}{20} \\ 2 & & 2\end{matrix}$$

Canceling the equal factors 4 and 7 in numerator and denominator, we have left in the numerator the factors 1, 1, and 3, and in the denominator the factors 2, 5, and 2.

Multiplying together the remaining factors in the numerator for the numerator of the product, and the remaining factors in the denominator for the denominator of the product, gives the result $\frac{3}{20}$.

The product of the sum of two numbers by a third number is the same as the sum of the products of each of the two numbers by the third number.

Sum $12 = 5 + 7$, two numbers.
Third number $2 \qquad 2$, third number.
Product of sum by 2, $\overline{24} = \overline{10 + 14} = 24$, sum of products.

5. Multiply $3\frac{2}{5}$ by 4 without reducing the mixed number to an improper fraction.

$$\begin{matrix}3\frac{2}{5} = 3 + \frac{2}{5} \\ \underline{4 \qquad\quad 4}\end{matrix}$$

Multiply and add the products.

Oral Drill

73. To the Teacher. Require pupils to read the examples, perform the indicated operations mentally, and give the result orally.

Put one of the examples on the board, and have each member of the class solve it mentally and raise a hand as soon as result is found. Follow with other examples.

Place ten examples on the board, and at a given signal have all members of the class begin to solve mentally, and as answers are obtained, write them on a piece of paper. Allow nothing but the answers to be written.

Test to see who will finish first. Test to see how many examples can be solved correctly by each in a given time, — one, two, or three minutes.

1. $\frac{2}{3} \times \frac{4}{5}$
2. $\frac{3}{5} \times \frac{5}{7}$
3. $\frac{5}{6} \times \frac{5}{8}$
4. $\frac{7}{8} \times \frac{3}{4}$
5. $\frac{2}{7} \times \frac{3}{10}$
6. $\frac{5}{9} \times \frac{2}{3}$
7. $\frac{4}{11} \times \frac{2}{5}$
8. $\frac{5}{6} \times \frac{7}{8}$
9. $\frac{4}{7} \times \frac{3}{5}$
10. $\frac{5}{12} \times \frac{1}{3}$
11. $2\frac{1}{2} \times 2$
12. $3\frac{1}{3} \times 3$
13. $4\frac{1}{5} \times 2$
14. $2\frac{3}{8} \times 2$
15. $3\frac{2}{5} \times 2$
16. $1\frac{1}{2} \times 3$
17. $5\frac{1}{3} \times 4$
18. $6\frac{2}{3} \times 3$
19. $7\frac{3}{8} \times 3$
20. $12\frac{1}{2} \times 2$
21. $\frac{2}{3} \times \frac{4}{5} \times \frac{1}{2}$
22. $\frac{3}{8} \times \frac{1}{2} \times \frac{8}{5}$
23. $\frac{4}{5} \times \frac{2}{3} \times \frac{7}{10}$
24. $\frac{3}{8} \times \frac{3}{4} \times \frac{9}{10}$
25. $\frac{2}{7} \times \frac{5}{8} \times 3$
26. $\frac{4}{5} \times \frac{3}{7} \times 2$
27. $\frac{3}{8} \times \frac{3}{5} \times \frac{3}{4}$
28. $2\frac{1}{2} \times 2 \times 3\frac{1}{3}$
29. $4\frac{1}{3} \times 3 \times 2$
30. $2\frac{2}{3} \times 3 \times \frac{1}{2}$

Written

31. $\frac{18}{36} \times \frac{24}{45} \times \frac{5}{12}$
32. $\frac{9}{10}$ of $\frac{14}{48} \times \frac{18}{35}$ of $\frac{8}{9}$
33. $\frac{45}{49} \times \frac{28}{15} \times \frac{7}{9}$
34. $12\frac{1}{2} \times 8\frac{1}{3} \times \frac{6}{50}$
35. $37\frac{1}{2} \times 6\frac{1}{4} \times \frac{7}{8}$
36. $33\frac{1}{3} \times \frac{63}{80}$
37. $16\frac{2}{3} \times 12\frac{1}{2} \times \frac{12}{25}$
38. $37\frac{1}{2} \times \frac{4}{7}$ of $\frac{28}{50}$
39. $66\frac{2}{3} \times \frac{12}{20} \times 3\frac{1}{8}$
40. $87\frac{1}{2} \times \frac{5}{6}$ of $\frac{36}{55}$

41.

$$796 \times 19\tfrac{2}{3}$$
$$796$$
$$19\tfrac{2}{3}$$
$$530\tfrac{2}{3} = 796 \times \tfrac{2}{3}$$
$$7164$$
$$796$$
$$15654\tfrac{2}{3}$$

42. $786\frac{1}{5} \times 16$ **43.** $3475 \times 62\frac{3}{4}$ **44.** $7847 \times 12\frac{1}{8}$

PROBLEMS

ORAL

74. 1. A family uses $\frac{7}{8}$ of a pound of sugar a day. How much will it use in a week?

2. John walks at the rate of $3\frac{3}{4}$ miles per hour. How far can he walk in 6 hours, at the same rate?

3. If a man saws $\frac{7}{8}$ of a cord of wood in a day, how much can he saw in $\frac{2}{3}$ of a day?

4. Find the cost of $2\frac{1}{2}$ pounds of rice at $7\frac{1}{2}$¢ a pound.

5. Find the cost of $3\frac{3}{4}$ cords of wood at $6 a cord.

6. Find the cost of $2\frac{5}{8}$ yards of velvet at $1.60 a yard.

7. Find the cost of $2\frac{1}{2}$ pounds of steak at $12\frac{1}{2}$ cents a pound.

8. I bought $\frac{3}{4}$ of a yard of satin at $2.60 a yard, and handed the clerk a two-dollar bill. How much change did I receive?

9. I sell $\frac{3}{4}$ of 70 bushels of potatoes at 50 cents a bushel. How much do I receive for them?

10. Three boys have thirty marbles; John owns $\frac{2}{3}$ of them; he gives $\frac{2}{5}$ of his share to Henry, and $\frac{1}{4}$ of his share to James. What part of all the marbles does he still own, and how many?

11. Ask the different questions that must be answered in solving problem 10.

12. A farmer put 600 sheep into 4 fields. He put $\frac{1}{6}$ of them into the first field, $\frac{1}{4}$ into the second field, and $\frac{3}{5}$ of the remainder into the third field. How many sheep were there in the fourth field?

Solve the problem by separating it into the different questions that must be answered.

13. $\frac{2}{3}$ of Henry's marbles exceeds $\frac{1}{3}$ of them by 4. How many has he?

14. $\frac{1}{5}$ of Henry's marbles is 5 more than $\frac{1}{8}$ of them. How many has he?

15. $\frac{5}{6}$ of Henry's marbles exceeds $\frac{1}{2}$ of them by 22. How many has he?

16. Mr. Jones spent $\frac{4}{5}$ of his money, and then found that he had $24 remaining. How much money did he spend?

17. I own $\frac{1}{4}$ of a mill and sell $\frac{9}{10}$ of my share for $450. At this rate, what is the value of the mill?

WRITTEN

18. A room is 16 ft. 8 in. wide and 18 ft. 6 in. long. What will be the cost of a picture molding extending around the room at 8¢ a foot? 8 in. is what part of a foot? 6 in. is what part of a foot?

19. If I sell $12\frac{3}{4}$ cords of oak at $4.50 a cord, $8\frac{1}{2}$ cords of maple at $5.25 a cord, and $16\frac{1}{4}$ cords of soft wood at $2.75 a cord, how much do I receive in all?

20. Make four questions of the 19th problem, and answer each.

21. If you sell to R. H. Fiske $12\frac{3}{4}$ yards of velvet at \$2.50 a yard, $20\frac{1}{4}$ yards of ribbon at 10 cents a yard, $6\frac{1}{3}$ dozen buttons at 15 cents a dozen, and $24\frac{1}{2}$ yards of lace at 56 cents a yard, how much will he owe you?

Make out a receipted bill. If Mr. Fiske hands you a fifty-dollar bill, what amount will you return to him?

22. What is the value of 500 dozen eggs at $12\frac{1}{2}$ cents a dozen?

23. Wheat weighs 60 pounds to the bushel. A farmer sells first 3640 pounds and then 3580 pounds of wheat at 75¢ a bushel. How much should he receive?

24. Corn weighs 56 pounds a bushel. If a farmer sells 6640 pounds of corn at 35¢ a bushel, and 4290 pounds of wheat at 75¢ a bushel, how much should he receive?

25. What is the cost of 1340 fence posts at \$7.50 per C? C means 100.

26. In the working days of a week, a man works $9\frac{1}{2}$ hr., $8\frac{3}{4}$ hr., $10\frac{3}{8}$ hr., $8\frac{3}{4}$ hr., $9\frac{3}{4}$ hr., and $8\frac{1}{4}$ hr. How much are his wages for the week, at 30¢ an hour?

27. At \$1.50 a day, how much does a man earn in 3 weeks, if he works $5\frac{3}{4}$ days the first week, $4\frac{3}{5}$ days the second week, and $5\frac{2}{3}$ days the third week?

28. A milkman sells $\frac{1}{2}$ of a pint of cream and $1\frac{1}{2}$ quarts of milk daily to each of three families. What is the value of the cream and milk sold to the three families in 4 days, if cream is worth 20¢ a quart and milk 6¢ a quart?

29. From a lot of $6\frac{1}{2}$ dozen pairs of shoes, $3\frac{1}{3}$ dozen pairs were sold at \$2.75 a pair, and the rest at \$2.50 per pair. How many pairs were sold at \$2.50 per pair? How much was received for the entire lot?

DIVISION OF FRACTIONS

Study Recitation

75. 1. In what ways may a fraction be divided by a whole number?

Illustrate by dividing $\frac{8}{9}$ by 4.

2. In what ways may a fraction be multiplied by a whole number?

Illustrate by multiplying $\frac{3}{10}$ by 5.

The expression $\frac{4}{5}$ equals 4 times $\frac{1}{5}$, or $\frac{1}{5}$ of 4, or $4 \div 5$.

3. To divide $\frac{8}{8}$ by $\frac{4}{5}$: The divisor is $\frac{4}{5}$, or $\frac{1}{5}$ of 4, or $4 \div 5$.

If $\frac{8}{8}$ is divided by 4, it is divided by a number 5 times as large as the correct divisor; the quotient, $\frac{8}{32}$, is, therefore, $\frac{1}{5}$ as large as the correct quotient.

How is the correct quotient obtained? What is the correct quotient?

How is the numerator of the correct quotient obtained? $3 \times 5 = 15$.

How is the denominator of the correct quotient obtained? $4 \times 8 = 32$. $\frac{8}{8} \div \frac{4}{5} = \frac{15}{32}$.

The same result is obtained if the divisor, $\frac{4}{5}$, is inverted, becoming $\frac{5}{4}$, and is then multiplied by $\frac{8}{8}$; thus,

$$\frac{8}{8} \times \frac{5}{4} = \frac{15}{32} = \frac{8}{8} \div \frac{4}{5}$$

4. Make and learn a rule for dividing a fraction by a fraction.

5. Since whole numbers and mixed numbers may be reduced to improper fractions, every case in division of fractions may be regarded as a case of dividing a fraction by a fraction; and may be changed to a case of multiplying a fraction by a fraction. How?

a.	$8 \div \frac{3}{7} = ?$	$\frac{8}{1} \div \frac{3}{7} = ?$	$\frac{8}{1} \times \frac{7}{3} = ?$
b.	$\frac{3}{7} \div 8 = ?$	$\frac{3}{7} \div \frac{8}{1} = ?$	$\frac{3}{7} \times \frac{1}{8} = ?$
c.	$7 \div 2\frac{1}{2} = ?$	$\frac{7}{1} \div \frac{5}{2} = ?$	$\frac{7}{1} \times \frac{2}{5} = ?$
d.	$2\frac{1}{2} \div 7 = ?$	$\frac{5}{2} \div \frac{7}{1} = ?$	$\frac{5}{2} \times \frac{1}{7} = ?$
e.	$2\frac{1}{3} \div 3\frac{1}{4} = ?$	$\frac{7}{3} \div 1\frac{3}{4} = ?$	$\frac{7}{3} \times \frac{4}{13} = ?$
f.	$\frac{3}{8} \div 4\frac{1}{5} = ?$	$\frac{3}{8} \div 2\frac{1}{5} = ?$	$\frac{3}{8} \times \frac{5}{21} = ?$
g.	$4\frac{1}{5} \div \frac{3}{8} = ?$	$2\frac{1}{5} \div \frac{3}{8} = ?$	$2\frac{1}{5} \times \frac{8}{3} = ?$
h.	$\frac{4}{5} \div \frac{3}{7} = ?$	$\frac{4}{5} \div \frac{3}{7} = ?$	$\frac{4}{5} \times \frac{7}{3} = ?$

a is a case of dividing a whole number by a fraction. What is *b*? *c*? *d*? *e*? *f*? *g*? *h*?

6. In dividing a mixed number by a whole number when the integral part of the mixed number is larger than the divisor, it is sometimes convenient to divide as follows:

$3\overline{)24\frac{3}{8}}$ $\quad 8\frac{1}{8}$

One third of 24 is 8; one third of $\frac{3}{8}$ is $\frac{1}{8}$; the quotient is $8\frac{1}{8}$.

$3\overline{)25\frac{3}{8}}$ $\quad 8\frac{11}{24}$

3 is contained in $25\frac{3}{8}$, 8 times and $1\frac{3}{8}$ over. $1\frac{3}{8} = \frac{11}{8}$. 3 is contained in $\frac{11}{8}$, $\frac{11}{24}$ times. The quotient is $8\frac{11}{24}$.

ORAL

Divide:

7. $\frac{7}{8} \div 6$	**15.** $\frac{7}{8} \div \frac{4}{5}$	**23.** $8 \div \frac{3}{4}$	**31.** $\frac{5}{6} \div 3$
8. $\frac{4}{5} \div 3$	**16.** $\frac{5}{6} \div \frac{2}{3}$	**24.** $7 \div \frac{2}{5}$	**32.** $\frac{3}{8} \div \frac{1}{4}$
9. $\frac{1}{2} \div \frac{2}{3}$	**17.** $6 \div \frac{2}{3}$	**25.** $\frac{1}{3} \div 2$	**33.** $9 \div \frac{3}{7}$
10. $4\frac{2}{5} \div 11$	**18.** $\frac{3}{11} \div 2$	**26.** $\frac{2}{3} \div 6$	**34.** $\frac{2}{3} \div 3\frac{1}{3}$
11. $3\frac{1}{3} \div \frac{2}{3}$	**19.** $\frac{5}{6} \div 3$	**27.** $\frac{3}{4} \div 8$	**35.** $2\frac{1}{2} \div 3\frac{1}{3}$
12. $3\frac{1}{3} \div 2\frac{1}{2}$	**20.** $\frac{2}{3} \div \frac{1}{2}$	**28.** $\frac{2}{5} \div 7$	**36.** $4\frac{2}{7} \div 6\frac{2}{3}$
13. $6\frac{2}{3} \div 4\frac{2}{7}$	**21.** $\frac{4}{5} \div \frac{7}{8}$	**29.** $2 \div 3\frac{1}{3}$	**37.** $3\frac{3}{4} \div 3\frac{1}{5}$
14. $3\frac{1}{5} \div 3\frac{3}{4}$	**22.** $\frac{7}{8} \div \frac{4}{7}$	**30.** $3\frac{1}{3} \div 2$	**38.** $3\frac{1}{5} \div 3\frac{3}{4}$

WRITTEN

39. $1\frac{2}{5} \div \frac{3}{7}\frac{6}{5}$	**43.** $62\frac{1}{2} \div 16\frac{2}{3}$	**47.** $87\frac{1}{2} \div 37\frac{1}{2}$
40. $16\frac{2}{3} \div 14\frac{2}{7}$	**44.** $16\frac{2}{3} \div 62\frac{1}{2}$	**48.** $28\frac{4}{7} \div \frac{2}{4}\frac{5}{9}$
41. $12\frac{1}{2} \div 8\frac{1}{3}$	**45.** $33\frac{1}{3} \div 4\frac{4}{5}$	**49.** $83\frac{1}{3} \div 3\frac{1}{3}$
42. $8\frac{1}{3} \div 12\frac{1}{2}$	**46.** $4\frac{4}{5} \div 33\frac{1}{3}$	**50.** $37\frac{1}{2} \div \frac{3}{4}$

51. $6393\frac{1}{6} \div 4$

$$4\overline{)6393\frac{1}{6}}$$
$$1598\frac{7}{24}$$

This is a convenient method of dividing a large mixed number by an integer. Dividing in the usual way gives 1598 as quotient and $1\frac{1}{6}$ remainder. $1\frac{1}{6} = \frac{7}{6}$, and this divided by 4 gives $\frac{7}{24}$.

52. $7854\frac{1}{5} \div 7$ **53.** $287\frac{4}{9} \div 11$ **54.** $1447\frac{2}{3} \div 30$

55. $\dfrac{2\frac{2}{3}}{\frac{1}{8}}$

This form of fraction is called a **complex fraction**, and is read $\frac{2}{3}$ over $\frac{1}{8}$, or $\frac{2}{3} \div \frac{1}{8}$. Complex fractions are simply indicated divisions of fractions, the heavy line separating the dividend from the divisor.

56. $\dfrac{2\frac{2}{3}}{3} = \frac{2}{3} \div 3$ **57.** $\dfrac{7}{\frac{3}{4}} = 7 \div \frac{3}{4} = ?$ **58.** $\dfrac{\frac{1}{2} + \frac{1}{3}}{\frac{4}{5} - \frac{2}{3}}$

SOLUTION (Ex. 58): $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ $\quad \frac{4}{5} - \frac{2}{3} = \frac{2}{15}$

$$\frac{\frac{5}{6}}{\frac{2}{15}} = \frac{5}{6} \div \frac{2}{15} = \frac{5}{\cancel{6}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{2}}} = \frac{25}{4} = 6\tfrac{1}{4}$$

59. $\dfrac{\frac{2}{3} + \frac{3}{4} - \frac{1}{2}}{2\frac{1}{3} + \frac{3}{5}}$ **60.** $\dfrac{5\frac{1}{3} \times \frac{7}{7}}{6\frac{2}{3} - \frac{4}{15}}$ **61.** $\dfrac{6\frac{2}{3} \div \frac{24}{50}}{17\frac{1}{2} \times \frac{26}{49}}$

Reduce the numerator to a single fraction, and the denominator to a single fraction before dividing.

62. $\dfrac{\frac{1}{2} + \frac{3}{4} + 6\frac{2}{3} \times \frac{9}{35}}{10\frac{2}{3} - \frac{7}{8} \times 3\frac{2}{7}}$

Be careful as to which operation is performed first. See p. 67.

ORAL

TO THE TEACHER. — In this drill work, when one fraction is to be divided by another, train pupils to think of the divisor as inverted, and then proceed as in multiplication of one fraction by another. Aim for accuracy and rapidity.

63. $\frac{8}{10} \div 4$	**66.** $\frac{18}{21} \div 6$	**69.** $\frac{7}{8} \div 2$	**72.** $\frac{2}{3} \div \frac{1}{2}$
64. $\frac{5}{6} \div 8$	**67.** $\frac{15}{20} \div 5$	**70.** $\frac{7}{12} \div 4$	**73.** $\frac{6}{8} \div \frac{3}{4}$
65. $\frac{3}{7} \div 5$	**68.** $\frac{12}{20} \div 3$	**71.** $\frac{5}{14} \div 2$	**74.** $\frac{5}{7} \div \frac{3}{10}$

PROBLEMS

Oral

76. **1.** In $2\frac{1}{4}$ acres, how many lots of $\frac{1}{8}$ of an acre each are there?

2. If it requires $5\frac{1}{3}$ yards of cloth for a skirt, how many skirts may be made from 32 yards?

3. If $\frac{3}{4}$ of a yard of ribbon is made into 6 badges of equal length, each badge is what part of a yard long? How many inches long?

4. Mrs. Brown filled 10 tumblers with $\frac{7}{8}$ of a gallon of jelly. What part of a gallon did she put in each?

5. James divided $6\frac{2}{3}$ dozen marbles equally among 10 schoolmates. What part of a dozen did each receive? How many marbles?

6. A family burns $10\frac{1}{2}$ tons of coal in 4 months. What is the average number of tons burned per month?

Written

7. 3 is what part of 5? *Ans.* $\frac{3}{5}$

3 is what part of $3\frac{1}{3}$? $\frac{3}{3\frac{1}{3}} = 3 \div 3\frac{1}{3} = 3 \div \frac{10}{3} = 3 \times \frac{3}{10} = \frac{9}{10}$.

3 is what part of any number? It is 3 divided by that number.

8. One rod equals $16\frac{1}{2}$ ft. How many rods are equal to 594 ft.?

9. If a barrel holds $2\frac{3}{4}$ bushels, how many barrels will be required for shipping 330 bushels of apples?

10. If I exchange $9\frac{3}{8}$ bushels of beans at \$1.60 a bushel for sugar at $5\frac{1}{3}$ cents a pound, how many pounds of sugar do I receive?

11. How many times the cost will be gained by buying apples at the rate of 3 for a cent, and selling them at 3¢ apiece?

12. A can do a piece of work in 8 days, B can do the same piece of work in 6 days, and C can do it in 4 days. In how many days can the three complete the work?

If A can do a piece of work in 8 days, what part can he do in 1 day? *Ans.* $\frac{1}{8}$.

If B can do a piece of work in 6 days, what part can he do in 1 day? *Ans.* $\frac{1}{6}$.

If C can do a piece of work in 4 days, what part can he do in 1 day? *Ans.* $\frac{1}{4}$.

If A can do $\frac{1}{8}$ of a piece of work in 1 day, B $\frac{1}{6}$ of it, and C $\frac{1}{4}$ of it, what part can they all do in 1 day? *Ans.* $\frac{13}{24}$ of it.

If A, B, and C can do $\frac{13}{24}$ of a piece of work in 1 day, how long will it take them to do $\frac{24}{24}$ of it? *Ans.* $\frac{24}{13}$, or $1\frac{11}{13}$ days.

The last question may be separated into two questions: If A, B, and C can do $\frac{13}{24}$ of a piece of work in 1 day, how long will it take to do $\frac{1}{24}$ of the work? *Ans.* $\frac{1}{13}$ of a day.

If A, B, and C can do $\frac{1}{24}$ of a piece of work in $\frac{1}{13}$ of a day, how long will it take them to do $\frac{24}{24}$ of the work? *Ans.* $\frac{24}{13}$, or $1\frac{11}{13}$ days.

13. The circumference of a bicycle wheel is $6\frac{3}{16}$ ft. How many revolutions will it make in going $\frac{3}{4}$ of a mile? A mile is 5280 ft.

14. How many strips of carpet $\frac{3}{4}$ of a yard wide will it take to cover a room $7\frac{1}{2}$ yd. wide?

15. How many bows, each requiring $\frac{3}{5}$ of a yard, can be made from a piece of ribbon 10 yd. long?

16. If 25 men earn $\$312\frac{1}{2}$ in $6\frac{1}{4}$ days, what is the wage of each per day?

17. At $\frac{7}{8}$ of a dollar a yard, how many yards of cloth can be bought for $8\frac{3}{4}$ dollars?

18. If 6 men can do a piece of work in $13\frac{3}{5}$ days, how long will it take 8 men to do it?

19. If an automobile goes a mile in $28\frac{1}{5}$ seconds, how many miles will it go in 1 hour, at the same rate?

20. If a 6-foot pole casts a shadow $8\frac{1}{2}$ feet long, what is the length of a pole which casts a shadow 34 feet long at the same time? Solve in two ways.

21. In the 20th problem, if the second pole is 34 feet long, what is the length of its shadow?

22. A rectangle is $2\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches wide. If one side represents a length of 40 rods, what length does each end represent?

23. If a horse travels $2\frac{1}{2}$ miles in 7 minutes, in how many minutes will it travel 1 mile? In how many minutes will it travel $4\frac{3}{4}$ miles?

24. If a train of cars runs $\frac{4}{5}$ of a mile in $1\frac{2}{3}$ minutes, in how many minutes will it run 4 miles?

25. If a train of cars runs $\frac{4}{5}$ of a mile in $1\frac{2}{3}$ minutes, how far will it run in 5 minutes?

26. How many tickets were sold for an excursion if $\$318\frac{3}{4}$ was received for all the tickets, and the price of each ticket was $\$3\frac{3}{4}$?

27. At $4\frac{1}{4}¢$ a pound, how many pounds of sugar can be bought for \$1.70?

28. If the circumference of a wheel is $5\frac{3}{4}$ ft., how many times will the wheel turn around in going one half a mile (2640 ft.)?

29. If a man's board costs him $\$1\frac{3}{4}$ a day, for how many days' board will $\$19\frac{1}{4}$ pay?

30. A man can do a piece of work in 6 days. What part of the work can he do in 2 days?

Ans. $\frac{2}{6} = \frac{1}{3}$. In 5 days? *Ans.* $\frac{5}{6}$. In $3\frac{1}{2}$ days?

Ans. $\frac{3\frac{1}{2}}{6} = 3\frac{1}{2} \div 6 = \frac{7}{12}$.

31. A tailor can make a suit in $6\frac{1}{4}$ days. What part of the work is completed when he has worked on one suit $2\frac{2}{5}$ days?

32. A train makes the run between two towns in $\frac{3}{4}$ of an hour. What part of the journey is finished in $\frac{2}{3}$ of an hour?

33. A builds a fence in 9 days. What part of it does he build in 1 day? In 3 days? In 8 days? In $\frac{2}{3}$ of a day? In $\frac{3}{4}$ of a day?

34. B builds a fence in $3\frac{1}{3}$ days. What part does he build in 1 day? In $\frac{1}{3}$ of a day? In 2 days? In $2\frac{1}{2}$ days?

35. A can dig a cellar in 12 days, and B in 8 days. What part can each finish in 1 day? What part can both finish in 1 day?

36. A does $\frac{1}{7}$ of a piece of work in 1 day. In how many days can he complete such a job?

37. B does $\frac{2}{5}$ of a piece of work in 1 day. In how many days can he complete such a job?

38. C does $\frac{7}{8}$ of a piece of work in 1 day. In how many days can he complete such a job?

39. D does $\frac{3}{51}$ of a piece of work in 1 day. In how many days can he complete such a job?

40. E does $\frac{4}{17}$ of a piece of work in 1 day. How long will it take him to do the whole work?

GENERAL PROBLEMS IN FRACTIONS

Study Recitation

77. Besides the operations of reduction, addition, subtraction, multiplication, and division of fractions, there are three general problems in fractions which should be mastered. They are:

	ILLUSTRATIONS
PROBLEM I. **To find any fractional part of a number.**	Find $\frac{3}{8}$ of 16.
PROBLEM II. **To find what fractional part one number is of another.**	6 is what fractional part of 16?
PROBLEM III. **To find a number when a given fractional part of it is known.**	6 is $\frac{3}{8}$ of what number?

Read the questions below, and state, in the language of the general problem, what is asked for in each case, and state also what is the particular thing asked for in each case; thus,

1. PROBLEM I. $\frac{3}{8}$ of 16 = what?

$\frac{3}{8}$ of 16 = 6.

Ans. 6. It is required to find *a fractional part* ($\frac{3}{8}$) of *a number* (16).

Read the above sentence, leaving out the numbers in parenthesis. Then read, leaving out the italicized words.

2. PROBLEM II. What fraction of 16 is 6? or, 6 is what fractional part of 16?

Ans. $\frac{3}{8}$. It is required to find what fractional part *one number* (6) is of *another number* (16).

3. PROBLEM III. $\frac{3}{8}$ of what number = 6? or, 6 is $\frac{3}{8}$ of what number?

Ans. 16. It is required to find a number when *a given fractional part* ($\frac{3}{8}$) of it is known (6).

Answer the following questions in the same way:

4. PROBLEM II. 14 is what fractional part of 21?
$14 = \frac{2}{3}$ of 21.

5. PROBLEM III. $14 = \frac{2}{3}$ of what number? or, $\frac{2}{3}$ of what number $= 14$?

6. PROBLEM I. What number $= \frac{2}{3}$ of 21? or, $\frac{2}{3}$ of $21 =$ what number?

7. PROBLEM III. $12 = \frac{3}{5}$ of what number?
$12 = \frac{3}{5}$ of 20.

8. PROBLEM II. $12 =$? of 20? State the question in full.

9. PROBLEM I. $? = \frac{3}{5}$ of 20? State the question in full.

Take notice that in each problem two things are given, and one is asked for. If any two of the three things are known, the other may be found.

The three things are:

a. A given number.

b. A second number, regarded as a part of the given number.

c. A fraction that expresses the part the second number is of the first number.

ORAL

78. PROBLEM I. To find any fractional part of a number.

1. $\frac{3}{4}$ of 20

SOLUTION: $\frac{1}{4}$ of $20 = 5$. $\frac{3}{4}$ of $20 = 3 \times 5 = 15$.

2. $\frac{5}{8}$ of 24
3. $\frac{4}{7}$ of 21
4. $\frac{3}{5}$ of 15
5. $\frac{7}{8}$ of 16
6. $\frac{5}{7}$ of 49
7. $\frac{6}{7}$ of 42
8. $\frac{5}{12}$ of 72
9. $\frac{4}{11}$ of 55
10. $\frac{7}{9}$ of 81
11. $\frac{3}{8}$ of 56
12. $\frac{7}{9}$ of 108
13. $\frac{8}{9}$ of 99
14. $\frac{5}{6}$ of 72
15. $\frac{4}{7}$ of 84
16. $\frac{3}{11}$ of 66

17. $\frac{3}{4}$ of $\frac{5}{7}$

SOLUTION: $\frac{1}{4}$ of $\frac{5}{7} = \frac{5}{28}$. $\frac{3}{4}$ of $\frac{5}{7} = 3 \times \frac{5}{28} = \frac{15}{28}$.

18. $\frac{4}{5}$ of $\frac{2}{3}$ 20. $\frac{2}{3}$ of $\frac{6}{7}$ 22. $\frac{3}{4}$ of $\frac{5}{7}$

19. $\frac{5}{12}$ of $\frac{7}{9}$ 21. $\frac{3}{7}$ of $\frac{5}{6}$ 23. $\frac{4}{9}$ of $\frac{18}{19}$

24. $\frac{3}{5}$ of $4\frac{1}{2}$

Reduce $4\frac{1}{2}$ to an improper fraction, and then analyze.

25. $\frac{3}{8}$ of $2\frac{1}{3}$ 26. $\frac{2}{7}$ of $4\frac{1}{2}$ 27. $\frac{8}{10}$ of $3\frac{3}{5}$

WRITTEN

28. $\frac{5}{16}$ of 96 30. $\frac{7}{12}$ of 288 32. $\frac{7}{15}$ of 225

29. $\frac{13}{14}$ of 112 31. $\frac{8}{21}$ of 252 33. $\frac{9}{28}$ of 529

34. $\frac{8}{12}$ of 246

Reduce the fraction to its lowest terms, and then solve.

35. $\frac{6}{12}$ of 42 36. $\frac{12}{20}$ of 45 37. $\frac{18}{21}$ of 637

38. $\frac{3}{8}$ of $46 = \frac{1}{8}$ of $3 \times 46 = \frac{138}{8} = 17\frac{1}{4}$

This mode of solving should be used when the whole number is not a multiple of the denominator.

39. $\frac{7}{8}$ of 29 40. $\frac{5}{12}$ of 274 41. $\frac{6}{17}$ of 24

42. $\frac{3}{8}$ of 24

What is done first in solving this example?

Make and learn a rule for finding a fractional part of a number.

43. $\frac{3}{7}$ of 34

What is done first in solving this example?

Make and learn a second rule for finding a fractional part of a number.

PROBLEMS

ORAL

79. 1. How much will $\frac{7}{8}$ of an acre of land cost at $42 an acre?

2. A man owning $\frac{5}{6}$ of a vessel sold $\frac{2}{3}$ of his share. What part of the vessel did he sell? What part had he left?

3. A man divides $\frac{8}{9}$ of an acre into 4 equal lots. What part of an acre is there in each lot?

4. What is the cost of $\frac{3}{4}$ of $9\frac{1}{3}$ tons of hay at $7\frac{1}{2}$ a ton?

5. Three men buy a farm for $2100. A is to own $\frac{1}{4}$ of it, B $\frac{3}{10}$ of it, and C the remainder. What part of the farm is C's share? How much does each pay for his share?

WRITTEN

6. What is the cost of $\frac{4}{5}$ of $6\frac{1}{2}$ tons of coal at $7\frac{1}{2}$ a ton?

7. At $7\frac{7}{8}$ each, how many books can be bought for $7\frac{7}{8}$?

8. How many pounds of tea, at $\$\frac{5}{12}$ a pound, must be given for 9 bushels of apples at $\$\frac{5}{16}$ a bushel?

9. If $\frac{3}{4}$ of 9 bushels of seed corn sells for $13\frac{1}{2}$, how much is $\frac{1}{2}$ bushel worth?

ORAL

80. PROBLEM II. **To find what fractional part one number is of another.**

1. 6 is what part of 18?

SOLUTION: $1 = \frac{1}{18}$ of 18. $6 = \frac{6}{18}$, or $\frac{1}{3}$, of 18.

2. 3 is what part of 7?

3. 4 is what part of 9?

4. 9 is what part of 15?

5. 7 is what part of 35?

6. 12 is what part of 13?
7. 12 is what part of 14?
8. What part of 9 is 7?
9. What part of 15 is 12?
10. What part of 24 is 20?
11. What part of 27 is 24?

In each of the examples above, tell which number is regarded as a part of the other.

ILLUSTRATIONS:

a. 5 is what part of 9? 9 is the whole, and 5 is regarded as a part of the whole.

b. What part of 10 is 7? 10 is the whole, and 7 is regarded as a part of the whole.

In solving the examples above, notice the answer before it is reduced to its lowest terms. In each fraction, what is the part? What is the whole?

To find what fractional part one number is of another.

RULE: *Write the number which is the part as the numerator and the number considered as the whole as the denominator of the fraction. Reduce the fraction to its lowest terms.*

Give the answers to the following without analyzing:

12. 6 is what part of 11?
13. 7 is what part of 12?
14. 9 is what part of 13?
15. 15 is what part of 20?

(Answer in lowest terms.)

16. 25 is what part of 40?
17. 15 is what part of 35?
18. 11 is what part of 14?
19. 27 is what part of 29?
20. 35 is what part of 44?
21. 36 is what part of 60?
22. 63 is what part of 70?
23. 24 is what part of 32?
24. What part of 36 is 7?

25. 2 is what part of $3\frac{1}{2}$?

SOLUTION: $3\frac{1}{2} = \frac{7}{2}$. $2 = \frac{4}{2}$. $\frac{4}{2}$ is $\frac{4}{7}$ of $\frac{7}{2}$.

26. 2 is what part of $3\frac{1}{2}$? SOLUTION: $\frac{2}{3\frac{1}{2}} = 2 \times \frac{2}{7} = \frac{4}{7}$.

27. 3 is what part of $8\frac{1}{3}$? **28.** 4 is what part of $6\frac{1}{2}$?

29. $2\frac{1}{2}$ is what part of $3\frac{1}{3}$?

SOLUTION: $2\frac{1}{2} = \frac{5}{2}$. $3\frac{1}{3} = \frac{10}{3}$. Reducing to a C. D. gives, $\frac{5}{2} = \frac{15}{6}$; $\frac{10}{3} = \frac{20}{6}$; $\frac{15}{6}$ is $\frac{15}{20}$, or $\frac{3}{4}$, of $\frac{20}{6}$.

30. $2\frac{1}{2}$ is what part of $3\frac{1}{3}$? SOLUTION: $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{5}{2} \times \frac{3}{10} = \frac{3}{4}$.

31. $2\frac{1}{4}$ is what part of $4\frac{1}{2}$? **32.** $2\frac{3}{4}$ is what part of $3\frac{1}{4}$?

PROBLEMS

ORAL

81. 1. In the month of November, James attended school 18 days. What part of the month did he attend school?

2. If there are 14 problems in to-day's lesson, and Mary solves 10 of them, what part of the lesson has she mastered?

3. A watch cost \$40 and a chain \$5. What part of the cost of both was the cost of each?

4. A boy hoes 12 rows of corn and has 4 rows yet to hoe. What part of the work has he done? What part has he still to do? What he has yet to do is what part of what he has done?

5. Rice that costs 8¢ a pound is sold for 10¢ a pound. The gain is what part of the cost? How many hundredths of the cost is the gain?

6. A grocer pays 25¢ for a gallon of vinegar and sells it for 30¢. What part of the cost does he gain? How many hundredths of the cost does he gain?

WRITTEN

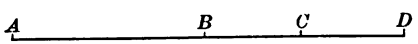
17. From a farm of 275 acres, there is sold to one man 40 acres, to another 85 acres. What part of the farm remains?

18. A man buys 5 horses at \$75 each and 25 cows at \$25 each. He sells the horses at \$85 each, and the cows at \$35 each. His gain is what part of the cost? How many dollars does he gain?

19. If wood is worth \$4$\frac{3}{4}$ a cord, and coal is worth \$7$\frac{1}{2}$ a ton, 5 cords of wood are worth what fraction of the value of 5 tons of coal? Work in two ways.

20. A father is 25 years older than his son. His son's age is 10$\frac{1}{2}$ years. What part of the father's age is the son's age?

21. The distance from A to B is 48 mi.; from B to C 24 mi.; from C to D 26 mi. The distance from A to B is what part of the distance from A to D? Make and answer three other questions on this problem.



ORAL

82. PROBLEM III. **To find a number when a given fractional part is known.**

1. 8 is $\frac{2}{3}$ of what number?

SOLUTION: If 8 is $\frac{2}{3}$ of a number, $\frac{1}{3}$ of that number = $\frac{1}{2}$ of 8, or 4. $\frac{3}{3}$ of the number is 3×4, or 12.

TO THE TEACHER. Pupils are apt to say, "If 8 is $\frac{2}{3}$ of some number, $\frac{1}{3}$ is $\frac{1}{3}$ of 8," etc. In such cases, put the first question thus: 8 is 2 parts of some number, what is 1 part? The answer will be "4." The second question is: If this part is $\frac{1}{3}$ of the number, what are $\frac{3}{3}$ of the number? Require the analysis in correct form.

2. 12 is $\frac{3}{7}$ of what number?

Analysis by question and answer:

1st question. If 12 is $\frac{3}{7}$ of some number, what is $\frac{1}{7}$ of that number? *Ans.* $\frac{1}{7}$ of 12, or 4.

To THE TEACHER. Do not allow pupils to use expressions like $\frac{1}{3}=4$. $\frac{1}{3}$ does not equal 4. $\frac{1}{3}$ of $12 = 4$.

2d question. If 4 is $\frac{1}{7}$ of some number, what is the number? *Ans.* $7 \times 4 = 28$.

When this form of analysis is used, each question should be so complete that it can be answered without reference to anything previously given.

3. 24 is $\frac{8}{9}$ of what number?

4. $\frac{3}{5}$ of what number is 12?

5. $\frac{5}{8}$ of a number is 15. What is the number?

6. 20 is $\frac{5}{6}$ of what number?

7. What is the number of which 6 is $\frac{3}{4}$?

8. What is the number of which 10 is $\frac{5}{7}$?

9. 28 is $\frac{4}{9}$ of what number?

10. 36 is $\frac{9}{10}$ of what number?

11. 75 is $\frac{3}{4}$ of what number?

12. 40 is $\frac{4}{9}$ of what number?

13. 80 is $\frac{4}{5}$ of what number?

14. 60 is $\frac{3}{5}$ of what number?

15. 40 is $\frac{2}{5}$ of what number?

16. State General Problem I, and give the rule for solving examples coming under it.

17. State General Problem II, and give the rule for solving examples coming under it.

18. State General Problem III, and give the rule for solving examples coming under it.

19. Make and solve an example coming under General Problem I. From this problem and its answer, make and solve an example under General Problem II. Make and solve another coming under General Problem III.

PROBLEMS

Oral

83. 1. If $\frac{3}{4}$ dozen eggs costs 12 cents, what is the price of $\frac{1}{4}$ dozen? $\frac{1}{2}$ dozen? 1 dozen? $\frac{2}{3}$ dozen?

2. If $\frac{7}{8}$ of a yard of cloth costs 21 cents, what is the cost of 1 yard? $2\frac{1}{2}$ yards? $\frac{3}{8}$ yards?

3. If $\frac{3}{5}$ of the cost of a hat equals 90 ¢, what is the cost?

4. If $\frac{2}{7}$ of the price of a house equals $6300, what is the price?

5. $\frac{5}{6}$ of the cost of an article equals 24 ¢. What is the cost?

6. John has $2\frac{1}{2}$ doz. marbles, which are $\frac{5}{6}$ of the number I have. How many have I?

7. $300 is $2\frac{1}{2}$ times the price of my watch. What is its price?

8. $3\frac{3}{4}$ acres are $\frac{3}{5}$ of Mr. Jay's land. How many acres has he?

9. A farmer sold 40 sheep, which were $\frac{5}{7}$ of his flock. How many sheep had he before the sale?

10. Mary's standing in arithmetic is 80, and Mary's standing is $\frac{8}{9}$ of her brother's. What is her brother's standing?

11. A room is 18 feet wide; its width is $\frac{3}{4}$ its length. How long is it?

12. When your age is $\frac{1}{3}$ greater than it is now, what fraction of your present age will it be? If your age then will be 16 years, what is it now?

13. The difference between $\frac{1}{3}$ and $\frac{3}{4}$ of a number is 15. What is the number?

14. A boy spent $\frac{3}{7}$ of his money for a suit of clothes and had $20 left. How much did his suit cost?

15. A fruit dealer sells in one day 77 apples and oranges. $\frac{2}{3}$ of the number of oranges equals $\frac{4}{5}$ of the number of apples. How many of each does he sell?

$\frac{1}{3}$ of the number of oranges equals how many fifths of the number of apples?

The oranges and apples together equal how many fifths of the apples?

16. If to A's age there is added $\frac{2}{3}$ and $\frac{2}{5}$ of his age, what fraction of his age will the sum be? If the sum is 62 years, what is his age?

WRITTEN

17. In the 5th grade in a school there are 20 boys; $\frac{3}{8}$ of all the pupils in the school are girls. How many girls are there in the 5th grade?

18. In 5 grades in the same school there are 216 boys, and $\frac{7}{15}$ of the pupils are girls. How many girls are there in the school?

19. $\frac{2}{3}$ of a ship is worth $20,000. Find the value of $\frac{2}{3}$ of it at the same rate.

20. A man sold $\frac{2}{3}$ of a flock of sheep to one man, and $\frac{1}{2}$ the remainder to another, and then had 44 sheep left. How many were there at first?

21. A pole stands $\frac{1}{7}$ of its length in mud, $\frac{2}{5}$ in water, and 32 ft. in air. How long is the pole?

22. A man owning $\frac{5}{8}$ of a ship sells $\frac{2}{7}$ of his share for $55,000; at the same rate what is the entire value of the ship?

23. If a number is diminished by $\frac{3}{8}$ of itself, 480 remains. What is the number?

24. If a number is increased by $\frac{3}{8}$ of itself, the sum is 165. What is the number?

25. If 4 men can do a piece of work in $9\frac{1}{2}$ days, how long will it take 1 man to do it? How long will it take 7 men to do it?

26. If 5 pounds of tea cost $\$3\frac{1}{8}$, how much will 24 pounds cost? Work in two ways.

27. If $2\frac{1}{2}$ bushels of corn cost $95¢$, how much will $3\frac{1}{4}$ bushels cost?

28. A man lost $\frac{5}{16}$ of his money, and afterward found $\frac{1}{5}$ of what he had lost. What part of the original amount had he then?

29. A cistern has two pipes; one will fill it in 5 hours, and the other in 4 hours. If both pipes are open, how long will it take to fill the cistern?

30. If one pipe fills the cistern in 4 hours, and the other empties it in 5 hours, how long will it take to fill the cistern if both pipes are open and the cistern is empty at the time the pipes are opened?

31. A can do a piece of work in $5\frac{1}{4}$ days. What part of the work can he do in $\frac{1}{4}$ of a day? In 1 day?

32. B can do a piece of work in $6\frac{2}{3}$ days, and C can do it in $7\frac{1}{2}$ days. What part of the work can they both do in 1 day? How many days will it take both to do the work?

33. If a man can do a piece of work in 5 days working $8\frac{1}{2}$ hours a day, in how many days can he do it working 9 hours a day?

34. A merchant sold goods at $\frac{1}{8}$ more than they cost, receiving for them $350. What was the cost? What part of the cost would he have gained if he had sold them for $375?

35. A man sold sheep at $4 each, and gained $\frac{1}{4}$ of the cost. What was the cost? If he had lost $\frac{1}{4}$ of the cost by selling them at $4, what would have been the cost?

36. A man sells two cows at $24 each; on one he gains $\frac{1}{4}$ of what it cost him, and on the other he loses $\frac{1}{4}$ of its cost. Does he gain or lose, and how much?

37. A bin containing 505 cubic feet is $\frac{3}{4}$ full of wheat. If a bushel is equal to $1\frac{1}{4}$ cubic feet, how much are the contents worth at 70 cents a bushel?

38. A bin 8 ft. square and 6 ft. deep is $\frac{3}{8}$ full of wheat. What is the value of the wheat at 75¢ a bushel?

RELATION OF FRACTIONS TO PERCENTAGE

Study Recitation

84. The three general problems in fractions are important because there are three general problems in percentage, each of which is a limited form of one of the three general problems in fractions.

Percentage deals with **hundredths.**

The term **per cent** is from the Latin words *per* and *centum*, meaning by the hundred.

25 per cent may also be written .25, $\frac{25}{100}$, or 25%, the character "%" being the sign of per cent.

25% of a number means $\frac{25}{100}$, or $\frac{1}{4}$ of it; 7% of a number means $\frac{7}{100}$ of it; 3% of a number means $\frac{3}{100}$ of it; 20% of a number means $\frac{20}{100}$, or $\frac{1}{5}$, of it. Any per cent of a number means that number of hundredths of it.

Following are the three general problems in fractions, and the corresponding general problems in percentage:

General Problem I

Fractions.	To find any	fractional part	of a number.
Percentage.	To find any	per cent	of a number.

General Problem II

Fractions.	To find what	fractional part	one number is of another.
Percentage.	To find what	per cent	one number is of another.

General Problem III

Fractions.	To find a number when a given	fractional part	of it is known.
Percentage.	To find a number when a given	per cent	of it is known.

Notice that the only new thing in the problems in percentage is the use of the term *per cent*, instead of the term *fractional part*, used in the problems in fractions. By substituting *per cent* for *fractional part* in the problems in fractions, the problems in percentage are formed. Since per cent means hundredths, the words *number of hundredths* may be substituted for the words *per cent* in the problems in percentage, or for *fractional part* in the problems in fractions.

In the problems in fractions, the *fractional part* may have any fractional unit.

In the problems in percentage, the fractional unit of the *per cent* is always one hundredth. From this it is seen that the percentage problems use some number of hundredths, or find some number of hundredths; while the fractional problems use any fraction or find any fraction.

This explains in what way the percentage problems are a limited form of the fraction problems.

By treating the given or required per cent in the percentage problems as a fraction having 100 for its denominator, or by that fraction in its lowest terms, the percentage problems may be changed to problems in common fractions.

PROBLEM I. **To find any per cent of a number.**

1. What is 20 % of 15?

Writing 20 % as a common fraction, $\frac{20}{100}$, we have,

a. What is $\frac{20}{100}$ of 15?

Reducing $\frac{20}{100}$ to lowest terms, $\frac{1}{5}$, we have,

b. What is $\frac{1}{5}$ of 15?

2. What is 5 % of 80?

Writing 5 % as a common fraction, $\frac{5}{100}$, we have,

c. What is $\frac{5}{100}$ of 80?

Reducing $\frac{5}{100}$ to its lowest terms, $\frac{1}{20}$, we have,

d. What is $\frac{1}{20}$ of 80?

$\frac{5}{100}$ of 80 may also be found by multiplying 80 by 5, and dividing the product by 100. Divide by 100 by cutting off two figures from the right of the product; thus, 4 | 00.

Notice that examples *a* and *b*, and *c* and *d*, fall under General Problem I, in fractions.

PROBLEM II. **To find what per cent one number is of another.**

3. 8 is what per cent of 10?

Substituting for *per cent* the words *fractional part*, we have,

8 is what fractional part of 10?

Solving as an example under General Problem II in fractions, we have,

8 is $\frac{4}{5}$ of 10. Since per cent, or hundredths, is asked for, $\frac{4}{5}$ must be reduced to 100ths. $\frac{4}{5} = \frac{80}{100}$, or .80, or 80 %.

4. 7 is what per cent of 14?

Substituting *number of hundredths* for *per cent*, we have,

a. 7 is what number of hundredths of 14? It is the same as asking what fractional part 7 is of 14, and then reducing that fraction to 100ths.

b. 7 is what fractional part of 14? $\frac{1}{2}$ = how many 100ths? 7 is $\frac{50}{100}$, or 50 %, of 14.

PROBLEM III. **To find a number when a given per cent of it is known.**

5. 18 is 25 % of what number?

Substituting $\frac{25}{100}$, or $\frac{1}{4}$, for 25 %, we have,

a. 18 is $\frac{1}{4}$ of what number? 18 is $\frac{1}{4}$, or 25 %, of 72.

6. 14 is 3 % of what number?

Substituting $\frac{3}{100}$ for 3 %, we have,

b. 14 is $\frac{3}{100}$ of what number?

SOLUTION: If 14 is $\frac{3}{100}$ of some number, $\frac{1}{100}$ of that number is $\frac{1}{3}$ of 14, or $\frac{14}{3}$, and $\frac{100}{100}$ of the number is $100 \times \frac{14}{3} = \frac{1400}{3} = 466\frac{2}{3}$.

ORAL

Change the following examples in percentage to examples in fractions. Reduce any number of hundredths to lowest terms:

7. What is 10 % of 45? What is $\frac{10}{100}$ of 45? What is $\frac{1}{10}$ of 45? Give the third form of question, omitting the second, in the following:

8. What is 15 % of 50?

9. Find 75 % of 24.

10. Find 30 % of 50.

11. 9 is what per cent of 36? 9 is what fractional part of 36? *Ans.* $\frac{1}{4}$. $\frac{1}{4}$ of a number is how many hundredths or what per cent of it?

12. 4 is what per cent of 10?

13. 6 is what per cent of 8?

14. 12 is what per cent of 16?

15. 8 is 20 % of what number? 8 is $\frac{20}{100}$ of what number? 8 is $\frac{1}{5}$ of what number? *Ans.* 40.

Omit the second form of question in the following:

16. 16 is 25 % of what number?

17. 30 is 10 % of what number?

18. 18 is 30 % of what number?

19. 15 is 75 % of what number?

Table of Per Cents and Equivalent Fractions

To the Teacher. Have pupils examine each statement to see how the final fraction is found in each case, and then require them to give the equivalent fraction in its lowest terms for each per cent; thus, $50\% = \frac{1}{2}$; $10\% = \frac{1}{10}$; $\frac{4}{5} = 80\%$, $\frac{1}{12} = 8\frac{1}{3}\%$; $33\frac{1}{3}\% = \frac{1}{3}$. Place the per cents on the board, and as you point to each, have pupils give the equivalent fraction from memory. The fractions should be placed on the board, and pupils required to give the equivalent per cents.

$50\% = \frac{50}{100} = \frac{1}{2}$ $\qquad$ $75\% = \frac{75}{100} = \frac{3}{4}$

$25\% = \frac{25}{100} = \frac{1}{4}$ $\qquad$ $5\% = \frac{5}{100} = \frac{1}{20}$

$4\% = \frac{4}{100} = \frac{1}{25}$

$10\% = \frac{10}{100} = \frac{1}{10}$

$20\% = \frac{20}{100} = \frac{1}{5}$

$30\% = \frac{30}{100} = \frac{3}{10}$

$40\% = \frac{40}{100} = \frac{2}{5}$

$60\% = \frac{60}{100} = \frac{3}{5}$

$70\% = \frac{70}{100} = \frac{7}{10}$

$80\% = \frac{80}{100} = \frac{4}{5}$

$90\% = \frac{90}{100} = \frac{9}{10}$

$15\% = \frac{15}{100} = \frac{3}{20}$

$35\% = \frac{35}{100} = \frac{7}{20}$

$45\% = \frac{45}{100} = \frac{9}{20}$

$55\% = \frac{55}{100} = \frac{11}{20}$

$8\frac{1}{3}\% = \frac{8\frac{1}{3}}{100} = \frac{25}{300} = \frac{1}{12}$

$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$

$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}$

$62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$

$87\frac{1}{2}\% = \frac{87\frac{1}{2}}{100} = \frac{175}{200} = \frac{7}{8}$

$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \frac{1}{3}$

$66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{200}{300} = \frac{2}{3}$

$16\frac{2}{3}\% = \frac{16\frac{2}{3}}{100} = \frac{50}{300} = \frac{1}{6}$

$6\frac{2}{3}\% = \frac{6\frac{2}{3}}{100} = \frac{20}{300} = \frac{1}{15}$

$6\frac{1}{4}\% = \frac{6\frac{1}{4}}{100} = \frac{25}{400} = \frac{1}{16}$

Oral

Give the per cents of the number at the right on the same line:

1. 20%, 25%, 10%, $33\frac{1}{3}$%, $16\frac{2}{3}$%, $66\frac{2}{3}$% of 30

Solution: 20% of 30 is $\frac{1}{5}$ of 30, or 6. 10% of 30 is $\frac{1}{10}$ of 30, or 3.

2. 10%, 30%, 40%, 60%, 80%, 90%, 50% of 25

3. $6\frac{1}{4}$%, $87\frac{1}{2}$%, $37\frac{1}{2}$%, $62\frac{1}{2}$%, $66\frac{2}{3}$%, 25% of 24

4. $8\frac{1}{3}\%$, $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, $66\frac{2}{3}\%$, 75%, 40%, 70% of 36

Give the numbers of which the number at the left in each line is a given per cent:

5. 8 is 20%, 25%, 10%, $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, of what number?

SOLUTION: 8 is 20%, or $\frac{1}{5}$, of 40.

6. 12 is $8\frac{1}{3}\%$, 75%, 40%, 60%, 80%, $37\frac{1}{2}\%$, of what number?

7. 24 is $66\frac{2}{3}\%$, $33\frac{1}{3}\%$, $62\frac{1}{2}\%$, $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, of what number?

8. $\frac{3}{4}$ is 20%, 25%, 30%, $37\frac{1}{2}\%$, 10%, $12\frac{1}{2}\%$, of what number?

Give the per cents the number at the left in each line is of the numbers at the right in the same line:

9. 8 is what per cent of 16? 20? 12? 10? 24? 32? 48?

SOLUTION: 8 is $\frac{1}{2}$ of 16, or 50% of 16.

10. 6 is what per cent of 8? 10? 9? 12? 15? 20? 36?

11. 12 is what per cent of 15? 16? 18? 20? 24? 32? 60?

12. $\frac{1}{4}$ is what per cent of $\frac{1}{2}$? $\frac{3}{4}$? $\frac{3}{8}$? $\frac{5}{12}$? $\frac{2}{3}$? $\frac{5}{6}$? $\frac{3}{2}$?

PROBLEMS

ORAL

85. Change the following problems in percentage into problems in fractions; the problems in fractions to problems in percentage, and solve each as a problem in fractions:

1. A boy bought a knife for $60¢$, which was 50% of his money. How much money had he?

Changed: A boy bought a knife for $60¢$, which was $\frac{1}{2}$ of his money. How much money had he?

2. There were 30 pupils in school on Monday. This number was $66\frac{2}{3}\%$ of the total enrollment. How many pupils were enrolled?

3. A boy's age is 20 % of his father's age. If the boy is 6 years old, how old is his father?

4. William is $\frac{2}{3}$ as old as his sister. William's age is 14 years. How old is his sister?

Changed: William is $66\frac{2}{3}\%$ as old as his sister. William's age is 14 years. How old is his sister?

5. A person who owns $\frac{2}{5}$ of a business sells $\frac{3}{4}$ of his share for $2100. What is the entire value of the business?

6. A man earns $30 a month. His expenses for the year are $240. What per cent of his earnings does he save?

7. A boy earns $6 a week, and pays $4 per week for living expenses. What part of his weekly wages does he save?

8. A girl saves $1.50 a week. Her savings are 75 % of her wages. How much does she receive per week?

9. If cloth is bought at 50 ¢ a yard and sold at 60 ¢ a yard, the gain is what part of the cost? The selling price is what fraction of the cost?

10. A produce dealer sells corn at 60 ¢ a bushel, and gains $\frac{1}{5}$ of what it cost him. What is the cost per bushel?

11. A farmer sold his farm for $50 an acre, and gained 25 % of the cost. How much did it cost him per acre?

12. A second-hand book is sold for 20 % less than it cost. If it is sold for 40 ¢, what is the cost?

13. If a milkman sells milk at 7 ¢ a quart, which cost 5 ¢, what per cent of the cost does he gain?

14. What is the cost of a horse, if a gain of $20 is made by selling it for 25 % more than it cost?

15. John and James receive $50 for doing a piece of work. John does $\frac{2}{3}$ as much work as James. How much money should each receive? Both receive what fraction of James's share?

16. A and B share $100. A receives 25 % as much as B. How much does each receive? What per cent of B's share do both receive?

REVIEW

86. **1.** What is a fractional unit? What is a fraction?

2. What are the terms of a fraction? Define each term.

3. Define proper fraction; improper fraction; mixed number.

4. What is meant by the reduction of a fraction?

5. How is a fraction reduced to a fraction of a higher denomination? Illustrate.

6. How is a fraction reduced to its lowest terms? Illustrate.

7. How is a whole number reduced to a fraction? Illustrate.

8. Illustrate the reduction of a mixed number to an improper fraction, and give the rule.

9. Illustrate the reduction of an improper fraction to a mixed number, and give the rule.

10. Reduce $\frac{5}{8}$ and $\frac{7}{12}$ to equivalent fractions having the least common denominator, and give the rule for reducing fractions to equivalent fractions having the least common denominator.

11. Add $\frac{5}{6}$ and $\frac{3}{4}$, and give the rule for addition of proper fractions.

12. Add $3\frac{2}{3}$ and $4\frac{3}{5}$, and give the rule for addition of mixed numbers.

13. Give the rule for subtracting one fraction from another. Illustrate.

14. In any fraction whose denominator is 4, the value of the numerator is 4 times the fraction. In the fraction $\frac{3}{4}$, the value of 3 is $\frac{1}{4}^2$. $\frac{1}{4}^2$ is 4 times $\frac{3}{4}$.

Show that in $\frac{2}{3}$ the value of the numerator is 3 times the fraction.

Give other examples to show that in any simple fraction, the numerator is as many times the fraction as there are units in the denominator.

How may a fraction be multiplied by a whole number? Illustrate.

How may a fraction be divided by a whole number? Illustrate.

If a multiplier is used that is 4 times the correct multiplier, the product is 4 times the correct product. What must be done to the first product to get the correct product?

15. In multiplying by a fraction, if we multiply by its numerator, the product obtained is as many times the correct product as its denominator is times 1; therefore this product must be divided by the denominator to secure the correct product.

16. Show the application of the statements in 15 in multiplying $\frac{5}{7}$ by $\frac{3}{4}$.

17. If a divisor is used that is 4 times the correct divisor, the quotient obtained is $\frac{1}{4}$ of the correct quotient. What must be done to the first quotient to get the correct quotient?

18. In dividing by a fraction, if we divide by its numerator, the quotient obtained is such a part of the correct quotient as 1 is a part of the denominator of the fraction. Therefore the first quotient must be multiplied by the denominator of the fraction to secure the correct quotient.

19. Show the application of the statements in 18 in dividing $\frac{5}{7}$ by $\frac{3}{4}$, and that it is the same as inverting the divisor and proceeding as in multiplication.

20. Give an example in which it is desired to find a fractional part of a number. Give the rule for finding a fractional part of a number. Change your example to an example in percentage and analyze it.

21. Give an example in which it is desired to find what fraction one number is of another. Give the rule for finding what fraction one number is of another. Change your example to an example in percentage and analyze it.

22. 8 is what fraction of 20? State the general problem that this example illustrates. Give the rule for solving examples coming under this general problem.

Change the problem to a question in percentage and analyze it.

DECIMAL FRACTIONS

87. A **decimal fraction** is a fraction whose denominator is 10 or some power of ten, expressed or merely indicated; thus, $\frac{3}{10}$, .3, $\frac{14}{100}$, .14, are decimal fractions.

Decimal fractions are usually called **decimals**.

Each decimal whose denominator is not expressed is preceded by a period called the **decimal point**; thus, .3, .025.

A **mixed decimal** is one having an integral and a decimal part; as, 12.75.

By **decimal places** we mean the number of figures to the right of the decimal point; thus, 2.06 has two decimal places; .0764, four.

SYSTEM OF DECIMAL NOTATION

Study Recitation

88. In integral or whole numbers, the value of a figure in any order is $\frac{1}{10}$ the value of the same figure in the next higher order, and 10 times the value of the same figure in the next lower order. In the following numbers, show that the value of any figure is $\frac{1}{10}$ the value of the figure in the next higher order, and 10 times the value of the figure in the next lower order:

1111 2222 3333 4444 5555 9999

In whole numbers, the name of the lowest order is units, or ones. In the number 1111, $\frac{1}{10}$ of the value of the figure in

the fourth order is 1 hundred, or the value of the figure in the third order; $\frac{1}{10}$ of the value of the figure in the third order is 1 ten, or the value of the figure in the second order; $\frac{1}{10}$ of the value of the figure in the second order is 1 unit, or the value of the figure in the first order; $\frac{1}{10}$ of the value of the figure in the first order is $\frac{1}{10}$. As a decimal it is written without the denominator by placing a decimal point at the right of the units' order and then writing 1 at its right; thus, 1111.1.

The name of the *first* order at the right of the decimal point is *tenths*. The value of .1 is $\frac{1}{10}$. $\frac{1}{10}$ of the value of 1 in tenths' order is $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, and is written at the right of the 1 in tenths' order; thus, .11. The name of the *second* order at the right of the decimal point is *hundredths*. $\frac{1}{10}$ the value of 1 in hundredths' order is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$, and is written at the right of the 1 in hundredths' order; thus, .111. The name of the *third* order at the right of the decimal point is *thousandths*.

In the same way it may be shown that $\frac{1}{10}$ of $\frac{1}{1000}$ is $\frac{1}{10000}$, that it is written at the right of the thousandths' order, and that the *fourth* order at the right of the decimal point is *ten-thousandths*.

The name of the *fifth* order is *hundred-thousandths*.

The name of the *sixth* order is *millionths*.

The name of the *seventh* order is *ten millionths*.

To the Teacher. The names of still lower orders may be given if the teacher so desires.

It will be seen that in writing decimal fractions, by using the decimal point, the fractions may be written in the same manner as whole numbers are written, the denominator being indicated by the number of places at the right of the decimal point.

Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	
$\frac{1}{10}$ =	.1						1 place or order is required to write tenths.
$\frac{1}{100}$ =	.0	1					2 places or orders are required to write hundredths.
$\frac{1}{1000}$ =	.0	0	1				3 places or orders are required to write thousandths.
$\frac{1}{10000}$ =	.0	0	0	1			4 places or orders are required to write ten-thousandths.
$\frac{1}{100000}$ =	.0	0	0	0	1		5 places or orders are required to write hundred-thousandths.
$\frac{1}{1000000}$ =	.0	0	0	0	0	1	6 places or orders are required to write millionths.
$\frac{1}{10000000}$ =	.0	0	0	0	0	0	1 7 places or orders are required to write ten-millionths.

The number of places required to write a decimal is the same as the number of zeros in its denominator.

It is rarely necessary to use decimal orders beyond millionths.

89. To read decimals.

RULE: *Read the decimal as though it were a whole number and give it the name of the right-hand order.*

1. Read .21567. The name of the right-hand order is hundred-thousandths. As a whole number, we read: Twenty-one thousand five hundred sixty-seven. Adding the name of the right-hand order, we read: Twenty-one thousand five hundred sixty-seven hundred-thousandths.

2. Read .0032. The name of the right-hand order is ten-thousandths. The decimal is read: Thirty-two ten-thousandths.

In reading a whole number and decimal combined, as, 156.27, read the whole number first; add the word "and," and then read the decimal, thus: One hundred fifty-six and twenty-seven hundredths.

Read the following. In examples 3–6 read down; in examples 7–18 read from left to right.

	3.	4.	5.	6.
7.	.5	.27	26.9	2.03
8.	.24	.9	900.064	6.3
9.	.06	.376	32.0007	7.83
10.	.325	$2.8\frac{1}{2}$ *	.9006	.0064
11.	.025	19.002	.7	.0000968
12.	.74	4.36463	.86076	.08
13.	.0004	86.00006	986.27	.001
14.	.4067	14.360076	$124.004\frac{1}{3}$ †	1.00032679
15.	875.64	800.08	9807.007	5647.0276
16.	3907.875	25.025	2000.0002	40.045
17.	1007.00107	$208.8\frac{1}{2}$	$560.37\frac{1}{2}$	5.0005
18.	2018.0001607	$563.005\frac{1}{2}$	$21708.006\frac{1}{4}$	27.003

90. To write decimals.

RULE: *Find the difference between the number of figures required to write the numerator and the number of zeros in the denominator. Write that number of zeros at the right of the decimal point and follow with the numerator.*

1. Write 56 thousandths. The numerator is 56; the denominator is 1000. How many figures are there in the numerator? How many zeros in the denominator? How many zeros must be placed at the right of the decimal point before the numerator? At the right of the decimal point write the required number of zeros and the numerator; thus, .056.

* Read 2 and $8\frac{1}{2}$ tenths. † Read 124 and $4\frac{1}{3}$ thousandths.

2. Write three thousand two hundred fifteen and 27 ten-thousandths. Write the whole number and place the decimal point at the right. How many figures are there in the numerator? How many zeros in the denominator? How many zeros must be placed at the right of the decimal point, before the numerator? Write them; follow with the numerator. The mixed decimal given is written 3215.0027.

In writing decimals, place the decimal point first and write from left to right. If there are more places in the decimal than there are figures in the numerator, write the necessary zeros before writing the numerator. This is important in writing decimals for addition and subtraction, in order to be sure that units of the same kind are in the same column.

ORAL

1. How many figures are required to write thousandths? Tenths? Millionths? Ten-thousandths? Hundredths? Ten-millionths? Hundred-thousandths? Hundred-millionths?

2. Numerate, or name, the decimal orders from the decimal point to the right, to the eighth order.

3. A decimal has four places. What is its denominator? Its denomination?

4. Give the denomination of a decimal of 3 places; 5 places; 2 places; 6 places; 4 places; 1 place.

5. Give the denominations of the following decimals, and then read them: .475; .0361; .00045; .05; .6; .275046.

WRITTEN

Write in decimal notation:

6. Three hundred six ten-thousandths.

7. Seventy-three and eight hundred-thousandths.

8. Nine hundred six and forty-three millionths.
9. Seven ten-thousandths.
10. 830 and 46 hundredths.
11. 47 thousandths.
12. $8\frac{2}{5}$ hundredths.
13. 834 hundred-thousandths.
14. 400 and 6 hundredths.
15. 1 and 15 millionths.
16. $\frac{7}{10}$, $\frac{82}{1000}$, $\frac{70}{10000}$, $\frac{5}{100}$.
17. Nine hundred eight thousand, and two thousandths.
18. Nine thousand, and nine ten-thousandths.
19. 864 million, and 84 thousandths.
20. 300 million 64, and 64 hundredths.
21. $\frac{83}{1000000}$, $\frac{7}{1000}$, $\frac{20}{10000}$.
22. $\frac{7}{1000}$, $\frac{83}{100}$, $\frac{65}{1000}$.
23. $\frac{875}{1000000}$, $96\frac{3}{10}$, $\frac{1}{100000}$.
24. Seven hundred four thousand one, and one thousandth.
25. Ten thousand, and four ten-thousandths.
26. Forty-five million, and forty-five thousandths.
27. 798 million 98, and 98 hundredths.

ADDITION AND SUBTRACTION OF DECIMALS

91. 1. Add: .27, 6.05, 3.756, .326.

$$\begin{array}{r} .27 \\ 6.05 \\ 3.756 \\ .326 \\ \hline 10.402 \end{array}$$

2. From 2.764 take .081.

$$\begin{array}{r} 2.764 \\ .081 \\ \hline 2.683 \end{array}$$

Read, write units of the same order in the same columns, and perform the required operations:

3. $3.26 + 83.08 + .9 + .026 + .07 + 3.0072 + 47.8305$.

4. Subtract 86.0764 from 193.08696.

5. Find the sum of .36, 3.7, 86.03, .476, 8.304, .0005.

6. Find the difference between 6.83 and 4.7638.

7. Add 7.5, .83, .936, 90., 8.7506, 12.065, 16.007.

8. From 78.00067 take 24.999999.

9. From three and eight tenths take sixty-four thousandths.

MULTIPLICATION OF DECIMALS

Study Recitation

92. 1. $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$; $.7 \times .3 = .21$

2. $18 \times \frac{2}{10} = 3\frac{6}{10}$; $18 \times .2 = 3.6$

3. $\frac{3}{10} \times \frac{7}{100} = \frac{21}{1000}$; $.3 \times .07 = .021$

4. $\frac{3}{100} \times \frac{4}{100} = \frac{12}{10000}$; $.03 \times .04 = .0012$

Compare the sum of the decimal places in the multiplicand and multiplier with those in the product. Compare the number of decimal places in each product with the number of 0's in the denominator of each product. Formulate a rule for pointing off in the product. Give the reason for your rule.

Written

5. Multiply 2.456 by .24.

$$\begin{array}{r} 2.456 \\ .24 \\ \hline 9824 \\ 4912 \\ \hline .58944 \end{array}$$

Multiply as in integers. The sum of the decimal places in the multiplier and multiplicand is 5; hence, count from the right 5 places and put the decimal point there.

6. Multiply .00176 by .015.

$$\begin{array}{r} .00176 \\ .015 \\ \hline 880 \\ 176 \\ \hline .00002640 \end{array}$$

Multiply as before. The sum of the decimal places in the multiplier and multiplicand is 8; there are but four figures in the product; hence, 4 zeros must be prefixed to the product obtained by the multiplication.

ORAL

Without multiplying, give the number of places to be pointed off in the products of the following:

7. $3 \times .4$; $.5 \times 4$
8. $.12 \times .2$; $.12 \times .2$
9. $.04 \times .3$; $.06 \times .04$
10. $1.2 \times .2$; $.2 \times 6$
11. $.07 \times 11$; $.11 \times .07$
12. $.012 \times .3$; $.012 \times .003$
13. $.016 \times .02$; $160 \times .0002$
14. $1.5 \times .04$; $1.5 \times .00004$
15. 14.2×2; $1.42 \times .04$
16. $.151 \times .03$; $1510 \times .0003$
17. $.7 \times 10$; $.07 \times 10$
18. 7.8×100; $.006 \times 10$
19. $.008 \times 100$; $.07 \times 1000$
20. $.87 \times 10$; $.87 \times 1000$

WRITTEN

Find the products:

21. $402.6 \times .0091$
22. $7.265 \times .018$
23. 30.007×7.25
24. 1907.06×32.06
25. $90.075 \times .756$
26. 2700×2.003
27. 6.706×230
28. $97.063 \times .00075$
29. $\$87.58 \times 1.756$
30. $\$126.05 \times .0839$
31. 7.763×10
32. $.3576 \times 100$
33. $.067 \times 1000$
34. $.00763 \times 100$
35. $.0000906 \times 1000$
36. $7.83 \times 10{,}000$

RULE: *To multiply a decimal by* 10, 100, 1000, *etc., move the decimal point as many places to the right as there are zeros in the multiplier, annexing zeros if necessary.*

DIVISION OF DECIMALS

Study Recitation

93. **1.** 1.28 } Factors
.012 }
.01536 Product

In each of the products in examples 1, 2, and 3, the number of decimal places is equal to the number of decimal places in both of the factors.

2. 1.23 } Factors
.04 }
.0492 Product

In example 1 there are 5 places in the product and 3 places in one of the factors. How many places are there in the other factor?

3. 15.6 } Factors
2.12 }
312
156
312
33.072 Product

In example 2 there are 4 places in the product and 2 places in one of the factors. How many places are there in the other factor?

In example 3 there are 3 places in the product and 1 place in one of the factors. How many places are there in the other factor?

In multiplication of decimals, every product has as many decimal places as the number of decimal places in both its factors.

If the number of decimal places in a product and in one factor are known, how can you find the number of decimal places in the other factor?

In division the dividend is always the product of the factors, — the divisor and quotient.

The dividend has as many decimal places as ——. Complete the statement.

The number of decimal places in the quotient is found how? Why?

Give rule for pointing off the quotient in division of decimals.

How many decimal places are there in the dividend? In the divisor? In the quotient?

4. .012).01536 5. .04).0492 6. 2.12)33.072

When the divisor is a whole number the quotient has the same number of decimal places as the dividend. The divisor may always be made a whole number by moving the decimal point to the right of its lowest order. When this is done, the decimal point in the dividend must be moved the same number of places to the right. Place the decimal point in the quotient below the decimal point in the dividend in short division, and above it in long division, before the division is begun. If this plan is followed, there is little likelihood of error in pointing off the decimal in the quotient.

ILLUSTRATIONS:

7. .012).01536

,012.)‚015.36
1.28

First step. Make the divisor a whole number by moving the decimal point to the right of the 2. Move the decimal point in the dividend 3 places to the right, placing it to the right of 5. In order to avoid confusion as to the place of the points, extend the original decimal points downward into a short line, as shown in the illustration. The divisor is now 12, the zero at its left not affecting its value. Place the decimal point for the quotient under the one in the dividend, before beginning the division, as shown.

Second step. The first partial dividend is 15; 12 in 15 gives 1 for the first quotient figure, which is placed under 5, the last figure in the first partial dividend; the remaining figures in the quotient are placed under the successive figures in the dividend.

The first step is the moving of the decimal points in divisor and dividend, extending the original points into lines, and placing the decimal point for the quotient under the new decimal point in the dividend.

The second step is the division, each quotient figure being written under the last figure of the partial dividend used to produce it.

In the actual work, the numbers need be written but once.

8. $2,12.\overline{)33,07.2}$ (15.6)

212
1187
1060
1272
1272

Example 8 as first written was 2.12)33.072. Without rewriting it, the decimal points were moved two places to the right in divisor and dividend, the first points extended downward into a short line, the decimal point for the quotient placed above the new point in the dividend, and the division performed. Each figure in the quotient is placed above the last figure of the partial dividend used to produce it.

9. What is the effect of moving the decimal point 1 place to the right in divisor and dividend? What, of moving it 2 places to the right? What, of moving it 3 places to the right?

10. Why is the quotient not affected by moving the decimal points the same number of places to the right in both divisor and dividend?

11. $.4\overline{).00136}$

$4.\overline{),0.0136}$

.0034

The following explanation may also be used. After moving the decimal points, it is required to find $\frac{1}{4}$ of .0136. $\frac{1}{4}$ of 0 tenths is 0, $\frac{1}{4}$ of 1 hundredth is 0, with 1 hundredth remaining. 1 hundredth = 10 thousandths; 10 thousandths and 3 thousandths = 13 thousandths; $\frac{1}{4}$ of 13 thousandths is 3 thousandths, with 1 thousandth remaining. 1 thousandth = 10 ten-thousandths. 10 ten-thousandths + 6 ten-thousandths = 16 ten-thousandths; $\frac{1}{4}$ of 16 ten-thousandths is 4 ten-thousandths.

12. $.008\overline{)25}$

$,008.\overline{)25,000.}$

It is required to divide the whole number, 25, by .008.

A decimal point is not usually written at the right of a whole number, but it may be regarded as having its place at the right of units.

When there are not as many decimal places in the dividend as the number of decimal places in the divisor, the decimal point must be moved to the right, the vacant places being filled with zeros.

EXAMPLES FOR PRACTICE

94. WRITTEN

1. $2.36 \div 2$	12. $.0025 \div 50$	23. $\$27.04 \div 2.42$
2. $.676 \div 6$	13. $.0782 \div .04$	24. $\$.306 \div .006$
3. $.07521 \div 3$	14. $.3276 \div 1.2$	25. $\$.316 \div 4.1$
4. $.00176 \div 4$	15. $.0804 \div .22$	26. $3 \div 8$
5. $7.64 \div .6$	16. $321.6 \div .021$	27. $4 \div 7$
6. $.0728 \div .8$	17. $3. \div .125$	28. $7 \div 9$
7. $.0237 \div .4$	18. $7. \div 8$	29. $5 \div 8$
8. $7.784 \div .12$	19. $24. \div 290.7$	30. $1 \div 12$
9. $27.076 \div .11$	20. $3.6 \div 90$	31. $2 \div 17$
10. $.045 \div 900$	21. $.78 \div 8.02$	32. $5 \div 28$
11. $\$8 \div \$.06\frac{1}{4}$	22. $\$9.24 \div .08$	33. $1 \div 16$

TO THE TEACHER. Pupils should be able not only to work the examples with book in hand, but they should also be able to write the numbers for division from the teacher's dictation.

95. To divide a number by 10 or any power of 10.

Multiply by moving the decimal point:

1. .27 by 10
2. .045 by 100
3. 2.006 by 1000
4. .15 by 1000

A number is multiplied by 10 or any power of 10 by moving the decimal point as many places to the right as there are zeros in the multiplier, zeros being annexed if necessary.

5. Divide each of the above products by the multiplier used in producing it, but without going through the process of division.

6. Make and learn a rule for dividing a number by 10 or any power of 10.

Write the quotient of each of the following numbers divided by 10; by 100; by 1000:

7. 27.5	9. 198.7	11. $462.5\frac{1}{2}$
8. 4.56	10. 27.56	12. $271.3\frac{1}{4}$

REDUCTION OF DECIMALS TO COMMON FRACTIONS

Study Recitation

96. Reduce:

1. $.4 = \frac{4}{10} = \frac{2}{5}$
2. $.28 = \frac{28}{100} = \frac{7}{20}$
3. $.025 = \frac{25}{1000} = \frac{1}{40}$
4. $.05\frac{1}{2} = \frac{5\frac{1}{2}}{100} = \frac{\frac{11}{2}}{100} = \frac{11}{2} \times \frac{1}{100} = \frac{11}{200}$
5. $.05\frac{1}{2} = .055 = \frac{55}{1000} = \frac{11}{200}$

Read the decimal in example 1.

What is its denomination? What is its denominator when it is written as a common fraction?

Read the decimal in example 3.

What denomination is expressed in the reading? How many units of this denomination are expressed in the reading?

What is the denominator of .025 when written as a common fraction? What is the numerator?

RULE: *Write the decimal as a common fraction of the denomination expressed in reading the decimal. Reduce the common fraction to its lowest terms.*

Reduce to common fractions:

6. .20; $.16\frac{2}{3}$	12. .50; .0375	18. $.87\frac{1}{2}$; $.05\frac{5}{9}$
7. $.6\frac{1}{4}$; $.08\frac{1}{3}$	13. .80; .96525	19. $.31\frac{1}{4}$; $.00\frac{1}{2}$
8. $.33\frac{1}{3}$; $.11\frac{1}{9}$	14. $.12\frac{1}{2}$; $.66\frac{2}{3}$	20. $.062\frac{1}{2}$; $.00\frac{5}{6}$
9. .25; .40	15. $.37\frac{1}{2}$; $.14\frac{2}{7}$	21. $.08\frac{1}{3}$; $.00\frac{3}{4}$
10. .075; .75	16. $.0012\frac{1}{2}$; $.06\frac{1}{4}$	22. $.43\frac{3}{4}$; $.58\frac{1}{3}$
11. .0005; .625	17. $.28\frac{4}{7}$; $.06\frac{2}{3}$	23. $.56\frac{1}{4}$; $.91\frac{2}{3}$

REDUCTION OF COMMON FRACTIONS TO DECIMALS

Study Recitation

97. This is a special case of the reduction of fractions to higher denominations. The denominator of a decimal is always 10 or some power of 10.

To reduce a common fraction to a decimal is to reduce the fraction to a fraction having 10 or some power of 10 for a denominator, writing the result as a decimal.

1. Reduce $\frac{3}{4}$ to hundredths.

1st SOLUTION: $1 = \frac{100}{100}$. $\frac{1}{4} = \frac{25}{100}$. $\frac{3}{4} = \frac{75}{100} = .75$.

2d SOLUTION: $3 = \frac{300}{100}$. $\frac{1}{4}$ of $\frac{300}{100} = \frac{75}{100} = .75$

3d SOLUTION: Since $\frac{3}{4}$ indicates the division of 3 by 4, example 1 may be stated: Divide 3 by 4, expressing the quotient as a decimal.

$$\begin{array}{r} 4)3.00 \\ \underline{.75} \end{array}$$

3 is not divisible by 4.

3 = 30 tenths. $\frac{1}{4}$ of 30 tenths = 7 tenths with 2 tenths remaining.

2 tenths = 20 hundredths. $\frac{1}{4}$ of 20 hundredths = 5 hundredths. $\frac{3}{4} = .75$.

As it is not always easy to tell to what decimal denomination a fraction is to be reduced before performing the operation, the process of division given last is the one ordinarily used.

To reduce a common fraction to a decimal.

RULE: *Place a decimal point at the right of the numerator, annex zeros, and divide by the denominator, pointing off as in division of decimals.*

Usually not more than three places are required in the decimal; if the division is not then exact, the remainder may be written as in division of whole numbers, or the + sign may be written at the right of the third figure in the quotient, to denote that the decimal is not exact, but somewhat larger.

2. Reduce $\frac{3}{7}$ to a decimal.

$$7)3.000 \qquad \text{or,} \quad 7)3.000$$
$$.428\frac{4}{7} \qquad .428^{+}$$

Reduce to decimals and memorize results. In these examples do not carry the decimal beyond the second place; give exact results by writing the remainders, if any, with the decimal:

3. $\frac{3}{4}$

4. $\frac{1}{2}$; $\frac{1}{4}$

5. $\frac{1}{5}$; $\frac{2}{5}$

6. $\frac{4}{5}$; $\frac{1}{6}$; $\frac{1}{3}$

7. $\frac{2}{3}$; $\frac{1}{7}$; $\frac{2}{7}$

8. $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$

9. $\frac{1}{9}$; $\frac{2}{9}$; $\frac{1}{12}$

10. $\frac{1}{16}$; $\frac{3}{16}$; $\frac{1}{15}$

11. $\frac{1}{20}$; $\frac{1}{40}$; $\frac{4}{25}$

Reduce to decimals of three places:

12. $\frac{7}{9}$; $\frac{5}{12}$; $\frac{6}{7}$

13. $\frac{9}{17}$; $\frac{25}{45}$; $\frac{11}{103}$

14. $\frac{27}{35}$; $\frac{19}{101}$; $\frac{77}{81}$

15. $\frac{9}{5}$; $\frac{26}{14}$; $\frac{93}{35}$

COST OF ARTICLES SOLD BY THE C, M, OR TON

98. To find the cost of articles when the number of units and price per hundred are given.

1. What is the cost of 7256 lb. of pork at \$4.50 per hundred pounds?

What is the unit? What is the given number of units? *Ans.* 7256.

What is the quotient of $7256 \div 100$? *Ans.* 72.56. To divide by 100, move the decimal point two places to the left.

What is the product of the cost per hundred by the number of hundreds? $72.56 \times \$4.50 = \$326.52$.

2. What is the cost of 1200 pounds of flour at \$3.50 per hundred pounds?

What is the first step required? What is the result?

What is the second step required? What is the result?

Give the rule. Make and solve three examples to which the rule applies.

99. To find the cost of articles when the number of units and price per thousand are given.

1. Change the rule called for in 2, exercise 98, p. 149, so as to make it apply to this case.

To divide by 1000, *move the decimal point three places to the left.*

2. Find the cost of 25575 bricks at $8.75 per M.

What is the first step required by the rule? What is the result? What is the second step required by the rule? What is the result? Give the rule. Make and solve three examples to which the rule applies.

100. To find the cost of articles sold by the ton or 2000 pounds.

Since there are 2 times 1000 pounds in a ton, the cost of 1000 pounds is $\frac{1}{2}$ the cost of a ton.

RULE I: *Divide the cost of* 1 *ton by* 2 *to find the cost of* 1000 *pounds, and apply the rule for finding cost of articles sold by the thousand.*

In a given number of pounds there are $\frac{1}{2}$ as many tons as there are times 1000 pounds, therefore

RULE II: *Divide the given number of pounds by* 1000, *and this result by* 2 *to find the number of tons. Find the product of the number of tons by the price per ton.*

1. What is the cost of 4425 pounds of coal at $8 per ton?

SOLUTION BY THE FIRST RULE: 2)$8 = $4, cost of 1000 pounds. 1000)4425 = 4.425, number of thousand. 4.425 × $4 = $17.70.

SOLUTION BY THE SECOND RULE: 1000)4425 = 4.425. 2)4.425 = 2.2125, number of tons. 2.2125 × $8 = $17.70.

2. How much are 8250 pounds of hay worth at $7.50 per ton?

Solve by the first rule, and state each step, giving the reason for it.

3. Make and solve three examples, solving each of them by both rules.

PROBLEMS

101. WRITTEN

1. A has 5 fields containing respectively 12.625 acres, 30.75 acres, 4.4 acres, 190.375 acres, and 12.125 acres. How many acres are there in the five fields?

2. From 24.875 tons of coal I sell 7.9 tons. How many tons remain?

3. A has 46.05 tons of hay; B has 2.375 tons less than A. How many tons of hay have A and B?

4. Find the cost of 24.375 yd. of cloth at $1.25 a yard.

5. If a ton of iron ore yields .65 of a ton of pure iron, how much iron will 376.48 tons yield?

6. James earned $38.65 and John earned $2.75 more than .40 as much as James. How much did John earn?

7. How far will a locomotive go in 6 hr., at the rate of 24.75 miles an hour, if 2 stops of 7.5 min. each are made?

8. A farmer exchanged 84.75 bushels of potatoes at $.45 a bushel for 450 pounds of sugar at $.06¼ a pound, and the balance in cash. How much cash did he receive?

9. J. R. Jones bought 24¾ yd. of silk ribbon at 12½¢ a yard, 12¼ yd. of alpaca at $1.20 a yard, 10½ yd. of silesia at 16¢ a yard, 2⅓ doz. buttons at 12½¢ a dozen, and 18 yd. of calico at 8¼¢ a yard. Write this out, foot, and receipt the bill.

10. A store valued at $12,000 is insured for .625 of its value. What is Mr. Smith's share of the insurance if he owns .375 of the store?

11. If the cost of any number of cords of wood and the number of cords are known, how can you find the cost of 1 cord? Illustrate.

The number of cords may be expressed as a whole number, 4 cords, a mixed number, $3\frac{3}{8}$ cords, a fractional number, $\frac{3}{4}$ cord, a decimal, .8 cord, or a mixed decimal, 2.2 cords.

12. If 4 cords of wood cost $11, what is the cost of 1 cord?

13. Make four examples by substituting for 4 in the last example each of the following numbers: $3\frac{3}{8}$, $\frac{3}{4}$, .8, 2.2. Solve them, showing that they are all solved like the 12th.

14. Make and solve similar examples in the same way, using other numbers and other expressions than "cords of wood."

Of the five different forms of numbers expressed in the statement preceding example 12, the first is a whole number; the other four may be expressed as common fractions; thus, $3\frac{3}{8} = \frac{27}{8}$, $\frac{3}{4}$, $.8 = \frac{4}{5}$, $2.2 = \frac{11}{5}$.

Analyze each of the examples called for under example 13, treating each number as a common fraction, and using the following form of analysis:

If $3\frac{3}{8}$ cords of wood cost $11, how much will 1 cord cost?

SOLUTION: $3\frac{3}{8} = \frac{27}{8}$. If $\frac{27}{8}$ cords of wood cost $11, $\frac{1}{8}$ of a cord costs $\frac{1}{27}$ of $11, or $1. If $\frac{1}{8}$ of a cord of wood costs $1, $\frac{8}{8}$ of 1 cord costs 3 times $1, or $3.

15. 6.5 yd. of cloth cost $3.25. What is the cost per yard?

16. How can you find the cost of 3.5 cords of wood from the cost of .4 of a cord? Suppose the cost of .4 of a cord to be $3.25, find the cost of 3.5 cords.

17. An engine pumped 37.75 barrels of 31.5 gallons each from a reservoir containing 31,500 gallons of water. How many barrels remained?

18. What is the cost of 850 fence posts at $7.50 per C?

SOLUTION: 100)850 × $7.50 = 8.5 × $7.50 = $63.75. *Ans.*
850 posts = 8.5 C; 8.5 times $7.50, the cost per C, gives $63.75.

19. Find the cost of 16375 bricks at $6.50 per M.

SOLUTION: 1000)16375 bricks = 16.375 M bricks.
16.375 × $6.50 = $106.44. *Ans.*

20. Find the cost of 5780 fence posts at $27 per C.

21. What is the cost of 3500 feet of lumber at $16.50 per M?

22. At $7.50 a ton, what is the cost of 4530 pounds of hard coal?

23. At $4.25 per ton, what is the cost of 6400 pounds of soft coal?

24. Bought of R. J. Green the following:

6000 lb. nut coal at $7.25 per ton.
4600 lb. soft coal at $4.50 per ton.
8750 lb. chestnut at $6.50 per ton.
4500 lb. screenings at $2.50 per ton.

Write out a receipted bill.

25. A contractor furnishes the following materials for the construction of a house:

32,600 bricks at $7.75 per M.
9800 ft. flooring at $18.50 per M.
12500 ft. lumber at $16 per M.
7200 shingles at $9.80 per M.
5400 laths at $.50 per C.

Find cost of the materials and make out bill.

26. A coal dealer sold to a lumber man 3 loads of coal weighing 4400 lb., 5775 lb., and 3800 lb. respectively. He took in exchange 3875 ft. of lumber at $16 per M, and 350 fence posts at $7.50 per C. If coal is $6.50 a ton, which is in debt to the other, and how much?

27. A sold B in exchange for potatoes at $.45 a bushel, 80 fence posts at $6 per C, and 450 ft. of lumber at $15 per M. How many bushels did B receive?

28. The product of four factors is 16.95. Three of the factors are 1.2, .25, and .113. What is the fourth factor?

29. Frame and solve two practical problems, each involving at least two operations with decimals.

RELATION OF DECIMALS TO COMMON FRACTIONS AND TO PERCENTAGE

Study Recitation

102. There are three general problems in decimals corresponding to the three general problems in fractions and percentage. They are:

PROBLEM I. **To find any decimal part of a number.**

PROBLEM II. **To find what decimal part one number is of another.**

PROBLEM III. **To find a number when a given decimal part of it is known.**

When the *decimal part* is expressed as hundredths, the problem is the same as the corresponding problem in percentage, except that the term *hundredths* is used instead of *per cent.*

It is often more convenient to express per cent as a decimal, and to perform the operation as in decimals, than to express it as a common fraction and to perform the operation as in common fractions.

103. GENERAL PROBLEM I. **To find any decimal part of a number.**

Study Recitation

1. A man has a farm of 125 acres; .15 of it is a wood lot. How many acres are there in the wood lot?

It is required to find .15 of 125 acres.

SOLUTION:

125 acres
.15
625
125
18.75 acres.

2. Substitute .125 for .15 in problem 1, and find .125 of 125 acres.

SOLUTION:

125 acres
.125
625
250
125
15.625 acres.

3. Substitute .7 for .15 in problem 1, and find .7 of 125 acres.

SOLUTION:

125 acres
.7
87.5 acres.

To find any decimal part of a number.

RULE: *Multiply the number by the decimal, according to the rule for multiplication of decimals.*

Problem 1 changed to a problem in percentage reads:

4. A man has a farm of 125 acres; 15% of it is a wood lot. How many acres are there in the wood lot?

It is required to find 15%, or .15, of 125 acres. Operation same as in problem 1.

Problems 2 and 3 may be changed to problems in percentage and solved in the same way, therefore,

To find any per cent of a number.

RULE: *Multiply the number by the rate per cent expressed as a decimal.*

5. Find 15% of 45.

SOLUTION: 15% of 45 is .15 of 45.
.15 of 45 is 15 times .01 of 45. .01 of 45 = .45.
15 times .45 = 6.75, or,

SECOND SOLUTION:
.15 of 45 is .01 of 15 times 45. 15 × 45 = 675.
.01 of 675 = 6.75

6. Find 27% of 36.
7. Find 13% of 128.
8. Find 24% of 56.
9. Find 75% of 86.
10. Find 10% of 75.
11. Find $12\frac{1}{2}$% of 126.
12. Find .125 of 205.
13. Find .007 of 165.
14. Find .08 of 24.
15. Find 12% of 3.8.
16. Find $37\frac{1}{2}$% of 15.6.
17. Find 25% of 275.
18. Find 3% of 125.
19. Find $87\frac{1}{2}$% of 160.

104. GENERAL PROBLEM II. **To find what decimal part one number is of another.**

Study Recitation

This is simply a case of reducing a common fraction to a decimal. It corresponds to the second general problem in fractions,—"To find what fractional part one number is of another."

When the decimal part one number is of another is to be expressed as hundredths, it corresponds to the second general problem in percentage,—"To find what per cent one number is of another."

1. 3 is what decimal of 4? Example in decimals.
   3 is what part of 4? Example in common fractions.
   3 is what per cent of 4? Example in percentage.

SOLUTION: 3 is $\frac{3}{4}$ of 4. $\frac{3}{4}$ reduced to a decimal = 4)3.00 = .75, therefore, 3 is .75, or 75%, of 4.

To find what decimal part one number is of another.

RULE: *Divide the number expressing the part by the number with which it is compared, continuing the division until the required number of decimal places is found.*

To find what per cent one number is of another.

RULE: *Divide the number expressing the part by the number with which it is compared, carrying the division to hundredths' place.*

To find what fractional part one number is of another.

RULE: *Make the number expressing the part, the numerator, and the number with which it is compared the denominator of a fraction, and reduce the fraction to its lowest terms.*

2. A man owns a farm of 125 acres; 18.75 acres of it is a wood lot. What per cent of the farm is the wood lot?

Apply Rule II. 18.75 is the part; 125 is the number with which the part is compared.

SOLUTION:

$$\begin{array}{r} .15 = 15\,\% \\ 125\overline{)18.75} \\ 125 \\ \hline 625 \\ 625 \\ \hline \end{array}$$

3. What per cent of 40 is 24?

SOLUTION:

$$\begin{array}{r} .60 = 60\,\% \\ 40\overline{)24.00} \\ 24\ 00 \\ \hline 0 \end{array}$$

On changing example 3 to an example in fractions, it reads: "What part of 40 is 24?" *Ans.* $\frac{24}{40} = \frac{3}{5}$.

On reducing $\frac{3}{5}$ to a decimal, the result is $\frac{3}{5} = .60 = 60\,\%$.

4. What decimal is 16 of 28? (Not more than 2 places required.)

SOLUTION: 16 is $\frac{16}{28}$ of 28. $\frac{16}{28}$ of a number, reduced to a decimal, equals .57$\frac{1}{7}$ of the number.

5. 7.5 is what decimal of 200?

6. 18.5 is what per cent of 740?

7. $12\frac{1}{2}$ is what per cent of 300?

Write $12\frac{1}{2}$ as a mixed decimal in performing the operation.

8. $\frac{1}{2}$ is what per cent of $\frac{3}{4}$?

Write the fractions as decimals in performing the operation. Perform also without reducing the fractions $\frac{1}{2}$ and $\frac{3}{4}$ to decimals.

9. .125 is what per cent of 2.5 %?

10. $.37\frac{1}{2}$ is what per cent of $.62\frac{1}{2}$?

11. What decimal of 16 is 6? What per cent?

12. What part of 18 is 16? What decimal? What per cent?

105. GENERAL PROBLEM III. **To find a number when a given decimal part of it is known.**

Study Recitation

When the given decimal is hundredths, the solution is simply a case of the application of the rule for General Problem III in percentage: "To find a number when a given per cent of it is known."

The solution of any example under this problem may always be regarded as an application of the rule for General Problem III in fractions: "To find a number when a given fractional part of it is known."

1. A wood lot of 18.75 acres is .15 of a man's farm. How many acres are there in the farm?

SOLUTION: The farm is 100 hundredths of itself; the wood lot is .15 of the farm. .01 of the farm is $\frac{1}{15}$ of 18.75 acres, or 1.25 acres. 100 hundredths of the farm = 100 × 1.25 acres = 125 acres.

Dividing a number by 15 and multiplying the product by 100 is the same as dividing the number by .01 of 15, or .15.

$$.15.\overline{)18.75.}\quad 125.$$

Problem 1, changed to a problem in percentage, reads:

2. A wood lot of 18.75 acres is 15% of a man's farm. How many acres are there in the farm?

SOLUTION: The number of acres in the farm is 100% of the farm. 18.75 acres = 15% of the farm; 1% of the farm = $\frac{1}{15}$ of 18.75 acres, or 1.25 acres. 100% of the farm = 100 × 1.25 acres, or 125 acres.

Dividing a number by 15, and multiplying the product by 100 is the same as dividing the number by .01 of 15, or .15.

$$.15.)\overline{18.75.}\quad 125.$$

It will be seen that problem 2 is solved in the same way as problem 1, that is, by dividing the known part, 18.75, by the given decimal .15.

To find a number when a given decimal part of it expressed in hundredths is known, or when a given per cent of it is known.

RULE: *Divide the number expressing the known part by the given decimal, or by the per cent, expressed as a decimal.*

3. In a spelling class there are 14 pupils, which is .35 of the pupils in the school. How many pupils are there in the school?

Analyze as a problem in decimals. Work without analyzing.

4. Make and solve five similar problems.

REVIEW

106. 1. Define decimal; mixed decimal.

2. What is the use of the decimal point in writing decimals?

3. The value of a unit in any order is —— the value of the same unit in the next lower order. The value of a unit in any order is —— the value of the same unit in the next higher order.

Fill the blanks and illustrate.

4. Show why the first order at the right of the decimal point is tenths; why the second order is hundredths.

5. Write a decimal of three places; write it as a decimal fraction with the denominator expressed. What is the relation of the number of places in a decimal to the number of places in its denominator?

6. How may any decimal be changed to a common fraction? Illustrate.

7. Annexing a zero to a decimal has what effect upon its denominator? Upon its denomination? Upon its value? Illustrate.

8. Cutting off a zero from the right of a decimal has what effect upon its denominator? Upon its denomination? Upon its value? Illustrate.

9. Moving the decimal point to the right has what effect upon the decimal? Illustrate.

10. Moving the decimal point to the left has what effect upon the decimal? Illustrate.

11. Moving the decimal point the same number of places to the right in both divisor and dividend has what effect upon each? Upon the quotient? Illustrate.

12. Give rule for writing decimals; for reading decimals.

13. Give rule for adding decimals; for subtracting decimals.

14. Give rule for multiplying decimals; for dividing decimals.

15. Give rule for changing a common fraction to a decimal. Illustrate. Analyze the operation.

16. How can you find the cost of articles sold by the hundred? Illustrate.

17. How can you find the cost of articles sold by the thousand? Illustrate.

18. How can you find the cost of articles sold by the ton? Illustrate.

19. Give the general problems in decimals, and in percentage corresponding to the following general problems in common fractions, and illustrate each.

I. To find any fractional part of a number.

II. To find what fraction one number is of another.

III. Knowing a fractional part of a number, to find the number.

20. Write five decimals of different denominations. Reduce them to the same denomination.

21. Why is $\frac{7}{10}$ a decimal fraction? Why is $\frac{5}{8}$ not a decimal fraction?

22. How may $\frac{5}{8}$ be changed to a decimal? What is done in making the change?

23. In 31.23, what is the value of the 3 at the right of the decimal point as compared with the 3 at the left of the decimal point?

PART II

DENOMINATE NUMBERS

107. A **denominate number** is a concrete number in which the unit of measure is established by law or custom.

A **simple denominate number** is one that is composed of units of only one denomination; as, 4 ounces, 6 hours, 8 feet, 20 pounds.

A **compound denominate number** is one that is composed of units of two or more denominations, each of which may be changed to the other; as, 4 bushels, 3 pecks; 2 feet, 3 inches; 3 gallons, 1 quart.

A **quantity** is anything that can be measured; as, money, grain, wood.

A **measure** is a standard, or fixed unit of some quantity; as, $1; 1 lb.; 1 in.

Measuring a quantity is the process of finding how many times the fixed unit of that quantity is contained in it.

Dollars and cents are measures of value; inches and feet, of length; ounces and pounds, of weight.

Usually not more than two denominations are used in stating the measure of any quantity; as, 2 pounds, 3 ounces; 6 feet, 2 inches.

In business transactions, one unit only is used, and this is the one in which the price of the quantity is quoted. To

illustrate: The butcher sells meat at a certain price per pound. He does not say, 3 pounds 12 ounces, but $3\frac{3}{4}$ pounds. The merchant says $8\frac{3}{4}$ yards of ribbon instead of 8 yards, 27 inches.

DRY MEASURE

108. Dry measure is used in measuring different kinds of vegetables, fruits, and other dry commodities. Some of these commodities, like grain, potatoes, fruits, and coal, are sometimes sold by weight.

TABLE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

NOTE. A bushel contains 2150.4 cubic inches, or $1\frac{1}{4}$ cubic feet, approximately. One quart, or $\frac{1}{32}$ of a bushel = 67.2 cubic inches.

LIQUID MEASURE

109. Liquid measure is used in measuring liquids.

TABLE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts, 8 pints, 32 gills = 1 gallon (gal.)

NOTES. 1. 1 gallon contains 231 cubic inches. 1 quart, or $\frac{1}{4}$ of a gallon, contains 57.75 cubic inches.

By comparing the number of cubic inches in a quart, Liquid Measure, with the number of cubic inches in a quart, Dry Measure, it will be seen that the latter is the larger by 9.45 cubic inches.

2. In measuring the capacities of cisterns, reservoirs, etc., $31\frac{1}{2}$ gal. make one barrel (bbl.) and 63 gal., a hogshead (hhd.); but in commerce the barrel and the hogshead vary in capacity.

ORAL

110. How many

1. pk. equal 6 bu.? 24 qt.? 8 bu.?

2. pt. equal 2 bu.? 72 qt.? 4 pk.?

3. qt. equal 2 gal.? 3 qt.? 5 gal. 1 qt.?

4. pt. equal 2 pk.? 7 qt. 1 pt.?

5. bu. equal 32 pk.? 37 pk.? 19 pk.?

6. gal. equal 2 bbl.? 2 bbl. 10 gal.? 2 hhd.?

7. When potatoes are selling at 20¢ a peck, what is the rate per bushel?

8. When apples are selling at 15¢ a half peck, what is the rate per bushel?

9. If milk is sold at 6¢ a quart, what is the value of 2 gal.? Of 2 gal. 3 qt.?

10. If chestnuts are worth 15¢ a quart, what is the price per peck?

11. If cream is sold at 80¢ per gallon, what is the cost of a pint?

12. What part of a gallon are 3 quarts? 2 quarts?

13. How many pints are there in a gallon? What part of a gallon are 4 pt.? 7 pt.?

14. How many quarts are there in a bushel? What part of a bushel are 7 qt.? 8 qt.? 9 qt.?

15. How many pints are there in a bushel? What part of a bushel are 3 pt.? 1 pt.?

16. How much will 4 gal. of vinegar cost at 5¢ a quart?

17. How much will 4 gal. 2 qt. of oil cost at 8¢ a quart?

18. How much will 5 gal. 3 qt. of oil cost at 10¢ a quart?

19. What is the cost of $4\frac{1}{2}$ gal. of sirup at 20¢ a quart?

20. The milkman sells 3 qt. 1 pt. of milk to one customer, and 3 pt. to another customer. How much does he sell to both?

21. What is the cost of 2 bu. 3 pk. of oats at 40¢ a bushel?

AVOIRDUPOIS WEIGHT

111. Avoirdupois weight is used in weighing most articles bought and sold by weight.

TABLE

7000 grains = 1 pound (lb.)
16 drams = 1 ounce (oz.)
16 ounces = 1 pound
100 pounds = 1 hundredweight (cwt.)
20 cwt. / 2000 pounds = 1 ton (T.)

NOTE. The denomination, dram, is not much used. Many scales do not show weight less than 1 oz. Some show divisions of the ounce, as $\frac{1}{2}$ oz. or $\frac{1}{4}$ oz.

NOTE. At the United States Custom House, and in weighing coal and iron at mines, the long ton of 2240 lb. is used.

The tables of Troy weight and Apothecaries' weight are so seldom used in ordinary business transactions, that they are not given here. They will be found in the Appendix.

112. TABLE OF MISCELLANEOUS WEIGHTS

200 pounds of pork = 1 bbl.
196 pounds of flour = 1 bbl.
100 pounds of nails = 1 keg
56 pounds of butter = 1 firkin

113. Most states have adopted the following:

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL

Oats,	32 lb.
Wheat,	60 lb.
Corn (shelled),	56 lb.
Corn on the cob,	70 lb.
Potatoes,	60 lb.
Rye,	56 lb.
Barley,	48 lb.
Clover seed,	60 lb.
Timothy seed,	45 lb.
Coal,	80 lb.

ORAL

114. 1. What is the value of $3\frac{1}{2}$ lb. of butter at 30¢ a pound?

2. What is the cost of $4\frac{3}{4}$ lb. of honey at 12¢ a pound?

3. How much will 12 oz. of tea cost at 60¢ a pound?

4. At 50¢ a bushel, what is the cost of 330 lb. of potatoes?

5. A load of hay weighs 3500 lb. Give its weight in tons.

6. How many pounds are there in $\frac{3}{4}$ of a ton? In $\frac{1}{2}$ of a ton?

7. How much heavier are 2 bu. of corn on the cob than 2 bu. of shelled corn?

8. How much will 2 bbl. of pork cost at 8¢ a pound?

9. How many pounds are there in 2 firkins of butter?

10. A hardware dealer buys 3 kegs of nails at $3 per keg, and sells them at 4¢ per pound. How much does he gain?

11. What is the weight of 1 pk. of wheat? Of 3 pk.?

12. What is the weight of 4 qt. of oats? Of 3 pk. of corn?

13. What part of a pound are 12 oz.? 6 oz.? 14 oz.? 8 oz.?

14. What part of a ton are 200 lb.? 1500 lb.? 500 lb.? 1250 lb.?

15. What is the value of 12 oz. of canary seed at 40¢ a pound?

16. Find the cost of $\frac{7}{8}$ lb. of pepper at 4¢ an ounce.

17. A teamster had a load of stone weighing $2\frac{3}{4}$ tons. How many pounds were there in the load?

18. Reduce 5 lb. 6 oz. to ounces.

19. Reduce 4 T. 653 lb. to pounds.

20. Reduce 16 cwt. 75 lb. to pounds.

MEASURE OF LENGTH

115. This measure is called Long Measure, or Linear Measure, and is used in measuring lengths or distances.

TABLE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft.,	= 1 rod (rd.)
320 rods 1760 yards 5280 feet	= 1 mile (mi.)

NOTE. The following measures are sometimes used: 4 in. = 1 hand, used in measuring the height of horses; 6 ft. = 1 fathom, used in measuring depths of water; 3 mi. = 1 league, used in measuring distances at sea; 40 rd. = 1 furlong.

Surveyors' Long Measure is a table not much used, therefore it is not given here, but will be found in the Appendix.

QUESTIONS ON UNITS USED IN MEASURING LENGTHS

116. 1. What is the unit used by the merchant in measuring cloth? By the carpenter in stating the size of a room? The height of a house? By the geographer in stating the height of a mountain? The distance between two cities? By the farmer in stating the length or breadth of a farm?

2. What are the dimensions of your arithmetic? The length, width, and height of the schoolroom? The width of the door? The height of the teacher's desk? The length of the school building? The length and width of the school yard? The distance to your home? The distance between the two rear wheels of a wagon?

TO THE TEACHER. — Each pupil should be provided with a ruler, and should be required to measure and give the length and width of the top of the desk; the height of the front edge of the desk from the floor; the height of the seat from the floor; the height and width of the windows and doors; the width and length of blackboards; the heights of different pupils. A yardstick should be provided for use in measuring yards. A cord 16½ ft. long should also be available.

These exercises should be given to make pupils familiar with different units of measurement, and skillful in estimating short distances. Each pupil should also fix in his mind two points in the locality which are one mile apart. This will serve him as a standard in estimating greater distances.

PROBLEMS

ORAL

117. 1. What part of a yard are 27 in.? 9 in.? 4 in.? 8 in.? 2 ft. 3 in.? 2 ft. 6 in.? 1 ft. 6 in.? 1 ft. 8 in.?

2. How many feet are there in a mile? In $\frac{1}{8}$ of a mile? In $\frac{1}{4}$ of a mile? How many rods are there in 66 ft.?

3. What would be the cost of picture molding around a room 14 ft. by 16 ft., at 10¢ per foot?

4. Jane had $\frac{8}{9}$ yd. of ribbon. After using $\frac{3}{4}$ yd., how many inches of ribbon had she left?

5. A milliner sold 27 in. from a piece of lace containing $8\frac{5}{8}$ yd. How many yards were left?

6. How many ribbons 9 in. in length may be cut from a bolt $9\frac{3}{4}$ yd. in length?

7. How many feet are there in 2 rd.? In 3 rd.? In 4 rd.?

8. How many rods are there in 1 mi.? In 3 mi.? In 2 mi.?

9. How many feet are there in 60 in.? In 98 in.?

10. There are four windows on one side of a room. Each window is 32 in. wide. What is its width in feet and inches? What is the combined width of the four windows?

11. In walking, the length of a man's step is 30 in. How many steps would he take in walking 10 ft.? 100 ft.?

12. If carpet is $\frac{3}{4}$ of a yard wide, how many strips will it take to carpet a room 18 ft. by 24 ft., if the strips are laid lengthwise of the room? If they are laid crosswise?

13. How many strips of wall paper 18 in. wide will be required to cover the walls of a room 18 ft. by 15 ft.? How many rolls will be required if the room is 8 ft. high?

NOTE. Wall paper is usually sold by the roll, each roll containing 8 yd. of paper $\frac{1}{2}$ yd. wide. Paper is laid on the wall in strips running up and down. As dealers do not sell less than a roll, a fractional part of a roll is regarded as an entire roll.

How many yards are there in the entire distance around the room? How many strips will it take to paper the room? What is the length of each strip? How many strips of this length will one roll of paper make? How many rolls of paper will be required?

RULE. *Find the distance in yards around the room, and reckon two strips for each yard. Divide this number of strips by the number of strips that can be made from a roll. The quotient is the number of rolls required.*

SURFACE OR SQUARE MEASURE

118. Square measure is used in measuring surfaces.

A **surface** is that which has length and breadth only.

An **angle** is the difference in direction of two lines.

If one straight line meets another straight line, so as to make two equal angles, each angle is a **right angle**. A right angle is formed by two lines meeting in a square corner.

A **square** is a plane surface having four equal sides and four right angles.

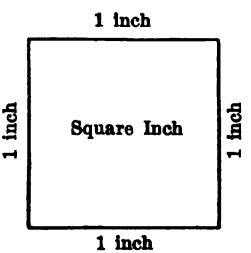
A **rectangle** is a plane surface having four sides and four right angles. The opposite sides are equal, but the length may be greater than the width.

TABLE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
30¼	square yards (272¼ sq. ft.)	= 1 square rod (sq. rd.)
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)
36	square miles or sections	= 1 township

A square inch is a plane surface one inch square.

A square foot is a plane surface one foot square.



TO THE TEACHER. Such a square should be drawn upon the blackboard, and divided into square inches, the number of square inches in the square foot ascertained, and the fact developed that the number of square inches in a square foot is found by multiplying the number of square inches in the row of squares on one side by the number of such rows.

1. Draw a square foot, and divide it into strips 1 in. wide. How many strips does it contain? How many square inches are there in each strip? How many square inches are there in a square foot?

2. Draw a rectangle 4 in. wide and 6 in. long. Divide it into inch squares. How many squares are there in the rectangle? How many square inches?

3. Draw a rectangle 8 in. wide and 12 in. long. How many strips 12 in. long and 1 in. wide does it contain? How many square inches are there in one strip? How many square inches are there in the rectangle? How may this number be obtained?

4. Draw another rectangle of the same size as the preceding one. How many strips 8 in. wide and 1 in. long does it contain? How many square inches are there in each strip? How many square inches are there in the rectangle? What is the unit of measurement? How can you find the area of rectangles?

A **square yard** is a plane surface one yard, or three feet, square.

TO THE TEACHER Such a square should be drawn upon the blackboard, and the number of square feet in its surface determined, and the mode of determining it found as in the case of the square foot.

A **square rod** is a plane surface one rod, or $16\frac{1}{2}$ feet, square, or $5\frac{1}{2}$ yards square.

TO THE TEACHER. Make clear to pupils that the number of square feet or square yards in a square rod is found in the same way as the number of square inches in a square foot or of square feet in a square yard.

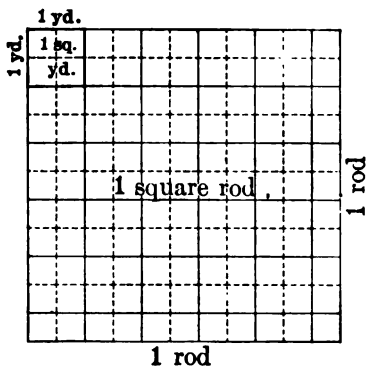
The unit of measurement in measuring surfaces is always a square. This square may be a square inch, a square foot, a square yard, etc.

What are the dimensions in yards of the square rod?

What are the dimensions in $\frac{1}{2}$ yards of the smallest of the squares? What are the dimensions in $\frac{1}{2}$ yards of the square rod?

The square $\frac{1}{2}$ yd. on each side is what part of the square yard?

How many of the $\frac{1}{2}$ yd. squares are there in the sq. rd.?



As there are 4 of the $\frac{1}{2}$ yd. squares in each sq. yd., how many square yards equal the whole number of $\frac{1}{2}$ yd. squares?

The dimensions of a surface are given in linear units. If the dimensions are in inches, the unit of measurement is the square inch, and the area or surface is given in square inches. If the dimensions are in feet, the area is square feet. If the dimensions are in yards, the area is square yards. If the dimensions are in rods, the area is square rods.

From the explanation already given of the mode of finding the number of square inches in a surface 1 ft. or 12 in. square, the number of square feet in a surface 1 yd. or 3 ft. square, it is seen that the number of square units in such a surface equals the product of the two numbers which give the dimensions in linear units; but what is really done is multiplying the number of square units on one side of the surface by the number of such units on one end of the surface. In actual work, the number of linear units in length is multiplied by the number of linear units in width, and the product is the number of square units whose sides are the linear units used in giving the dimensions.

"Multiplying feet by feet" does not give square feet, but multiplying together two numbers each representing feet, gives as product a third number representing square feet.

PROBLEMS

ORAL

119. 1. What is the area of the floor of a room 10 ft. wide and 12 ft. long?

2. What is the area of a blackboard that is 4 ft. wide and 16 ft. long? What is the unit of measurement in each case?

NOTE. Estimates for painting are by the square of 100 sq. ft., that is, a square surface 10 ft. on each side.

3. What is the cost of painting a solid board fence 10 ft. high and 150 ft. long at 50¢ per square?

4. The surface of a slate contains 96 sq. in.; the slate is 8 in. wide. How long is it?

5. What must be the width of a floor 6 yd. long in order to contain 30 sq. yd.? Give the width first in yards and then in feet.

6. A room is 9 ft. high and 12 ft. wide. How many square yards are there in the two ends?

If the room is 15 ft. long, how many square yards are there in the two sides? How many square yards are there in the ceiling? How many square yards are there in the walls and ceiling? How many square yards of carpet will it take to cover the floor?

7. How many square rods are there in half an acre? In $\frac{3}{4}$ of an acre? In $\frac{3}{8}$ of an acre?

8. How many acres are there in a square mile? In $\frac{1}{2}$ of a square mile? In $\frac{1}{8}$ of a square mile? In $\frac{3}{4}$ of a square mile? In $\frac{5}{8}$ of a square mile?

QUESTIONS ON UNITS OF MEASUREMENT IN SURFACES

120. 1. A book is 7 in. wide and 9 in. long. What is the unit used in expressing its surface?

2. If the floor of the schoolroom is 20 ft. wide and 30 ft. long, in what units should you express its area?

3. A field is 80 rd. long and 40 rd. wide. Its area is found in what units? These units may be changed to what other units?

4. A wall is 9 ft. high and 16 ft. long. What is its area? What is the unit of area? To what other units of area may it be changed?

5. If a sheet of paper is 23 in. by 26 in., what is its area? If its area in square feet had been called for, how many operations would be necessary to find it?

6. If the dimensions of a surface are given in inches and its area is to be expressed in square feet, what must be done?

7. If the dimensions of a wall are given in feet and its area in square yards is asked for, how is it found?

8. If the dimensions of the same wall had been given in inches, and its area in square yards asked for, how many operations would be necessary?

9. What unit is used in giving the area of a farm? Of a carpet? Of the surface of a table?

10. May the area be given in more than one kind of surface units? Illustrate.

11. The number expressing the area of a surface in square units is the product of the two numbers expressing the length

and breadth in linear units of the same name. If the area of a rectangular surface and the dimension of one side are given, how can you find the dimension of the other side? Illustrate by example.

12. If the area of a rectangular surface is given in square units, and the length of one side in linear units of a different name, what must be done in order to find the other side?

13. The area of a blackboard is 4 sq. yd.; it is 3 ft. wide. How long is it?

The name of the square units expressing the area is yards. The name of the linear units expressing the width is feet. The length may be found in two ways:

The units, feet, expressing the width, may be changed to the unit 1 yd. The problem then becomes, "The area of a blackboard is 4 sq. yd.; it is 1 yd. wide. How long is it?"

The second way is by changing the surface expressed in square yards to a surface expressed in square feet, and the problem becomes, "The area of a blackboard is 36 sq. ft.; it is 3 ft. wide. How long is it?"

14. What is the difference between 3 sq. in. and 3 in. square? Show it by drawings.

LUMBER MEASURE

121. The board foot, or a piece of board 1 ft. long, 1 ft. wide, and 1 in. or less in thickness, is the unit of lumber measure.

If a board is 1 in. or less in thickness, the number of board feet it contains is the number of square feet on one side of the board. If the board is an inch and a half thick, the number of board feet is $1\frac{1}{2}$ times the number of square feet on one side of the board. If the board is 2 inches thick, the number of board feet is 2 times the number of square feet on one side of the board, and so on.

ORAL

122. 1. A piece of 1-inch board is 10 ft. long, 1 ft. wide. How many board feet does it contain? If it is 6 in. wide, how many board feet does it contain? If 4 in. wide? If 3 in. wide? If 8 in. wide? If 9 in. wide?

2. A piece of board 16 ft. long is 4 in. wide and 1 in. thick. How many board feet does it contain? What part of a board foot is 1 ft. of the length of the board? If the board is 2 in. thick, how many board feet does it contain?

In lumber more than 1 in. thick, the number of board feet in any piece may be found by multiplying the length in feet by the width and thickness in inches, and dividing the product by 12. Explain why this gives the correct result.

WRITTEN

3. How many board feet are there in an inch board 16 ft. long and 8 in. wide?

4. How many board feet are there in 24 boards $\frac{7}{8}$ in. thick, 10 in. wide, and 16 ft. long?

5. How many feet of lumber are there in 30 scantlings 2×4 and 14 ft. long?

2×4 means 2 inches thick and 4 inches wide.

6. What is the cost of 20 boards $\frac{3}{4}$ in. thick, 16 ft. long, and 9 in. wide, at \$26 per M?

7. What is the cost of 60 planks 2 in. thick, 10 in. wide, and 14 ft. long, at \$24 per M?

8. What is the cost of sixty $1\frac{1}{2}$ in. flooring planks, 8 in. wide, 16 ft. long, at \$33 per M?

9. A 4-ft. sidewalk is to be built for a 50-ft. lot. 2-in. planks 6 in. wide are used for the surface, and two stringers 4×4 as supports. Find cost of material at \$24 per M.

123. MEASURE OF VOLUME, OR CUBIC MEASURE

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (Cd.)

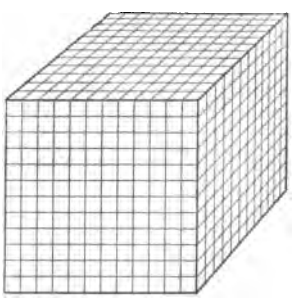
A **solid body** has three dimensions, length, breadth, and thickness. It is also called a **volume.**

A **line** has but one dimension, length; a **surface** has two dimensions, length and width.

A **rectangular solid** is a solid bounded by six rectangular surfaces; its corners are square.

The surfaces of a solid are called **faces.**

The lines in which the faces meet are called **edges.**



How many edges has a crayon box? How many faces? How many corners?

A **cube** is a rectangular solid, all of whose faces are square.

A **cubic inch** is a cube whose edges are each one inch long. What is the area of each face? What is the entire area of an inch cube?

A **cubic foot** is a cube whose edges are each one foot long. What is the area of each face? What is the entire area of a cube whose edges are each one foot?

What is a cubic yard? What is the area of each face? What is the area of the top and bottom of a cube whose edges are each one yard long? What is the area of the six faces of such a cube?

ORAL

124. 1. How many cubic feet are there in a block of wood 1 ft. long, 1 foot wide, and 1 foot high? In a block of wood 2 ft. long, 1 ft. wide, and 1 ft. high?

2. How many cubic feet are there in a block of wood 3 ft. long, 1 ft. wide, and 1 ft. high? In a block 3 ft. long, 2 ft. wide, and 1 ft. high? 3 ft. long, 3 ft. wide, and 1 ft. high?

3. If the block of wood is 3 ft. long, 3 ft. wide, and 2 ft. high, how many cubic feet does it contain? If it is 3 ft. long, 3 ft. wide, and 3 ft. high?

The number of cubic feet in each case equals the product of what three numbers?

4. A solid 1 in. long, 1 in. wide, and 1 in. high is a cubic inch. How many cubic inches are there in a solid 12 in. long, 1 in. wide, and 1 in. high?

Make a drawing showing such a solid, marked off into cubic inches.

5. How many cubic inches are there in a solid 12 in. long, 2 in. wide, and 1 in. high? Make a drawing showing such a solid, marked off into cubic inches.

6. If a solid is 12 in. long, 12 in. wide, and 1 in. high, how many cubic inches does it contain, or what is its volume?

7. If a solid is 12 in. long, 12 in. wide, and 2 in. high, how many cubic inches are there in its volume?

8. If a solid is 12 in. long, 12 in. wide, and 12 in. high, how many cubic inches does it contain?

Such a solid being 1 ft. square is a cubic foot. The number expressing its volume in cubic inches is the product of what three numbers?

9. How many cubic inches are there in a brick 8 in. long, 4 in. wide, and 2 in. thick?

SOLUTION: If the brick were 1 in. long, 1 in. wide, and 1 in. thick, it would contain 1 cu. in.; if it were 8 in. long, 1 in. wide, and 1 in. thick, it would contain 8 cu. in.; if it were 8 in. long, 4 in. wide, and 1 in. thick, it would contain 32 cu. in. Since it is 8 in. long, 4 in. wide, and 2 in. thick, it contains 64 cu. in.

The operations performed in the analysis are multiplying 1 cu. in. by 8; multiplying the product 8 cu. in. by 4; multiplying this product, 32 cu. in., by 2; the result is 64 cu. in.

What is the unit of measurement?

In finding the volume of the brick without going through the analysis, what is to be done?

Notice that the dimensions are given in linear inches; the volume is given in cubic inches. If the dimensions had been linear feet, the volume would be expressed in what units?

10. A pile of 2-foot wood is 10 ft. long, 4 ft. high. How many cubic feet are there in the pile? Analyze. Solve without analyzing.

11. How many cubic feet are there in a block of granite 8 ft. long, 3 ft. wide, and 4 ft. high? Analyze. Solve without analyzing.

12. How many cubic feet are there in $\frac{1}{2}$ of a cord of wood? In $\frac{1}{4}$ of a cord?

13. The volume of a brick is 64 in. It is 8 in. long and 2 in. thick. How wide is it?

14. Having the number of cubic units in a rectangular solid, and the number of linear units in its length and thickness, how can you find its width?

15. Having the volume of a rectangular solid and any two dimensions given, how can you find the other dimension?

TIME MEASURE

125. Time measure is used in measuring time.

TABLE

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
365 days (or 52 weeks, 1 day)	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

The solar year, or the time required for the earth to revolve around the sun, is 365 days, 5 hours, 48 minutes, and 46 seconds nearly. This is nearly ¼ of a day more than 365 days. This fractional part of the day is omitted each year until the fourth year, when the time lost amounts to nearly a day. In every year, the date number of which is divisible by four, one day is added to the month of February, and that year is called **leap year**, except in the *centennial* years, in which case the date number must be divisible by four hundred to be a leap year. This is because, in adding one day every four years, the error amounts to something over three days in four centuries, and is corrected by making the centennial year a leap year once in four hundred years only.

ORAL

126. 1. How many minutes are there in 2 hours? In half an hour? In an hour and a half? In 10 hours?

2. How many minutes are there from 9 A.M. to 11.30 A.M.? From 10 A.M. to 2.30 P.M.?

3. How many months are there in $3\frac{1}{2}$ years? In $4\frac{1}{3}$ years?

4. A man lived 15 years more than half a century. At what age did he die?

5. How many days are there in the spring months? In the winter months of 1908? Of 1909?

CIRCULAR MEASURE

127. 1. Circular measure is used in measuring arcs of circles and angles, and in estimating latitude and longitude.

TABLE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (deg. or °)
360 degrees = 1 circumference

LONGITUDE AND TIME

128. 1. The **longitude** of a place is its distance east or west from a given meridian. The meridian most commonly used for this purpose is the one passing through the observatory at Greenwich, England. It is called the prime meridian. All places on it have 0° longitude. Longitude is reckoned in degrees, minutes, and seconds, and ranges from 0° to 180° east or west. The earth revolves on its axis from west to east once in every twenty-four hours, and this causes the sun to appear to revolve around the earth from east to west in the same time. Since the sun travels through the entire circumference, or 360°, in twenty-four hours, in one hour it travels $\frac{1}{24}$ of 360°, or 15°; in one minute it travels $\frac{1}{60}$ of 15°, or 15′, and in one second it travels $\frac{1}{60}$ of 15′, or 15″.

Places to the *east* of any meridian have *later* time, those to the *west*, have *earlier* time, since the sun appears first to those in the east. When the sun is directly overhead, it is noon on that meridian.

TABLE OF LONGITUDE AND TIME

A difference of 15° of longitude makes a difference of 1 hour of time.
A difference of 15′ of longitude makes a difference of 1 minute of time.
A difference of 15″ of longitude makes a difference of 1 second of time.

ORAL AND WRITTEN

129. 1. How many degrees of longitude make a difference of 1 hour in time? 12 hours? 30 minutes? 20 seconds?

2. When it is noon at Chicago, what is the time 15° east? 15° west? 30° east? 30° west? 60° east? 60° west?

3. What is the difference in longitude between two cities, if the difference in time is 2 hours? 3 hours, 30 minutes?

4. If I start at Denver and travel until my watch is 1 hr. 30 min. too fast, in what direction and how far do I travel?

5. The first shock of the San Francisco earthquake, April 18, 1906, was recorded at 5.12 A.M. in observatories in 120° W. and at 19 min. 20 sec. past 8 A.M. in 75° W. What length of time was consumed by the shock in passing between the two observatories?

6. The longitude of Philadelphia is 75° 10′ 0″ west, and of Vienna 16° 20′ 22″ east. What is the time in Vienna when it is 9 A.M. in Philadelphia?

	16°	20′	22″ E.
	75°	10′	0″ W.
15)	91°	30′	22″

6 hr. 6 min. $1\frac{7}{15}$ sec.

9 A.M. + 6 hr. 6 min. $1\frac{7}{15}$ sec. = 3 hr. 6 min. $1\frac{7}{15}$ sec. P.M. *Ans.*

Since one place is in east and the other in west longitude, the difference in longitude is found by *adding*, which gives 91° 30′ 22″. From the table on p. 182 it follows that the difference in time is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude, or 6 hr. 6 min. $1\frac{7}{15}$ sec. Since Vienna is in east longitude, and Philadelphia in west longitude, the sun rises earlier in Vienna, and hence we must add the difference in time to 9 A.M., which gives 6 min. $1\frac{7}{15}$ sec. past 3 P.M.

7. The difference in longitude of two places is 40°. What is the difference in time between the places?

8. The longitude of New Orleans is 90° west. Find the difference in time between this place and Philadelphia, which is in longitude 75° 10′ 0″ west.

9. London is on the prime meridian, and Washington is 77° 3′ 6″ west. What change must a traveler make in his timepiece in going from Washington to London?

10. Paris is 2° 20′ east longitude, and San Francisco 122° 26′ 15″ west. What time is it in San Francisco when it is 7:00 P.M. in Paris?

11. Rome is 12° 27′ east, and Manila 120° 58′ 6″ east. Dewey began firing on the defenses of Manila at 5:41 A.M., May 1, 1898. At what time, by timepieces in Rome, was the fight begun?

12. The difference in time between Philadelphia and Cincinnati is 37 min. 20 sec. What is the difference in longitude?

37 min. 20 sec.
15
9° 20′ 0″. *Ans.*

Since a difference of 1 minute of time corresponds to a difference of 15′ of longitude, etc., 15 times the number of seconds of time will represent seconds of longitude, (300″) and 15 times the number of minutes of time, the number of minutes of longitude (555′), 555′ + 300″ = 9° 20′ 0″.

13. The time at St. Louis is 53 min. earlier than the time at Washington. What is the difference in longitude?

14. In traveling from New York to Cincinnati I find that my watch is 41 min. 32 sec. too fast. Find the difference in longitude.

15. In traveling on a parallel of latitude from 89° 32′ 30″ west longitude, I notice that my watch has lost 1 hr. 25 min. 30 sec. In what direction am I traveling, and what is the longitude of the second place?

16. The longitude of New York is 74° 3′ west, and of Jerusalem 35° 13′ east. When it is 4:30 A.M. in New York, what is the time in Jerusalem?

17. The longitude of Boston is 71° 4′ west, and of New Orleans 90° west. When it is noon in Boston, what time is it in New Orleans?

18. Formulate a problem in longitude and time, getting the data from your geography. Solve the problem.

STANDARD TIME

130. In the work on longitude and time, it was shown that all places differing in longitude differ in time as reckoned by the sun. In order to avoid the confusing differences in time when each place has its own local time, the railroads of Canada and the United States in 1883 formulated a system of **standard time**. In this system, meridians that are multiples of 15° are selected as principal meridians, and the belts $7\frac{1}{2}$° on either side have the local time of the principal meridian. In practice, however, the time belts are not of uniform width, since the division terminals of railroads often extend beyond, or fall short of, the halfway points between the principal meridians. The time belts used in standard time are the Atlantic, Eastern, Central, Mountain, and Pacific, with principal meridians of 60, 75, 90, 105, and 120 degrees respectively.

A traveler leaves New York at 8 A.M. As he leaves Pittsburg, he notices that his watch, an accurate timekeeper, is an hour fast. Pulling into Ellis, Kansas, his watch is still

an hour fast, but as the train leaves the station he notes that his watch is two hours fast. When leaving Sparks, Nevada, he notes that his watch has gained another hour. His timepiece now is three hours fast, and remains so during the rest of the trip to San Francisco. Explain.

MISCELLANEOUS TABLES

131. NUMBER

12 units or things = 1 dozen (doz.)
12 dozen = 1 gross
12 gross = 1 great gross
20 units or things = 1 score

132. PAPER

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

133. BOOKS

A sheet folded in two leaves is called a folio.
A sheet folded in four leaves is called a quarto, or 4to.
A sheet folded in eight leaves is called an octavo, or 8vo.
A sheet folded in twelve leaves is called a duodecimo, or 12mo.
A sheet folded in sixteen leaves is called a 16mo.
A sheet folded in twenty-four leaves is called a 24mo.

134. TABLE OF UNITED STATES MONEY

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

United States money consists of coin and paper money.

The gold coins are the double eagle ($20), eagle ($10), and half-eagle ($5). The quarter-eagle ($2.50) and dollar, formerly coined in small numbers, are no longer issued.

The silver coins are the dollar, half-dollar, quarter-dollar, and dime. The silver five-cent piece and three-cent piece were formerly coined, but are no longer issued.

The five-cent piece now coined is made of copper and nickel; the one-cent piece is made of bronze.

NOTE. There are 23.22 grains of pure gold in a gold dollar and nearly 16 times as many grains of pure silver, 371.25 grains, in a silver dollar. This is what is meant by "the ratio of 16 to 1."

Paper money is issued in denominations of $1, $2, $5, $10, $20, $50, $100, $500, and $1000.

The mill is not coined, but is used in calculations as the tenth part of a cent. In business, the seller usually regards any part of a cent as an additional cent; but in making computations in interest, etc., anything less than five mills is disregarded, and anything between five mills and one cent is regarded as an additional cent. Thus, $2.574 is usually regarded as $2.57, $2.575 as $2.58.

In writing dollars, cents, and mills, cents are written as hundredths of a dollar, and mills as thousandths of a dollar. All operations involving the expression of dollars, cents, and mills are treated as decimals, and need not be considered as special cases.

Aliquot Parts. An aliquot part of a number is one of its equal parts. Thus, 2 is an aliquot part of 8 because it is one of the four equal parts of 8. 5 is an aliquot part of a dollar because it is one of the twenty equal parts of a dollar. Aliquot parts of a dollar are frequently used in business computations.

TABLE OF ALIQUOT PARTS OF A DOLLAR FREQUENTLY USED

This table should be learned:

10 cents = $\frac{1}{10}$ of a dollar
12½ cents = $\frac{1}{8}$ of a dollar
16⅔ cents = $\frac{1}{6}$ of a dollar
20 cents = $\frac{1}{5}$ of a dollar
25 cents = $\frac{1}{4}$ of a dollar
30 cents = $\frac{3}{10}$ of a dollar
33⅓ cents = $\frac{1}{3}$ of a dollar
37½ cents = $\frac{3}{8}$ of a dollar
50 cents = $\frac{1}{2}$ of a dollar
62½ cents = $\frac{5}{8}$ of a dollar
75 cents = $\frac{3}{4}$ of a dollar
87½ cents = $\frac{7}{8}$ of a dollar

ORAL

135. 1. At 25¢ a pound, how much will 16 lb. of butter cost?

SOLUTION: 25¢ = $\frac{1}{4}$ of a dollar. 16 lb. of butter will cost 16 × $\frac{1}{4}$ of a dollar, or $4.

2. At 87½¢ a bushel, how many bushels of wheat may be bought for $7?

SOLUTION: 87½¢ = $\frac{7}{8}$ of a dollar. As many bushels can be bought as $\frac{7}{8}$ is contained times in $7. $7 \times \frac{8}{7} = 8$, the number of bushels.

3. How much will 240 lb. of raisins cost at 12½¢ a pound? At 16⅔¢ a pound? At 25¢ a pound?

4. How much will 48 lb. of grapes cost at 12½¢ a pound?

5. What is the cost of 4 doz. eggs at $16\frac{2}{3}¢$ a dozen? At $33\frac{1}{3}¢$ a dozen?

6. A dealer buys 120 readers at $37\frac{1}{2}¢$ each, and sells them at $50¢$ each. How much does he gain?

7. At $37\frac{1}{2}¢$ an hour, how much will a man earn in 2 days of 10 hours each?

8. What part of a day are 9 hours? 6 hours? 10 hours? 12 hours?

9. What part of a day is the time from 6 A.M. to 10 A.M.? From 6 A.M. to 10 P.M.?

10. How many hours are there in the month of June? In the month of July?

Answer the last question without multiplying the number of hours in a day by the number of days in July.

11. If the cost of printing a pamphlet is $16\frac{2}{3}¢$, how much will 1000 cost?

12. At $37\frac{1}{2}¢$ a yard, how many yards of lace can be bought for $3?

13. At $16\frac{2}{3}¢$ a dozen, how many dozen eggs can be bought for $2?

REDUCTION OF DENOMINATE NUMBERS

136. Reduction of denominate numbers is the process of changing the denomination without changing the value.

137. To reduce a simple denominate number of a given denomination to a lower denomination.

RULE: *Multiply the given number by the number of units of the next lower denomination it takes to make one of the given denomination.*

138. To reduce a compound denominate number to the lowest denomination given, or to a still lower denomination.

RULE: *Multiply the given number of units of the highest denomination by the number of units of the next lower denomination it takes to make one of the highest denomination, and to the product add the number of units of the lower denomination, if any. Proceed in like manner with this result and each successive result thus obtained, until the number is reduced to the denominations required.*

NOTE. In applying this rule, it does not matter which factor is used as the multiplier.

ORAL

139. 1. In 2 bu. how many pecks are there?

SOLUTION: In 1 bu. there are 4 pk. In 2 bu. there are 2 times 4 pk., or 8 pk. Or,

In 1 bu. there are 4 pk. In 2 bu. there are as many pecks as 4 times 2, or 8 pk.

2. In 8 pk. how many quarts are there?

3. In 64 qt. how many pints are there?

In the above examples, is the reduction from a given denomination to a higher or to a lower denomination? What operation is required in solving the examples?

4. Reduce 3 lb. to ounces.
5. Reduce 2 da. to hours.
6. Reduce 5 gal. to quarts.
7. Reduce 4 ft. to inches.
8. Reduce 4 yd. to inches.
9. Reduce 12 pk. to bushels.
10. Reduce 16 pt. to quarts.
11. Reduce 21 da. to weeks.
12. Reduce 18 ft. to yards.
13. Reduce 24 qt. to gallons.

In examples 4–8 is the reduction to a lower or to a higher denomination?

In examples 9–13 is the reduction to a lower or to a higher denomination?

What operation is required in solving examples 4–8? In solving examples 9–13?

Study Recitation

Reduction of a denominate number to a lower denomination is called **reduction descending.**

What is the reduction of a denominate number to a higher denomination called?

By what operation is reduction descending performed?

By what operation is reduction ascending performed?

Give illustrations of each kind of reduction.

14. Reduce 8 gal. 3 qt. 1 pt. to pints.

gal.	qt.	pt.
8	+ 3	+ 1
	4	
	32	
	3	
	35	
	2	
	70	
	1	
	71	

In 1 gal. there are 4 qt. In 8 gal. there are 8 × 4 qt., or, it may be stated: In 8 gal. there are as many quarts as 4 × 8, or 32 qt. 32 qt. + 3 qt. = 35 qt. In 1 qt. there are 2 pt. In 35 qt. there are 35 × 2 pt., or there are as many pints as 2 × 35 pt., or 70 pt. 70 pt. + 1 pt. = 71 pt.

Notice that in the work of multiplying, the multiplier is 4, the multiplicand 8, and the product 32, all abstract numbers. The number 32 is the same as the number of quarts which would be obtained by taking 4 qt. 8 times, that is, 32.

In our explanation of the work, in order to know what denomination we have as the result of the reduction, we make the statement as though 8 were used as the multiplier, and 4 quarts as the multiplicand, or we say that the number of quarts in 8 gal. is the same as the number obtained by multiplying 8 by 4, or 32 qt.

In order to avoid making mistakes in the denominations as the reduction goes on, it is well to write at the right of the number which we

actually use as the multiplier its denomination, and remember that the product has the same denomination, indicating it at its left.

In explaining, use either form of statement the teacher may think preferable.

To the Teacher. Explanations should be required until pupils are able to give them accurately and readily, and then only the results of each operation should be required: In 8 gal. there are 32 qt. 32 qt. + 3 qt. = 35 qt. 35 qt. = 70 pt. 70 pt. + 1 pt. = 71 pt. Finally, require the last result only, as, 71 pt.

Written

Reduce the following to the required denomination:

15. 24 bu. 2 pk. to pecks.

16. 25 bu. 3 pk. to quarts.

17. 3 pk. 5 qt. 1 pt. to pints.

18. 8 da. 23 hr. to hours.

19. 7 da. 5 hr. to minutes.

20. 8 sq. yd. 9 sq. ft. to square feet.

21. 30 sq. ft. 72 sq. in. to square inches.

22. 24 lb. 12 oz. to ounces.

23. 3 T. 1500 lb. to pounds.

24. 4 cwt. 75 lb. to pounds.

25. 4 Cd. 112 cu. ft. to cubic feet.

26. 24 cu. ft. 110 cu. in. to cubic inches.

27. 10 rd. 3 yd. to yards.

28. 11 rd. 4 yd. to yards.

rd. yd.
11 + 4

5½ yd.

5½

55

60½ yd.

4

64½ yd.

If it is required to reduce the ½ yd. to lower denominations, it may be reduced by multiplication as though it were a whole number.

½
3 ft.
1½ ft.

½ ft. = ½ × 12 in. = 6 in.

The entire result is, yd. ft. in.
64 + 1 + 6.

140. To reduce a denominate number of a given denomination to higher denominations.

RULE: *Divide the given number by the number of units it takes of its denomination to make one unit of the next higher denomination, and place the remainder, if any, at the right. Divide the quotient thus obtained by the number of units of its denomination it takes to make one of the next higher denomination, and write the remainder, if any, at the right. Proceed in the same manner until the number is reduced to the required denomination. The last quotient, with the several remainders annexed in proper order, will be the answer required.*

WRITTEN

1. In 56 pt., how many gallons and pints are there?
2. In 225 in., how many yards, feet, and inches are there?
3. In 2475 min., how many days, hours, and minutes are there?
4. Reduce 475 cu. ft. to higher denominations.
5. Reduce 247 qt. to pecks and bushels.
6. Reduce 150 ft. to yards and rods.
7. Reduce 20516 ft. to rods and miles.
8. Reduce 2560 lb. to hundredweight.
9. Reduce 75 oz. to pounds.
10. In 778 pt. how many bushels are there?
11. Reduce 64,922 oz. to tons.
12. In 6400 sq. rd. how many acres are there?

141. Reduction of denominate fractions, common and decimal.

A denominate number expressed either as a common fraction or as a decimal is reduced to lower denominations in the same way as integral denominate numbers are reduced, by multiplication.

Oral and Written

1. What part of a peck is $\frac{5}{12}$ of a bushel, is the same as the following: Reduce $\frac{5}{12}$ of a bushel to pecks.

Any number of bushels is reduced to pecks by multiplying by 4. Any fraction of a bushel is reduced to pecks in the same way.

Solution: $\frac{5}{12} \times 4 = \frac{5}{3} = 1\frac{2}{3}$. The answer is $1\frac{2}{3}$ pk.

2. Reduce $\frac{3}{8}$ of a peck to quarts. How are pecks reduced to quarts? What is the result?

3. .75 of a foot equals how many inches? How are feet reduced to inches? Apply the process.

4. Reduce $\frac{7}{8}$ of a gallon to lower denominations.

Solution: $\frac{7}{8} \times 4 = 3\frac{1}{2}$, the number of quarts. $\frac{1}{2}$ qt. reduced to pints $= \frac{1}{2} \times 2 = 1$, the number of pints. The answer is 3 qt. 1 pt.

5. Reduce $\frac{4}{5}$ of a yard to lower denominations.

Solution: $\frac{4}{5} \times 3 = 2\frac{2}{5}$, the number of feet. $\frac{2}{5} \times 12 = 4\frac{4}{5}$, the number of inches. *Ans.* 2 ft. $4\frac{4}{5}$ in.

6. Reduce $\frac{7}{8}$ of a day to lower denominations.

7. In $\frac{3}{4}$ of a cord, how many cubic feet are there?

8. Reduce $\frac{1}{8}$ of a barrel to lower denominations.

9. Reduce $\frac{7}{8}$ of a yard to lower denominations.

10. Reduce $\frac{3}{4}$ of a gallon to lower denominations.

11. Reduce .7 of a pound to ounces.

$$\begin{array}{r} 16 \\ .7 \\ \hline 11.2 \end{array}$$

In .7 of a pound there are .7 of 16 oz., or 11.2 oz.

$$\begin{array}{r} 16 \\ .2 \\ \hline 3.2 \end{array}$$

In .2 of an ounce there are .2 of 16 drams, or 3.2 drams.

Ans. 11 oz. 3.2 drams.

12. What decimal of an hour is .07 of a day? .15 of a day?

13. What decimal of a quart is .75 of a peck? .45 of a peck?

14. Reduce $\frac{3}{4}$ of a quart to the fraction of a gallon. How are quarts reduced to gallons? Apply the process in the above example.

SOLUTION: $\frac{3}{4} \div 4 = \frac{3}{16}$, the fraction of a gallon. The answer is $\frac{3}{16}$ gal.

15. What part of a gallon is $\frac{3}{5}$ of a pint?

SOLUTION: First find what part of a quart $\frac{3}{5}$ of a pint is, then find what part of a gallon the resulting fraction is.

16. How can you reduce days to weeks? What decimal of a week is .21 of a day?

17. 30.8 rd. is what decimal of a mile?

18. What part of a gallon is 2 qt. 1 pt.?

SOLUTION

2 qt. 1 pt. = 5 pt.
1 gal. = 8 pt.

The question now becomes, 5 pt. is what part of 8 pt.? *Ans.* $\frac{5}{8}$.

19. What part of a rod is 2 yd. 2 ft.?

SOLUTION

2 yd. 2 ft. = 8 ft.
1 rd. = $16\frac{1}{2}$ ft.

The question now becomes, 8 ft. is what part of $16\frac{1}{2}$ ft.? *Ans.* $\frac{16}{33}$.

20. What part of a day is 12 hr. 30 min.?

12 hr. 30 min. = how many minutes?
1 day = how many minutes?
How may the question now be stated?

21. What part of 2 bu. is 2 pk. 2 qt.?

2 bu. = how many quarts?
2 pk. 2 qt. = how many quarts?
How may the question now be stated? Solve.

22. What part of $5\frac{1}{2}$ yd. is 2 yd. 2 ft.?

23. What part of a ton of hay is 1800 lb.? What decimal?

ADDITION AND SUBTRACTION OF COMPOUND DENOMINATE NUMBERS

142. In adding compound numbers, as in simple numbers, the numbers to be added must be of the same denomination. In adding simple numbers, the sum of the numbers of any denomination is divided by ten, the quotient is added to the next higher denomination, and the remainder is written under the numbers added. In adding compound numbers, the sum of the numbers of any denomination is divided by the number of units of that denomination it takes to make one of the next higher denomination; the remainder, if any, is written under the numbers added, as in simple numbers, and the quotient is added to the number of the next higher denomination, as in simple numbers.

Add:

	wk.	da.	hr.
1.	7	6	12
	5	4	13
	3	5	10
	9	2	11
	26	4	22

Adding the hours, gives the sum 46 hr. Dividing 46, the number of hours, by 24, to reduce to days, gives 1 da. and the remainder 22 hr. Write the remainder 22 under the hours added. Add the 1 da. to the numbers representing days; the sum is 18 da. Dividing 18, the number of days, by 7, to reduce to weeks, gives 2 wk. with 4 da. remainder. Write the 4 under the days added. Add the 2 wk. to the numbers representing weeks; the sum is 26 wk. The entire sum is 26 wk. 4 da. 22 hr.

	rd.	yd.	ft.	in.
2.	8	+ 5		
	3	+ 2		
	4	+ 3		
	16	+ 4	+ 1	+ 6

Adding the yards, gives the sum 10 yd. Dividing 10, the number of yards, by $5\frac{1}{2}$, to reduce to rods, gives 1 rd. with $4\frac{1}{2}$ yd. remainder. Write the $4\frac{1}{2}$ yd. under the numbers added. Add the 1 rd. to the rods; the sum is 16 rd. The entire sum is 16 rd. $4\frac{1}{2}$ yd.

The $\frac{1}{2}$ yd. may be reduced to lower denominations by multiplication. $\frac{1}{2}$ yd. $= 1\frac{1}{2}$ ft. $\frac{1}{2}$ ft. $= 6$ in. Instead of writing $4\frac{1}{2}$ yd., the sum may be written 4 yd. 1 ft. 6 in., the entire sum being 16 rd. 4 yd. 1 ft. 6 in.

	rd.	ft.
3.	5	+ 14
	2	+ 13
	8	+ 10
	17	+ 4

Adding the feet gives the sum, 37 ft. Divide 37, the number of feet, by $16\frac{1}{2}$, the number of feet in a rod, by reducing $16\frac{1}{2}$ to $\frac{33}{2}$ and 37 to $\frac{74}{2}$. Dividing $\frac{74}{2}$, the number of feet, by $\frac{33}{2}$ gives 2, the number of rods, and $\frac{8}{2}$ ft., or 4 ft. remaining. Write the 4 ft. under the number of feet added, and add the 2 rd. to the rods. The entire sum is 17 rd. 4 ft.

4.			**5.**			**6.**		
mi.	rd.	yd.	mi.	rd.	ft.	bu.	pk.	qt.
2	+ 120	+ 5	1	+ 180	+ 15	8	+ 3	+ 5
1	+ 250	+ 3	2	+ 120	+ 13	25	+ 2	+ 7
1	+ 17	+ 1	3	+ 240	+ 9	16	+ 3	+ 6

	bu.	pk.	qt.
7. From	8	+ 1	+ 5
take	5	+ 3	+ 7
	2	+ 1	+ 6

7 qt. cannot be taken from 5 qt. Add to the 5 qt., 8 qt., the number of quarts in 1 pk.; from the sum, 13 qt., subtract 7 qt., and write the remainder, 6 qt., underneath. We have added 1 pk. to the minuend. We must, therefore, either take 1 pk. from it or add 1 pk. to the subtrahend, in order that the result may be correct. Add 1 pk. to the 3 pk. in the subtrahend; the sum is 4 pk. 4 pk. cannot be taken from 1 pk.; add 4 pk., the number of pecks in a bushel, to the 1 pk., and from its sum, 5 pk., subtract 4 pk. The remainder is 1 pk. Write the 1 pk. underneath. Add 1 bu. to the 5 bu. in the subtrahend, making it 6 bu.; subtract 6 bu. from 8 bu. The remainder is 2 bu. The entire answer is 2 bu. 1 pk. 6 qt.

8.	9.	10.
bu. pk. qt.	gal. qt.	yd. ft. in.
From 9 + 3 + 2	From 8 + 2	From 9 + 2 + 7
take 4 + 2 + 6	take 2 + 3	take 2 + 2 + 9

If there is a fraction in the remainder, reduce it to lower denominations, and write the numbers of the lower denominations as a part of the remainder.

143. Finding the difference between dates.

In finding the difference between two dates, treat the number of the year as so many years, the number of the month as so many months, and the day of the month as so many days. In subtracting, count thirty days to the month and twelve months to the year.

Find the difference in time between Sept. 8, 1901, and Nov. 21, 1896.

	yr.	mo.	da.
1.	1901 +	9 +	8
	1896 +	11 +	21
	4 +	9 +	17

The later date is written as 1901 yr. 9 mo. 8 da. The earlier date is written as 1896 yr. 11 mo. 21 da. The subtraction is performed in the same way as in other cases of subtraction in compound numbers.

2. A man was born Nov. 3, 1844; he died Jan. 18, 1900. At what age did he die?

3. What was the date of your birth? What is your age to-day?

4. If you have a brother who is 4 yr. 6 mo. 10 da. older than you are, what was the date of his birth?

5. A man gave a note Aug. 10, 1904; he paid it June 7, 1906. How long did the note run?

6. If the fall term of school begins September 4, and the school year closes on the first day of June following, how many months and days are there from the date of opening to the close?

7. A man gave a note for $1000 dated Sept. 15, 1900; he made a payment of $100 June 20, 1901; a second payment of $100 the 4th of April, 1902. He paid the balance due on the note Sept. 15, 1904. What time elapsed from the date of the note to the date of the first payment? From the date of the first payment to the date of the second payment? From the date of the second payment to the date of settlement?

	yr.	mo.	da.	
(4)	1904	+ 9	+ 15	Date of settlement.
(3)	1902	+ 4	+ 4	Date of second payment.
(2)	1901	+ 6	+ 20	Date of first payment.
(1)	1900	+ 9	+ 15	Date note was given.
		9	+ 5	Time from date of note to first payment.
		9	+ 14	Time from first payment to second payment.
	2	+ 5	+ 11	Time from second payment to date of settlement.

Write the dates for subtraction as shown above.

Subtract (1) from (2) and write difference below.

Subtract (2) from (3) and write difference below the first difference.

Subtract (3) from (4) and write the difference below the second difference.

8. A note was given bearing date July 1, 1896. Upon this note partial payments were made Aug. 10, 1896; June 6, 1897; Dec. 12, 1898. The balance due was paid July 1, 1899.

Find the time which elapsed between the date the note was given and the time of the 1st payment; between the 1st payment and the 2d payment; between the 2d payment and the 3d payment; between the 3d payment and the date of settlement.

9. Make a similar example involving four partial payments.

MULTIPLICATION AND DIVISION OF COMPOUND DENOMINATE NUMBERS

144. 1. Multiply 3 ft. 8 in. by 4.

ft.	in.
3	+ 8
	4
14	+ 8

4 × 8 in. = 32 in.

Reduce to feet; 32 in. = 2 ft. 8 in. Write the 8 in. under the denomination inch, and add the 2 ft. to the product of 4 × 3 ft. Write the sum, 14 ft., under the denomination feet. The product is 14 ft. 8 in.

2. Multiply 2 rd. 12 ft. 4 in. by 4.

rd.	ft.	in.
2	+ 12	+ 4
		4
10	+ 16	+ 4

4 × 4 in. = 16 in. = 1 ft. 4 in. Write the 4 in. under inches, and add the 1 ft. to the product of 4 × 12 ft. The sum is 49 ft. Divide 49 ft. by 16½, the number of feet in a rod, by reducing both to half feet.

16½ ft. = $\frac{33}{2}$ ft. 49 ft. = $\frac{98}{2}$ ft.

$\frac{33}{2}$ ft. is contained in $\frac{98}{2}$ ft. 2 times and $\frac{32}{2}$ ft. remaining. $\frac{32}{2}$ ft. = 16 ft. Write the remainder, 16 ft., under the denomination feet, and add 2 rd. to the product of 4 × 2 rd. The sum is 10 rd. The entire product is 10 rd. 16 ft. 4 in.

3. Multiply 7 rd. 11 ft. 6 in. by 15.

4. What is the product of 25 bu. 3 pk. by 14?

5. Divide 15 bu. 3 pk. by 4.

bu.	pk.	qt.
4)15	+ 3	
3	+ 3	+ 6

¼ of 15 bu. = 3 bu. with 3 bu. remaining undivided. 3 bu. = 12 pk. 12 pk. + 3 pk. = 15 pk. ¼ of 15 pk. = 3 pk., with 3 pk. remaining undivided. 3 pk. = 24 qt. ¼ of 24 qt. = 6 qt. The quotient is 3 bu. 3 pk. 6 qt.

6. Divide 115 rd. 2 yd. 1 ft. 6 in. by 15.

7. A man has a farm of 160 A. 100 sq. rd., which he wishes to divide into 6 equal lots. What is the area of each lot?

8. If $12\frac{1}{2}$ lb. of spices are put up into 8 packages of equal size, what is the weight of each package?

9. Make a rule for multiplication of compound denominate numbers. For division of compound denominate numbers.

MISCELLANEOUS PROBLEMS

WRITTEN

145. 1. A merchant sells 5 cwt. 25 lb. of sugar at $5.50 per hundredweight. How much does he receive for it?

2. A wholesale merchant sold the following amounts of sugar: 7 cwt. 94 lb.; 4 cwt. 65 lb.; 3 cwt. 29 lb.; 5 cwt. 65 lb.; the selling price was $5.50 per hundredweight. What was the entire amount sold? What was the value?

3. A grocer bought 5 cwt. 75 lb. of sugar and sold 3 cwt. 95 lb. How many pounds remained?

4. A butcher bought 1600 cwt. 72 lb. of beef at $8\frac{3}{4}$¢ a pound and sold it at 12¢ a pound. How much did he gain? Work in two ways.

5. A farmer sold a load of corn weighing 2748 lb., and a load of wheat weighing 2560 lb. For the wheat he received 76 cents a bushel, and for the corn 55 cents a bushel. How much did he receive for both?

6. An ocean steamer during a voyage of 3025 miles burns coal at the rate of 1 T. 95 lb. for every 25 miles. How many tons of coal are used up in the voyage?

7. What is the cost of the following coal bill, at $7.50 per ton: 3650 lb., 2720 lb., 5000 lb., 1960 lb., 2750 lb.?

8. A machinist ships twelve machines weighing 3 T. 750 lb. each. How much is paid for freight at 22¢ per hundredweight?

9. A dealer sold 420 T. 1250 lb. of coal at $5.50 per ton, and a number of tons of other kinds at $7.50 per ton. In all, he received $4780. How many tons of the second kind did he sell?

10. Reduce 33,000 in. to higher denominations.

11. How many steps of 30 inches each must be taken in walking 4 rods?

12. How many steps of 33 in. each must be taken in walking 2 mi. 300 rd.?

13. How many times will a bicycle wheel 88 inches in circumference turn in going one mile?

14. How many times will a bicycle wheel 88 inches in circumference turn in going $12\frac{3}{8}$ miles?

15. How many strips of carpet 30 in. wide are required to cover a floor 20 ft. × 25 ft., if the strips run lengthwise? Crosswise?

16. How many strips of carpet 30 in. wide are required to cover a floor 18 ft. × 22 ft., if the strips run lengthwise? Crosswise?

17. In carpeting the floor in example 16, if the strips are laid lengthwise, when seven strips have been laid, what is the width of the portion of the floor not carpeted?

In buying carpet what must be bought to cover the remaining portion of the floor? Draw a figure on the board representing the floor on the scale of 1 inch to a foot. Draw lines showing the lines of joining of the strips.

18. How many square rods are there in a piece of land 24 rd. long and 12 rd. wide?

19. In 288 sq. rd., how many acres are there?

20. How many acres are there in a farm 134 rd. long and 52.5 rd. wide?

21. From a lot containing 15 acres of land, a man sold 1400 sq. rd. How many acres did he have left?

22. Draw a rectangle 6 in. by 8 in. Suppose this rectangle represents a flower bed 6 ft. by 8 ft. It is desired to make a walk 1 ft. wide around the bed. Draw lines to represent this walk. What is the length of the walk on two sides of the bed? What is the length of the remaining parts of the walk on the two ends? How many square feet are there in the bed? In the walk?

23. A lot 120 ft. long and 80 ft. wide is surrounded by a sidewalk 8 ft. wide. What will be the cost of the sidewalk at $1.50 per square yard?

24. A door 7 ft. high and 3 ft. wide has a 6 in. casing around it. How many square inches are there in the surface of the casing? How many board feet are there in the casing?

25. If a rug 12 ft. long and 10 ft. wide is laid on a floor so as to leave a margin of 2 ft. in width all around the rug, what is the area of the floor? What is the area of the part not covered by the rug? Draw a plan of the floor and rug on a scale of 1 inch to the foot.

26. From a lot of land 80 rd. square, 80 sq. rd. were sold. What was the value of the remainder at $50 per acre?

27. A pasteboard box is 8 in. long, 6 in. wide, and 4 in. high. How many square inches of pasteboard will it take to make the sides of the box? To make the ends? To make the top and bottom? How much more than a square foot of pasteboard will be required?

28. A room is 18 ft. long, 15 ft. wide, and 9 ft. high. How many square yards are there in the walls, floor, and ceiling of the room?

29. A city lot is 30 ft. front by 60 ft. deep. If it sells at $5 per square foot, how much is the lot worth? If it sells at $75 per front foot, how much is it worth?

30. A garden 40 ft. by 60 ft. has a gravel walk 4 ft. wide around it. How many square feet are there in the walk? Illustrate by a diagram.

31. A room is 15 ft. by 18 ft.; how many square yards of plastering are there in the ceiling? How many yards of carpeting $\frac{3}{4}$ of a yard wide will it take to cover the floor? Which way should the carpet be laid?

32. A wall is 20 ft. long and 10 ft. 6 in. high, with an 18 in. base board. How much will it cost to plaster the wall at 12¢ a square yard? If this wall is one side of a room 16 ft. wide, how much will it cost to plaster the walls and ceiling at the same rate, making no allowance for doors or windows?

33. How many tiles 6 in. by 4 in. will it take to cover the floor of a hall 8 ft. by 6 ft.?

34. How much glass is there in 12 windows of 12 panes each if each pane contains 2 sq. ft. 8 sq. in.?

35. How many feet of lumber are there in ten boards 14 ft. long, 6 in. wide, and 1 in. thick?

36. How many feet of lumber are there in nine joists 18 ft. long, 8 in. wide, and 2 in. thick?

37. Find the cost at $24 per M of 17 boards 16 ft. long, 10 in. wide, and 1 in. thick, of 8 scantlings 14 ft. long, 4 in.

wide, and 2 in. thick, and of 18 joists 12 ft. long, 6 in. wide, and $2\frac{1}{2}$ in. thick. Put in the form of a receipted bill.

38. How much will it cost for the flooring for a floor 21 ft. by 16 ft. at \$25 per M, allowing nothing for waste?

Flooring is supposed to be 6 in. in width.

39. If a thousand shingles are needed to cover 100 sq. ft. of roof, how many thousand shingles are required to cover a gable roof 50 ft. long, each of the two sides being $22\frac{1}{2}$ ft. wide? What will be the cost of the shingles at \$4 per M?

40. Measure a door in the schoolroom and the width of its casing, and find the number of square feet in the surface of the door and in the surface of the casing.

41. How many strips of carpet 1 yd. wide will be required for a floor 24 ft. wide and 30 ft. long if the strips are laid lengthwise? If laid crosswise? Remember that carpet is sold by the number of yards in length without regard to its width.

42. A room 20 ft. by 18 ft. is to be covered with carpet 27 in. wide, worth \$1.50 per yard. After the first strip, it is necessary to cut off 9 in. from each of the remaining strips in order to match the figures. How much will it cost to carpet the room?

43. How many square feet are there in a floor 14 ft. 7 in. wide and 19 ft. 4 in. long?

44. At $12\frac{1}{2}$¢ a square yard, how much will it cost to sod a lawn 40 ft. long by 36 ft. wide?

45. A string 40 ft. long was used to outline a square flower bed, the string reaching entirely around it. What were the dimensions of the bed? What was its area? If the bed had the same area and was 20 ft. long, what length of string would be required to go around it?

46. The distance between two cities is 55 miles. How many times will an engine wheel 18 ft. 6 in. in circumference turn in making the trip?

47. From a cask of oil containing 33 gal., 18 gal. 3 qt. were drawn. How much remained?

48. How many baskets each holding 3 pk. can be filled with 12 bu. of apples?

49. How many ties will be laid on 20 ft. of a railroad, if the ties are laid 2 ft. from center to center? Draw a figure showing that your answer is correct. How many ties will there be on 24 ft. of the road? On 32 ft. of the road? How much will it cost for the ties for one mile of such a road at $.40 per tie, if the ties are laid 2 ft. from center to center?

50. How many barrels, each holding 2 bu. 3 pk., will it take to hold 100 bu. of apples? How many pecks will there be in the last barrel?

51. If it requires 12 qt. of oats per day to feed a horse, how many days will 6 bu. 3 pk. last a span of horses?

52. If a train travels 85 miles in 2 hr. 30 min., what is its rate per hour?

53. At $7 a cord, what is the cost of a load of wood 12 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. high?

54. A farmer sells 3 loads of hay, each containing 1785 lb. If he sells it at $7 per ton, how much does he receive for it?

55. If 12 equal bins hold 430 bu. 2 pk. of wheat, how much wheat is there in each bin?

56. How many bushels of wheat are there in 12 bins, each containing 124 bu. 3 pk. 5 qt.?

57. Reduce 2 pk. 7 qt. to the decimal of a bushel.

58. If 9 bu. 2 pk. of potatoes cost $3.80, what is the average cost per bushel? Per peck?

59. If a school uses 24 crayons a week, it will use how many gross of crayons in 36 weeks?

60. If a dealer buys pencils at $3.50 per gross and sells them at 5¢ apiece, how much does he gain on each gross? On 30 dozen?

61. A tank 2 ft. wide, 4 ft. high, and 6 ft. long will contain how many gallons? How many bushels?

62. A cube is 4 in. square. If all the sides are covered with paper, how many square inches of paper are required?

63. How many cubic yards of earth will be taken out in digging a cellar 20 ft. long, 12 ft. wide, and $5\frac{1}{2}$ feet deep?

64. How many bushels of oats will a bin 8 ft. by 6 ft. by 5 ft. contain?

65. During a heavy rain, 2 in. of water fell. How many gallons of water fell on a garden 60 ft. by 80 ft.?

66. If a schoolroom is 24 ft. wide, 36 ft. long, and 10 ft. high, how many cubic feet of space will there be to each of 40 pupils?

67. How many feet of boards are required for a coal bin in the corner of a cellar, so constructed that the walls of the cellar form two sides of the bin, and the floor of the cellar, the bottom of the bin, the bin being 12 ft. long, 6 ft. wide, and 8 ft. high?

68. A cask holds 52 gallons of water. The contents will fill how many bottles, if it requires 5 bottles to hold one gallon of water?

69. Sound travels 1120 ft. per second. A hunter sees the flash of a gun at a distance, and 4 seconds later hears the report. What fraction of a mile distant is he from the gun?

70. When shingles are laid 4 in. to the weather, how many shingles will cover the two sides of a roof, each side being 40 ft. by 20 ft.?

NOTE. Shingles are 16 in. long and of different widths, but in reckoning, each width of 4 in. is considered one shingle, and a bundle of shingles is considered as made up of 250 shingles 16 in. long and 4 in. wide. If a shingle is laid 4 in. to the weather, then each shingle will cover 16 sq. in. of space.

71. If a shingle is laid $\frac{1}{4}$ to the weather, how many shingles does it take to cover a square foot? When the length and width of a roof are given, and shingles are laid $\frac{1}{4}$ to the weather, what is the process of finding the number of shingles required to cover the roof? How can you find the number of bundles of shingles required to cover the roof?

REVIEW

146. 1. State the difference between a simple denominate number and a compound denominate number. How may the latter be changed to the former?

2. Define quantity; measure. What is meant by measuring a quantity?

3. Define surface; angle; right angle. Give an example of each.

4. What is the difference between a square and a rectangle?

5. How many dimensions has a line? A square? A solid?

6. What is a cube? A rectangular solid? Give an example of a rectangular solid that is not a cube.

7. How can you find the area of a rectangle when its length and breadth are known? Make and solve an example illustrating your statement.

8. When one dimension and the area of a rectangle are given, how can you find the other dimension? Illustrate by an example.

9. How can you find the volume of any rectangular solid? Illustrate by an example.

10. Having two dimensions and the volume of a rectangular solid, how can you find the other dimension?

11. The dimensions of a square, A, are one half the dimensions of a square, B; the surface of A is what part of the surface of B? Show by a diagram.

If the dimensions of A are one third the dimensions of B, the area of B is how many times the area of A? Show by a diagram.

12. If two places are in east longitude, or in west longitude, how can you find their difference in longitude? If one of the places is in east longitude and the other is in west longitude, how can you find their difference in longitude?

13. What is the greatest longitude a place can have?

A is in longitude 120° east, and B is in longitude 90° west. Draw a circle and divide it into two equal parts by a vertical line. Starting from the top of this line, locate A and B on the circumference of the circle. Measuring from B to A around the top of the circle, how many degrees of longitude apart are they? Measuring around the bottom of the circle,

how many degrees of longitude apart are they? The answer to the last question may be found by subtracting the sum of 120° and 90° from 360°. Why?

Meridians are imaginary lines on the surface of the earth passing directly from pole to pole. Starting on any meridian, a person may reach any other meridian by traveling east or west. The distance traveled in one direction will be greater than the distance traveled in the other direction, unless the meridians are 180° apart. In giving the distance in longitude between two places, the less of the two distances is the one to be used.

14. How can you reduce a compound denominate number to a simple denominate number?

15. How can you reduce a simple denominate number to a compound denominate number of higher denominations?

16. How do addition and subtraction of compound denominate numbers differ from addition and subtraction of simple denominate numbers?

17. Multiply 52, or $\overset{\text{tens}}{5}\ \overset{\text{units}}{+\ 2}$, by 6. Multiply $\overset{\text{pk.}}{5}\ \overset{\text{qt.}}{+\ 2}$ by 6.

6 times 2 units = 12 units, or 1 ten + 2 units. 6 times 2 qt. = 12 qt., or 1 pk. + 4 qt.

What is done with the product in each case before multiplying the number, 5, of the next higher denomination by 6?

6 times 5 tens = 30 tens. 6 times 5 pk. = 30 pk.

30 tens + 1 ten = 31 tens, or 3 hundred + 1 ten. 30 pk. + 1 pk. = 31 pk., or 7 bu. + 2 pk.

3 hundred + 1 ten + 2 units = 312, product. 3 bu. + 7 pk. + 4 qt. = product.

Show similarities and differences in the two operations.

PERCENTAGE

147. Express the following per cents as common fractions: as decimals:

1. 1 %, 2 %, 4 %, 5 %, 10 %, 12½ %, 20 %
2. 25 %, 30 %, 33⅓ %, 37½ %, 40 %, 50 %
3. 60 %, 62½ %, 66⅔ %, 70 %, 75 %, 80 %
4. 90 %, 100 %, 125 %, 150 %, 110 %, 120 %
5. 16⅔ %, 8⅓ %, 14⅔ %, 1 %, ½ %, ¾ %

Review pp. 124–130 and pp. 154–160.

6. Any per cent of a number is what part of it? Illustrate.

7. How can you reduce a given per cent to a common fraction? Illustrate.

8. How can you reduce a fraction to an equivalent per cent? Illustrate.

9. How can you find any fractional part of a number? Illustrate.

10. How can you find any per cent of a number without reducing the per cent to a fraction? Illustrate.

11. How can you reduce a given per cent, written in words or with the sign %, as five per cent, or 5 %, to a decimal? Illustrate.

12. How can you find what fraction one number is of another? Illustrate.

How can you find what per cent one number is of another? Illustrate.

13. 160 is $\frac{4}{5}$ of what number? Change to a question in percentage.

14. How can you find a number when a given fractional part of it is known? Illustrate.

15. How can you find a number when a given per cent of it is known? Illustrate.

Study Recitation

148. ILLUSTRATIVE SOLUTION.

1. What is 25% of 48?

Analysis by question and answer.
What part of a number is 25% of it? *Ans.* $\frac{1}{4}$ of it.
What is $\frac{1}{4}$ of 48? *Ans.* 12.

Analysis by statement.
25% of a number is $\frac{1}{4}$ of it. $\frac{1}{4}$ of 48 is 12.

Solution, using the given per cent as a decimal.
25% of a number is .25 of it. .25 of 48 is 48.

$$\begin{array}{r} .25 \\ \hline 12. \end{array}$$

2. 60 is what per cent of 160?

Analysis by question and answer.
60 is what part of 160? *Ans.* $\frac{60}{160} = \frac{3}{8}$ of it.
$\frac{3}{8}$ of a number is what per cent of it? *Ans.* $37\frac{1}{2}\%$.

Analysis by statement.
60 is $\frac{60}{160}$, or $\frac{3}{8}$, of 160.
$\frac{3}{8}$ of a number is $37\frac{1}{2}\%$ of it.

Solution by direct reduction to a decimal.
60 is $\frac{60}{160}$ of 160. $\frac{60}{160}$ reduced to a decimal $= .375 = 37\frac{1}{2}\%$.

3. 24 is 15 % of what number?

Analysis by question and answer.

15 % of a number is what part of it? *Ans.* $\frac{15}{100}$, or $\frac{3}{20}$, of it.

24 is $\frac{3}{20}$ of what number? *Ans.* 160. Or,

If 24 is $\frac{3}{20}$ of some number, what is $\frac{1}{20}$ of the number? *Ans.* $\frac{1}{3}$ of 24, or 8.

If 8 is $\frac{1}{20}$ of a number, what is the number? *Ans.* 20 × 8, or 160.

Analysis by statement.

If 24 is 15 % of a number, or $\frac{3}{20}$ of it, $\frac{1}{20}$ of the number = $\frac{1}{3}$ of 24, or 8.

$\frac{20}{20}$ of the number = 20 × 8, or 160.

Solution by using the given per cent as a decimal.

If 24 is 15 % of a number, the number is 24 ÷ .15, or 160

$$.15\overline{)24.00}\quad 160$$

ORAL

149. Analyze, using the given per cents as common fractions.

Solve, using the given per cents as decimals.

What is:

1. 5 % of 40?
2. $8\frac{1}{3}$ % of \$36?
3. $37\frac{1}{2}$ % of 48 lb.?
4. 50 % of 13 oz.?
5. $12\frac{1}{2}$ % of 88 da.?
6. 60 % of \$18?
7. 75 % of 12 mo.?
8. $\frac{1}{2}$ % of \$400?

First find 1 % of \$400, and then $\frac{1}{2}$ %.

9. $\frac{2}{3}$ % of \$1200?
10. 200 % of 6 cents?
11. $6\frac{1}{4}$ % of 64?
12. $1\frac{1}{2}$ % of \$100?
13. $2\frac{1}{2}$ % of 200?
14. 90 % of 81?
15. $\frac{5}{7}$ % of 91?
16. 4 % of \$250?
17. 10 % of 87.50 acres?
18. 100 % of 12 eggs?

19. 150 % of 12 eggs?

20. 600 % of 6 cents?

21. 2 is what per cent of 4?

22. 9 is what per cent of 10?

23. 3 is what per cent of 9?

24. 4 is what per cent of 16?

25. 12 is what per cent of 96?

What per cent of:

26. 16 is 10?

27. 8 is 5?

28. $\frac{1}{2}$ is $\frac{1}{4}$?

29. 15 is 15?

30. 6 is 9?

31. 4 is 16?

32. 16 is 50 % of what number?

33. 3 is $33\frac{1}{3}$ % of what number?

34. 80 is 40 % of what number?

35. 10 is $62\frac{1}{2}$ % of what number?

36. 300 is 150 % of what number?

37. 6.50 is 10 % of what number?

38. $\frac{1}{2}$ is $6\frac{1}{4}$ % of what number?

39. $\frac{3}{4}$ is 75 % of what number?

40. $\frac{7}{8}$ is $37\frac{1}{2}$ % of what number?

41. What is the difference between $\frac{1}{8}$ % of \$720 and $33\frac{1}{3}$ % of \$720? Write both per cents as mixed decimals of two places.

42. A farmer having 240 chickens sold 60 of them. What part remained unsold? What per cent?

43. A man paid \$910 for 13 % of a farm. What was the value of the farm at that rate?

44. I spend in one year \$1260, which is $66\frac{2}{3}$ % of my income. What is my income?

45. 8 hours is what per cent of a day? 15 seconds is what per cent of an hour?

46. If $\frac{1}{4}$ of a box of lemons is sold, and $\frac{1}{3}$ of the remainder is unsalable, what per cent of the lemons is not salable?

47. A watch costing \$60 was sold for \$45. What part of the cost was lost? What per cent?

48. Of 144 yd. of ribbon, 108 yards are sold. What per cent of the ribbon is sold? What per cent of it remains?

49. $62\frac{1}{2}\%$, or 120 acres, of my farm, is marsh. How many acres are there in the farm?

50. $\frac{1}{5}$ of my salary is spent for rent, $16\frac{2}{3}\%$ for provisions, and 40% for all other expenses. I save \$210 annually. Find the amount of my salary.

WRITTEN

51. A speculator bought 1275 acres of land, and sold $66\frac{2}{3}\%$ of it to one man, and $12\frac{1}{2}\%$ of the remainder to another. How many acres did each purchase, and how many remained?

52. If bread made from a sack of flour weighs $33\frac{1}{3}\%$ more than the flour, what is the weight of the bread made from a sack of flour weighing 50 lb.?

53. What number increased by $37\frac{1}{2}\%$ of itself equals 440?

54. What number diminished by 40% of itself equals 783?

55. John earned \$45, which was 30% of what he had in the bank. How much money had he in the bank?

56. Mrs. Jones owned $\frac{7}{8}$ of a ship worth \$24,640, and sold $16\frac{2}{3}\%$ of her share. What part of the ship did she still retain, and what was its value?

57. I paid $\frac{2}{5}$ of my money for a watch and $62\frac{1}{2}\%$ of the remainder for a chain, and then had \$9 left. What was the cost of the watch? Of the chain?

58. A man walked $\frac{5}{12}$ of his journey the first day, $37\frac{1}{2}\%$ of it the next day, and then had 24 miles to travel. How long was the journey?

59. $12\frac{1}{2}\%$ of 4% of A's money is $200. What is the amount of his money?

60. What per cent of 16 is 21? Of 21 is 16? What per cent of $\frac{3}{8}$ is $\frac{7}{8}$? Of $\frac{7}{8}$ is $\frac{3}{8}$?

61. What per cent of a yard is 2 ft. 9 in.?

62. I spent $\frac{1}{4}$ of my money for groceries, 20% for house rent, $\frac{3}{8}$ for clothing, and the remainder, $132, for sundries. How much had I at first?

63. What per cent of 8 is .375.? Of 1 ton is 75 lb.? Of $27.50 is $12\frac{1}{2}$ cents?

64. A piece of property increased in value 25% in one year, and was then worth $20,000. How much was it worth at first?

65. 5 is what per cent of 4? If A's money is 25% more than B's, what per cent of A's money is B's?

66. A lost $33\frac{1}{3}\%$ of his money and then found that $16\frac{2}{3}\%$ of the remainder was $30 less than he lost. How much did he lose, and how much had he at first? Change to a problem in fractions and solve mentally.

67. A merchant's debts are $12,500, and his property amounts to $9500. What per cent of his debts can he pay? How much will Mr. Smith, who has a claim of $750, receive?

68. Two merchants, A and B, have 300 cords of wood each. A sells 60% of his wood, and B 50% of his. What per cent of A's sale is B's? Of B's is A's?

69. Frame two questions in percentage and solve them.

150. Application of the three general problems of percentage not involving the element of time.

1. Profit and loss.
2. Trade discount.
3. Commission and brokerage.
4. Insurance.
5. Taxes.
6. Duties and customs.

The only new things in these applications are the definitions of new terms and the business usages or principles governing the transactions.

PROFIT AND LOSS

151. The **cost** of anything is the amount paid for it by the buyer.

The **selling price** of anything is the amount received for it by the seller.

The **gain** or **profit** is the excess of the selling price over the cost.

The **loss** is the excess of the cost over the selling price.

The **list price** of an article is the amount asked for it.

A **discount** is a per cent deducted from the list price or asking price.

The gain or loss per cent is always reckoned upon the cost price in finding the gain or loss.

The cost is always 100 % of itself.

PROBLEMS

Oral

152. 1. A clothing merchant asks $20 for coats for which he paid $15. In order to close out his stock he offers the coats at 10 % less than the list price, and at this price they are all sold. What is the cost price to the buyer of one of these coats? What was the cost price to the merchant?

What is the merchants' gain? What amount did he reduce the price by making a reduction of 10%? What per cent did he gain?

2. I bought a horse for $120 and sold it at a profit of 25%. Find the gain and the selling price.

Analysis by direct statement.

1. 25% of the cost = ¼ of it.
2. ¼ of $120 = $30, gain.
3. $120 + $30 = $150, selling price.

Analysis by question and answer.

What is 25% of $120? *Ans.* $30. $30 = gain. If a horse was bought for $120 and sold at a profit of $30, what was the selling price? *Ans.* $150.

3. I bought a horse for $120 and sold it for $150. Find the gain per cent.

Analysis by direct statement.

1. $150 − $120 = $30, gain.
2. $30 is $\frac{30}{120}$, or ¼, of $120.
3. ¼ of 100% = 25%, gain.

Analysis by question and answer.

If a horse is bought for $120 and sold for $150, what is the gain? *Ans.* $30. $30 is what per cent of $120? *Ans.* 25%.

4. I sold a horse for $150, thereby gaining 25%. Find the cost.

Analysis by direct statement.

1. 100% of the cost + 25% of it = 125% of the cost = selling price, $150.
2. 125% of the cost = ⅘ of it.
3. ⅘ of the cost = $150.
4. ¼ of the cost = 4 × ⅕ of $150, or $120, cost.

Analysis by question and answer.

If a horse is sold at a gain of 25%, for what per cent of the cost is it sold? *Ans.* 125%. 125% of a number is what fraction of it? *Ans.* ⅘ of it. $150 is ⅘ of what number? *Ans.* $120. $120 = cost.

To the Teacher. Lead pupils to see what is given and what is asked for in each of the three problems above, and how each problem may be derived from either of the others. Require pupils to give original problems like any one of the three, and then to derive from these original problems two others like the two others above.

Each of the following problems may be changed so as to become a problem under one or more of the three general problems in percentage, decimals, or fractions.

Change to problems in fractions and solve:

5. A lot costing $1600 is sold at a profit of $12\frac{1}{2}\%$. What is the gain? The selling price?

6. A 50-cent knife is sold for 70 cents. Find the gain per cent.

7. A man makes $16\frac{2}{3}\%$ of a 72-mile journey in 2 hours. How many miles remain to be finished?

8. A jeweler sold a watch for $32, thereby gaining $33\frac{1}{3}\%$. Find the gain and the cost.

9. By selling corn at 54 cents a bushel, I lose 10%. Find the cost and the loss.

10. What per cent do I lose when I buy at 20 cents and sell at 10 cents? What per cent is gained in buying at 10 cents and selling at 20 cents?

11. A 60-acre farm costing $4800 is sold at a gain of $12\frac{1}{2}\%$. Find the selling price.

12. A merchant buys pianos at $250 apiece. How much must he charge for them in order to make a profit of 20%?

13. What must be the list price of a watch in order that I may give a discount of 10% and yet receive $36 for it?

WRITTEN

14. For how much must apples costing $2.40 a barrel be sold to bring a gain of $37\frac{1}{2}\%$?

15. I lose 30% in selling a suit for $21.70. What is the cost?

16. I lose 10% in selling an article for $15.30. What would have been the selling price had I gained 10%?

17. A house costing $3750 is sold for $3975. What is the gain per cent?

18. An article costing $7.50 is sold at a loss of 20%. The purchaser sells it for $10. What is his gain per cent?

19. A certain piano costs the manufacturer $150. He sells it at a profit of 20%. The retail merchant who buys the piano pays $5 freight, and sells it at a profit of 25%. Find the selling price.

20. A merchant sells carpet at 75 cents a yard, and gains 20%. What would be the gain if the carpet were sold at 85 cents a yard?

21. By selling an article for $7.20 I gain $12\frac{1}{2}$%. What per cent should I gain by selling it for $7.50?

22. A dealer sells two wagons for $75 each. On one he gains 25%, and on the other he loses 25%. Does he gain or lose by the transaction, and how much?

Analysis by question and answer.

A gain or loss of 25% is what part of the cost? *Ans.* $\frac{1}{4}$ the cost. If a gain of $\frac{1}{4}$ the cost is made by selling a wagon for $75, $75 is what fraction of the cost? *Ans.* $\frac{5}{4}$ of the cost. $75 is $\frac{5}{4}$ of what sum? *Ans.* $60. $60 = cost.

If a loss of $\frac{1}{4}$ the cost results from selling a wagon for $75, $75 is what fraction of the cost? *Ans.* $\frac{3}{4}$ of the cost. $75 is $\frac{3}{4}$ of what sum? *Ans.* $100, or cost. ($100 + $60), cost, − ($150), selling price, = $10, loss.

23. I sell two machines at $150 each. On the first I gain 25%, and on the second I lose 20%. Do I gain or lose, and how much?

24. A boy buys 6 apples for a dime, and sells them at the rate of 2 for 5 cents. What is his gain per cent?

25. A boy buys apples at the rate of 5 for 5 cents, and sells them at the rate of 3 for 5 cents. What is his per cent of gain?

26. A boy buys 120 oranges at the rate of 4 for 10 cents, and sells them at the rate of 3 for 10 cents. What is his per cent of gain or loss? What is his per cent of gain or loss if 10 % of the oranges are unsalable?

27. A farmer sells 75 sheep for what 80 cost. Find his gain per cent.

28. A merchant buys bankrupt stock at 45 cents on the dollar, and sells it at 10 % below the original list price. What is his gain per cent? What is meant by "buying at 45¢ on the dollar"?

29. I buy an article for $10 and sell it for 10 cents. What is the loss per cent? Another article costing 10 cents is sold for $10. What per cent is gained?

30. Mr. Lewis bought 12,000 bushels of wheat at $.60, and 6500 bushels of corn at $.55 a bushel. In selling, he gained $16\frac{2}{3}$ % on the wheat and lost 20 % on the corn. Did he gain or lose on the whole transaction, and how much? What per cent?

31. At what price must goods costing $40 be listed so that a discount of $16\frac{2}{3}$ % may be allowed and yet a profit of 20 % may be realized?

Make two problems of the foregoing. Thus, at what price must goods costing $40 be sold to realize a profit of 20%? *Ans.* $48. If $48 are received for goods sold at a discount of $16\frac{2}{3}$ % from the list price, what is the list price?

32. How must I mark a suit costing $16 that I may make a profit of 25 % after giving a discount of 20 % ?

A dealer "marks" his goods, or puts a price upon them, which he asks the buyer. He may give a discount from this price. Goods are frequently marked at a higher price than the merchant expects to receive, so that he may give the buyer a discount, and thus make him think he is getting a "bargain."

33. A merchant sold cloth at $1.50 a yard, thereby gaining 20 %. The cloth did not sell well, and he listed it to sell at a loss of 20 %. What was the second list price?

34. How many bushels of wheat, at $.60 a bushel, must I purchase, so that if I sell at a profit of $8\frac{1}{3}$ %, a gain of $250 may be made?

35. A merchant lists his goods $33\frac{1}{3}$ % above cost, and then gives a discount of 10 %. What is his per cent of gain? He gains $8 on an article. Find the list price and the cost of this article.

36. Fifty pounds of soap are purchased at 5 cents a pound. In a damp cellar it absorbs water equal to 10 % of its weight, and is then sold at 6 cents a pound. What is the gain? The gain per cent?

37. I sell $\frac{3}{5}$ of an article for what $\frac{5}{6}$ of it costs. Find the gain per cent.

38. A tailor purchases cloth at $3 a yard. How much must he charge per yard in order to make a profit of 20 % should the cloth shrink 10 % in length?

39. Gloves that cost $18 per dozen pairs were sold at $2.25 per pair. What per cent was gained?

40. Construct two problems involving profit and loss, and solve them.

TRADE DISCOUNT

Study Recitation

153. Manufacturers and merchants frequently deduct a certain per cent from the list price of goods when selling them, or offering them for sale. This deduction is called a **discount.** We studied it in preceding problems. Many business houses give two or more discounts off the list price on account of payment in cash, changes in market price, or for other reasons.

These deductions are called **trade discounts** or **commercial discounts.**

NOTE. — Discounts made for payments in cash are sometimes called **cash discounts**, and discounts made for payment within a certain time, as 10 days, 1 month, etc., are called **time discounts.**

The **net price** is the list price less the discounts; it is the price the buyer pays.

Business usage or principle.

In trade discounts, the first discount is computed on and deducted from the list price; the second is computed on and deducted from the resulting remainder, etc.

A piano was catalogued at $600 with a discount of $33\frac{1}{3}\%$ and another discount of 10 % for cash. Find the net price.

$33\frac{1}{3}$ of a number $= \frac{1}{3}$ of it.	3)$600
$\frac{1}{3}$ of $600 = $200, first discount.	200
$600 − $200 = $400, remainder.	10) 400
10%, or $\frac{1}{10}$, of $400 = $40, second discount.	40
$400 − $40 = $360, net price. *Ans.*	$360

A discount of $33\frac{1}{3}\%$, or $\frac{1}{3}$, of the cost, leaves $\frac{2}{3}$ of the cost. A further discount of 10%, or $\frac{1}{10}$, of the remainder, leaves $\frac{9}{10}$ of $\frac{2}{3}$, or $\frac{3}{5}$, of the cost; or, net price $= \frac{3}{5}$ of $600 = $360.

PROBLEMS

WRITTEN

154. The solution of these problems involves nothing new.

1. Discounts of 20 % and 10 % are allowed on goods listed at $60. Find the discounts and the net price.

2. A gun was listed at $120, with trade discounts of $33\frac{1}{3}$ %, 25 %, and 10 %. What was the selling price?

3. A merchant sold a bill of goods listed at $200 with 20 % and 5 % off for cash. How much did he receive?

4. Which is better for the purchaser, 40 % and 20 % off, or a single discount of 53 %?

5. Find a single discount equivalent to 25 %, 20 %, and 10 % off, on a bill of goods listed at $120.

6. A merchant buys goods at 50 % and 20 % off, and sells them at 10 % and 20 % off the list price. What is his gain per cent?

7. Construct and solve two problems involving trade discount.

COMMISSION

155. An **agent** is a person authorized to buy, sell, or transact any business for another.

An agent who buys or sells goods is often called a **commission merchant.**

An agent who buys and sells real estate, bonds, and stocks for others is called a **broker.**

Commission is a per cent paid to an agent for the transaction of business. The commission paid to a broker is often called **brokerage.**

The **proceeds** is the sum left after the commission and other expenses have been deducted.

Business usage or principle.

In selling, commission is computed on the sum received for the goods; in buying, on the sum paid for the goods; in collecting, on the sum collected. The commission, or brokerage, paid a broker for buying or selling stocks and bonds is usually a certain per cent of their par value.

PROBLEMS

Written

156. 1. An agent sells goods to the amount of $2450. What is his commission at $2\frac{1}{2}\%$, and what are the proceeds?

SOLUTION:

$2\frac{1}{2}\%$ of a number = .02½ of it.
$2450 × .02\frac{1}{2}$ = $61.25, commission.
$2450 − $61.25 = $2388.75, proceeds.

2. A merchant received $61.25 as his commission for selling goods to the amount of $2450. What was the rate of his commission?

3. An agent collected a $565 debt. What was his commission at 5%, and how much should he remit to the creditor?

4. Mr. B. sold 16,000 bu. of wheat at $.75 a bushel on a commission of $2\frac{1}{2}\%$. What was his commission?

5. A real estate man received $6800 to be invested in lots. He bought five lots at $1200 each and charged 5% commission. What was his commission, and how much did he return to his employer?

6. An agent charged 2% commission for selling a quantity of goods. He received $6 for his services. What

was the selling price of the goods? How much did the party for whom the goods were sold, receive?

Analyze the problem, beginning with: 2% of the selling price = $6.

7. An agent charging 2½% commission, retained $648.20 from the sale of cotton. What was the selling price, and how much was remitted to his employer?

$648.20 is 2½% of what number? State the general problem in percentage under which this example falls. Give rule for its solution. Analyze.

8. If a commission merchant receives 3% for selling goods, what per cent of the selling price does the owner receive? Illustrate by an original problem.

9. A collector remits $4750 after taking out his commission of 5%. Find the amount of the debt.

10. A commission merchant receives turkeys weighing 3140 lb. He sells them at 11¢ a pound. How much should he send to the man for whom he made the sale, after taking out his commission of 5% and paying $5.60 for freight and $6.75 for storage?

11. If a commission merchant receives 2½% commission for buying goods, how much does each $1's worth of goods cost his principal, or the party for whom he is acting?

12. A man sends his agent $820 to be invested in goods, and to pay the commission of 2½% on the amount of money used in making the purchase. What is the value of the goods? What is the amount of the commission? Prove the correctness of your results.

13. A commission firm is ordered to buy 5000 bu. of wheat at 75¢. The commission is ⅛¢ per bushel. What is the cost of the wheat, including the commission?

INSURANCE

157. **Insurance** is security against financial loss on account of destruction or loss of property, health, or life. We shall consider property insurance only.

The **policy** is the contract between the insured and the insurance company.

The **premium** is the sum paid the insurance company for taking the risk.

Business usage or principle.

Premiums are computed on the face of the policy, usually at a certain sum per hundred dollars. Thus, a premium of \$.80 per hundred for three years means that the company charges \$.80 for every \$100 insurance for three years.

PROBLEMS

Written

158. 1. A house valued at \$24,000 is insured for three years for $\frac{2}{3}$ of its value at 80¢ a hundred. What is the amount of premium to be paid?

Solution

$\frac{2}{3}$ of \$24,000 = \$16,000, face of policy.

$\frac{\$16{,}000}{100} \times .80 = \$128$, premium.

2. How much must I pay for a policy of \$1750 on my furniture at 50¢ a hundred for one year?

3. A ship worth \$780,000 was insured for $\frac{7}{8}$ of its value at \$1.25 per hundred. Find the premium.

4. Mr. Brown insured his house for \$3500, his barn for \$850, and his personal property for \$1250, at 90¢ a hundred for three years. Find the premium.

5. A company charges a premium of $19.60 on a risk of $2450. What is the premium per hundred? The rate per cent?

6. An insurance company whose rate is 80 cents a hundred charges Mr. Goodyear $278.20 premium. Find the face of the policy.

7. State and solve two problems in insurance.

TAXES

159. A **tax** is a sum of money assessed on property for public use.

The **assessed valuation** of property is its estimated value for the apportionment of taxes.

The **rate of taxation** is the amount of tax per dollar of assessed valuation. Thus, a rate of .0285 means that $.0285 are charged on every dollar of assessed valuation.

Principle.

The tax is computed on the assessed valuation.

PROBLEMS

WRITTEN

160. 1. In a township whose assessed valuation is $625,000, a tax of $15,000 is to be raised. What is the rate of taxation, and what is A's tax if his property is assessed at $10,500?

SOLUTION

$15,000 is $\frac{15000}{625000}$, or $\frac{1}{125}$, of $625,000.

$\frac{1}{125}$ of $100\% = 2\frac{2}{5}\%$, rate of taxation.

$2\frac{2}{5}\%$ of a number $= .024$ of it, expressed as the decimal of a dollar. $.024 is the rate of taxation in cents and mills per dollar.

4. $10,500 × .024 = $252, A's tax.

2. What is B's tax if the rate of taxation is $1\frac{1}{8}\%$ and his assessed valuation is $7850?

3. In a certain city whose assessed valuation is $2,645,000, a tax of $6575 is to be raised. Find the rate of taxation and Mr. Healy's tax on property assessed at $6000.

4. In a certain township the tax was $.01786 on a dollar. What was the tax on a farm assessed at $5750?

5. State and solve a problem in taxes.

DUTIES

161. **Duties** or **customs** are taxes levied by the government on imported goods.

An **ad valorem duty** is a tax computed upon the price paid for the goods; as, an ad valorem duty of 75% on the cost.

A **specific duty** is a tax on the quantity of goods without regard to their cost; as, a specific duty of 15 cents a yard, or 25 cents per pound.

An **invoice** is a bill accompanying the imported goods, showing their quantity and value.

Principle.

Ad valorem duties are computed at a certain per cent of the value of the goods, and specific duties at a certain sum per unit of weight, length, etc.

PROBLEMS

WRITTEN

162. 1. What is the duty at 40% ad valorem on 16,500 clocks invoiced at $1.60 each?

SOLUTION

16,500 × $1.60 = $26,400, price paid.
40% of a number = .40 of it.
$26,400 × .40 = $10,560, duty.

2. What is the duty, at 55¢ a pound, on 45 cases of tobacco of 185 lb. each?

SOLUTION

45 × 185 lb. = 8325 lb.
8325 × 55¢ = $4578.75.

3. S. M. Giles of Chicago imported from Geneva 150 watches invoiced at $55 each, and 250 diamonds at $80 each. What was the duty, the rate being 40% ad valorem on watches and 10% on diamonds?

4. Find the duty on 2260 lb. of cheese at 6¢ a pound.

5. What is the duty on blankets valued at $225 and weighing 500 lb., at 35% ad valorem and 33¢ per pound?

6. A painting valued at $1800 is imported from Florence, Italy. At 20% ad valorem find the total price the purchaser has to pay for it in New York.

7. Stewart & Co. import from England 18 doz. silk umbrellas invoiced at $2 each. How much must they pay for the invoice, including ad valorem duty at 50 %?

8. Find the duty on an invoice of $300 worth of blankets weighing 1000 lb., at 30 % ad valorem and 22¢ per pound.

9. What is the duty on 500 dozen table knives valued at $7 a dozen, at 15% ad valorem and 16¢ each?

10. Fields, Ransom & Co. of New York imported from Havana 75 hhd. of molasses, 63 gal. each; 125 hhd. of sugar, 500 lb. each; 800 boxes of cigars (each box weighing 1 lb. and containing 100 cigars), at $8 per box. 7% was allowed for leakage on the molasses. Find the duty at 3¢ a gallon on the molasses, 1.95¢ per pound on the sugar, and $4.50 per pound, plus 25% ad valorem, on the cigars.

11. Construct and solve a problem in customs and duties.

INTEREST

163. Applications of percentage, with the element of time.
1. Interest. 2. Partial payments. 3. Bank discount.

164. **Interest** is a sum paid for the use of money.

Simple interest is interest on the principal only.

Compound interest is interest on the amount of principal and interest, if the interest is unpaid when due.

Usury is a rate greater than the **legal rate**, which is the rate established by law. In most states usury is prohibited.

The **principal** is the sum of money for the use of which interest is paid.

The **amount** is the sum of the principal and interest.

The **rate of interest** is the per cent of the principal paid for its use for *one year*. Thus, the expression, "interest at the rate of 5%," means that the interest for *one year* equals 5% of the principal.

SIMPLE INTEREST

PROBLEMS

ORAL

165. 1. What is the interest on $250 for one year at the rate of 5%? The amount?

SOLUTION

Interest at 5% = .05 of the principal per year.
$250 × .05 = $12.50, interest.
250.00, principal.
262.50, amount.

2. What is meant by interest at 4%? 5%? 7%? $2\frac{1}{2}$%? $4\frac{1}{2}$%?

3. What is the interest on \$100 for one year at 6%? 5%? $3\frac{1}{2}$%? $6\frac{1}{2}$%? 10%?

4. Find the interest on \$400 for one year at 4%, 5%, $6\frac{1}{2}$%.

5. Answer the foregoing when the interest runs for 2 years.

6. Find the interest on \$500 for 3 years at 4%.

7. I borrow \$300 at 5%, and at the expiration of 4 years pay the interest. How much interest do I pay?

8. Find the amount of \$800 at 6% for 2 years.

9. What is the interest on \$600 at 6% for 8 months?

10. Find the amount of \$1200 at 5% for 2 years.

11. Frame a problem in interest, and solve it.

Study Recitation

166. Finding the interest on any sum of money for 1 year at any rate is the same as finding any per cent of a number.

The interest on any sum of money for any number of years is the interest for 1 year multiplied by the number of years.

The interest for 1 month is $\frac{1}{12}$ of the interest for 1 year. The interest for any number of months is the interest for 1 month multiplied by the number of months.

Any number of months and days may be expressed as months and tenths of a month.

To reduce any number of days to a fraction of a month.

RULE: *Divide the number of days by* 3 *and call the result tenths of a month.*

1. 3 days = .1 of a month.
2. 6 days = .2 of a month.
3. 12 days = .4 of a month.
4. 1 day = .0⅓ of a month.
5. 2 days = .0⅔ of a month.
6. 5 days = .1⅔ of a month.
7. 10 days = .3⅓, or ⅓ of a mo.
8. 26 days = .8⅔ of a month.
9. 17 days = ? of a month.
10. 29 days = ? of a month.
11. 9 days = .3 of a month.
12. 18 days = .6 of a month.
13. 11 days = .3⅔ of a month.
14. 28 days = .9⅓ of a month.
15. 20 days = ? of a month.
16. 23 days = ? of a month.
17. 6 months and 6 days = 6.2 months.
18. 7 months and 16 days = 7.5⅓ months.
19. 5 months and 13 days = ? months.
20. 4 months and 14 days = ? months.
21. 3 months and 20 days = ? months.
22. 4 years, 3 months, and 7 days = 4 years, 3.2⅓ months.
23. 6 years, 7 months, and 13 days = 6 years, 7.4⅓ months.
24. 9 months and 18 days = 9.6 months.
25. 11 months and 23 days = ? months.
26. 3 months and 17 days = ? months.
27. 5 months and 26 days = ? months.
28. 1 month and 10 days = ? months.

PROBLEMS

WRITTEN

167. 1. What is the interest on $150 for 6 yr. 7 mo. 13 da. at 5% per annum?

SOLUTION:

$150
.05
Interest on $150 for 1 yr. at 5% = $7.50
6
Interest on $150 for 6 yr. = $45.00
12)$7.50
Interest on $150 for 1 mo. = $.625
7 mo. and 13 da. = 7.4⅓ mo.
7.4⅓
2081/3
2500
4375
Interest on $150 for 7.4⅓ mo. = $ 4.6458⅓
Interest on $150 for 6 yr. = $45.00
Interest on $150 for 6 yr. 7 mo. 13 da. = $49.65

2. Find the amount of $250 for 3 yr. 9 mo. 17 da. at 7%.

SOLUTION:

$250
.07
Interest on $250 for 1 yr. at 7% = $17.50
3
Interest on $250 for 3 yr. = $52.50
12)$17.50
Interest on $250 for 1 mo. = $1.458+
9 mo. 17 da. = 9.5⅔ mo.
9.5⅔
972
7290
13122
Interest on $250 for 9.5⅔ mo. = $ 13.9482+
Interest on $250 for 3 yr. = $ 52.50
Interest on $250 for 3 yr. 9 mo. 17 da. = $ 66.45
Principal $250.00
Amount of $250 for 3 yr. 9 mo. 17 da. at 7% = $316.45

Find the interest on:

3. $2500 for 60 days at 5 per cent.

4. $125 for 2 years, 9 months, at 7 per cent.

5. $900 for 90 days at $6\frac{1}{2}$ per cent.

6. $600 for 2 years, 8 months, at 5 per cent.

7. $850 for 1 year, 10 days, at 6 per cent.

8. $625.80 for 8 months, 15 days, at 5 per cent.

Find the amount of:

9. $750 for 4 years at 6 per cent.

10. $4200 for 60 days at $7\frac{1}{2}$ per cent.

11. $75,000 for 1 year, 6 months, 15 days, at 7 per cent.

12. Mr. H. H. Jones borrows $1200 June 5, and pays the debt October 23. What is the interest at $4\frac{1}{2}$ per cent?

13. Find the interest on a $900 debt bearing 5 per cent interest, from Jan. 5, 1900, to Dec. 30, 1900.

14. Find the amount due at 6 per cent on a $360 note, given July 7, 1899, and paid Jan. 20, 1900.

15. Frame and solve two problems involving the finding of time between two dates.

Six Per Cent Method

168. Another method of computing interest, called the 6 % method, is frequently employed.

The interest on $1 for 1 year = 6 cents.

The interest on $1 for 2 months = 1 cent. Why?

The interest on $1 for 1 month, or 30 days = 5 mills. Why?

The interest on $1 for 6 days = 1 mill. Why?

The interest on $1 for 1 day = $\frac{1}{6}$ of a mill. Why?

To find the interest on $1 for any period at 6 %.

RULE: *Take 6 times as many cents as there are years plus $\frac{1}{2}$ as many cents as there are months plus $\frac{1}{6}$ as many mills as there are days.*

ORAL

169. What is the interest on $1 at 6 % for the following periods of time:

1.	2 years?	8 months?	2 years, 8 months?		
2.	8 years?	10 months?	8 years, 10 months?		
3.	3 years?	3 months?	3 years, 3 months?		
4.	9 years?	7 months?	9 years, 7 months?		
5.	10 years?	11 months?	10 years, 11 months?		
6.	18 days?	15 days?	13 days?	23 days?	2 days?
7.	24 days?	21 days?	14 days?	26 days?	5 days?
8.	12 days?	27 days?	16 days?	29 days?	4 days?
9.	1 day?	7 days?	17 days?	11 days?	6 days?
10.	9 days?	8 days?	19 days?	3 days?	22 days?
11.	10 days?	25 days?	28 days?	30 days?	20 days?

12. 4 months, 12 days? 5 months, 7 days? 5 months, 29 days?

13. 7 months, 18 days? 3 months, 21 days? 6 months, 14 days?

14. 9 months, 24 days? 8 months, 23 days? 9 months, 13 days?

15. 10 months, 6 days? 7 months, 25 days? 9 months, 25 days?

16. 10 months, 9 days? 9 months, 16 days? 10 months, 10 days?

17. 2 years, 6 months, 12 days? 4 years, 11 months, 12 days?

18. 3 years, 5 months, 18 days? 12 years, 7 months, 3 days?

19. 4 years, 7 months, 24 days? 5 years, 8 months, 18 days?

20. 1 year, 6 months, 7 days? 7 years, 9 months, 15 days?

21. 5 years, 8 months, 27 days? 8 years, 11 months, 10 days?

To find the interest on any sum, for any time, at 6 %.

RULE: *Multiply the interest on* $1, *at* 6 %, *for the given time, by the number of dollars.*

WRITTEN

Find the interest on each of the following amounts, for the given time, at 6 % :

22. $250 for 3 yr. 6 mo. 18 da.

23. $175 for 2 yr. 9 mo. 24 da.

24. $215 for 4 yr. 10 mo. 12 da.

25. $125 for 6 yr. 6 mo. 9 da.

26. $120.75 for 3 yr. 7 mo. 18 da.

27. $118.50 for 4 yr. 8 mo. 15 da.

28. $500 for 8 yr. 2 mo. 4 da.

29. $625 for 1 yr. 1 mo. 1 da.

170. To find the interest on any sum, for any time, at any rate per cent.

1. The interest on a given sum of money, for a given time, at 6 %, is $24.

Interest on the same sum for the same time, at 1 %, is $\frac{1}{6}$ of $24, or $4.

2. If the interest at 1% is multiplied by 7, by 5, by 4, by 8, or by any other number, the product will be the interest at what rate?

3. If the interest at 1% is added to the interest at 6%, the sum will be the interest at what per cent?

4. If the interest at 1% is subtracted from the interest at 6%, the difference will be the interest at what per cent?

5. If the interest at 1% is multiplied by 10, the product will be the interest at what per cent?

6. Having found the interest on any sum for any time at 6%, how can you find it for 7%? for 5%? for 10%? for 8%? for 4%? for 9%? for 3%?

7. If the interest on any given sum at 6% is divided by 2, the quotient will be the interest at what per cent? If this quotient is added to the interest at 6%, the sum will be the interest at what per cent?

If it is subtracted instead, the difference will be the interest at what per cent?

8. Ask and answer questions similar to those in Ex. 7, using 3 for a divisor.

9. Give other rules for finding interest on any sum at 8%, 9%, 4%, 3%, when the interest at 6% has been found.

10. $1\frac{1}{2}$ is what part of 6? If this part of the interest at 6% is taken, the result is the interest at what per cent? If the result is added to the interest at 6%, the sum is the interest at what per cent? If the interest at $1\frac{1}{2}$% is subtracted from the interest at 6%, the result is the interest at what per cent?

Solve examples on page 235 by the 6% method.

EXACT INTEREST

171. Exact interest, or **accurate interest**, as it is sometimes called, is found by taking the *exact* number of days between two dates, and reckoning 365 days to a common year, and 366 days to a leap year.

Interest is usually reckoned on the basis of 12 months of 30 days each, or 360 days to the year. If the time for which the principal is on interest is stated in years, or in years and months, the usual method also gives the exact interest; but if the time the principal is on interest is given in days, the usual method does not give the exact interest. If the time the principal is on interest is less than 1 year, and if the dates between which the principal draws interest are given, the exact number of days may be found, but the usual method will not give the exact interest.

The interest on $1460 for 60 days, at 6%, is found by the usual method to be $14.60.

This is upon the supposition that 60 days are $\frac{60}{360}$ of a year. But 360 days are exactly $\frac{360}{365}$, or $\frac{72}{73}$, of a common year; and $\frac{360}{366}$, or $\frac{60}{61}$, of a leap year. If a man pays the interest for the use of money for 360 days, which he should pay for the use of the money for 365 days or for 366 days, he pays $\frac{5}{365}$, or $\frac{1}{73}$, too much in the common year, and $\frac{6}{366}$, or $\frac{1}{61}$, too much in the leap year.

To find exact interest when the time is in days.

RULE: *Diminish the interest as found in the usual way by $\frac{1}{73}$ of itself if the time is a part of a common year; diminish it by $\frac{1}{61}$ of itself if the time is a part of a leap year.*

If the time is given in years, months, and days, find the interest for the years and months by the usual method, and to it add the exact interest for the days.

To find the exact number of days between two dates.

1. How many days are there between May 20 and November 15 of the same year?

May 20–31	= 11 da.
June	30 da.
July	31 da.
August	31 da.
September	30 da.
October	31 da.
November	15 da.
	179 da.

RULE: *To the number of days remaining in the earliest month given, add the number indicated by the date of the latest month given, and also the number of days in each of the intervening months.*

The number of days remaining in May is 11. The latest month named is November, and the date given is 15. The number of days in each of the intervening months, respectively, is 30, 31, 31, 30, 31. The sum of all is 179.

2. Find the time from June 15, 1886, to Sept. 20, 1890.

From June 15, 1886, to June 15, 1890, it is 4 years.

From June 15, 1890, to Sept. 20, 1890, it is 15 + 31 + 31 + 20 da. = 97 days.

The entire time is 4 years, 97 days; counting 1 leap year, it is 4 years, 98 days.

PROBLEMS

WRITTEN

172. Find the exact interest due on the following:

1. $75 for 36 days at 6%.
2. $125 for 66 days at 6%.
3. $250 for 24 days at 8%.
4. $325 for 33 days at 7%.
5. $115.25 for 80 days at 5%.
6. $1000 for 90 days at 4%.
7. $3500 for 30 days at 3%.

8. $275 from Jan. 12, 1892, to May 13, 1892, at 6%.

9. $315 from September 24 to December 28 of the same year at 7%.

10. $425 from April 20, 1888, to Nov. 12, 1891, at 4%.

PROMISSORY NOTES

173. A **promissory note** is a written promise made by one person to pay a specified person a given sum on demand or at a definite time.

The **maker** of a note is the person who signs it, the one who is to pay it.

The **payee** is the person to whom it is payable.

The **holder** is the person who owns it.

The **face** of a note is the sum named in it.

Days of grace are three days allowed by law in some states for the payment of a note after the expiration of the time specified in the note. In this book, days of grace will not be reckoned unless so stated. In most of the states, days of grace are not allowed by law.

A note is said to **mature** at the expiration of the time specified in the note, or three days later than that time in states where days of grace are allowed. A note is due at **maturity.**

A **negotiable note** is a note made payable to the "bearer" or "to the order" of the payee; it is not negotiable when made payable to the payee only. A negotiable note may be transferred from one person to another.

In some states, the laws require other words to be added in order to make the note negotiable. In Pennsylvania, the words "without defalcation" and in New Jersey, the words "without defalcation or discount," must be added to "value received."

Study Recitation

174. 1. $500.00 ST. PAUL, MINN., Aug. 14, 1908.

Six months after date, for value received, I promise to pay to J. C. Root, or order, Five Hundred Dollars, with interest at the rate of 6 per cent per annum.

J. H. JOHNSON.

2. $200.00 MILWAUKEE, WIS., Sept. 15, 1907.

Thirty days after date, for value received, I promise to pay to John Jones, or bearer, Two Hundred Dollars, with interest at 7 %.

HENRY SMITH.

Both of the above notes are *negotiable*. What words should be omitted from each note to make it *non-negotiable?*

Name the *maker*, the *payee*, and the *holder* of each note.

What is the *face* of each note? When does it *mature?*

The **indorser** is the person who writes his name on the back of a note to transfer it, if it is made payable to his order, or to guarantee its payment in case the maker fails to pay it when due.

To hold the indorser responsible for the payment of the note, notice must be sent him of its non-payment by the maker, on the day the note matures.

If the holder of the first note given above wishes to transfer it to John Doe, he must indorse it, that is, write his name on the back of the note.

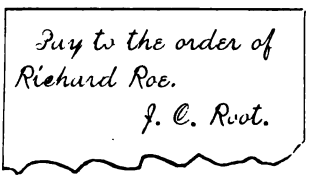
INDORSEMENT IN BLANK

J. C. Root.

If he writes his name "J. C. Root" and nothing else on the back of the note, it is an **indorsement "in blank,"** and the note is then payable to the holder. The indorsement by J. C. Root renders him liable for the payment in case J. H. Johnson is unable to pay the note.

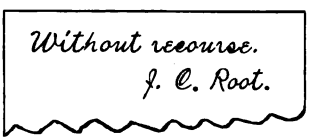
If Mr. Root writes on the back of the note "Pay to the order of Richard Roe, J. C. Root," it is an **indorsement "in full,"** and the note is then payable to Richard Roe, or to the person to whom he may transfer it by his indorsement. The indorser is liable for payment of the note in case the maker fails to pay it.

INDORSEMENT IN FULL



If Mr. Root writes his name on the back of the note, as in either of the above cases, and adds the words "without recourse," it is an **indorsement without recourse,** and the indorser is not liable for payment of the note in case the maker fails to pay it.

QUALIFIED INDORSEMENT



DIFFERENT FORMS OF NOTES

3. $100.25 CHICAGO, ILL., June 1, 1909.

On Aug. 10, 1909, I promise to pay John Jones, or order, One Hundred and $\frac{25}{100}$ dollars, with interest, value received.

A. J. HENRY.

When a note is drawn "with interest," but with no rate specified, the legal rate of the state in which the note is drawn is understood.

4. $150.00 CLEVELAND, O., April 4, 1909.

On demand, for value received, I promise to pay A. H. George, or order, One Hundred Fifty Dollars, with interest at 6 %.

A. B. SMITH.

When is the above note due?

If a note makes no mention of interest, no interest can be collected until it matures ; if not paid at maturity, it draws legal interest from that date to the date of payment.

5. $150.00 MILWAUKEE, WIS., May 5, 1908.

Three months from date, I promise to pay John Jones, or order, One Hundred Fifty Dollars, value received.

JAMES JOHNSON.

When did this note mature?

It was not paid until Dec. 5, 1908. For what length of time did it draw interest? Why? At what rate? What amount was required to pay the note?

6. $100.00 MADISON, WIS., Feb. 12, 1909.

Six months after date, for value received, we jointly and severally promise to pay James Smith, or order, One Hundred Dollars, with interest at 7 %.

JOHN DOE,
RICHARD ROE.

The above note is called a **joint and several note**, and the makers of the note, John Doe and Richard Roe, are jointly and singly liable for its payment. If the words "and severally" were omitted, the note would be a **joint note**, and each of the makers would be liable for payment of only one half the amount due at maturity.

A **protest** is a written notice sent by a notary public in a manner provided by law to the indorsers, stating that the maker has failed to pay the note.

The words "value received" are usually written in a note, though they are not essential. A note maturing on Sunday or on a legal holiday is made payable by law in most states the day before it matures.

7. Write a negotiable note for $750, making yourself the payee and your teacher the maker, interest at 6 %. Find the amount due at maturity.

8. Write a negotiable note payable on demand for $175.25, with interest at 7%, making one of your classmates the payee and yourself the maker.

Have the payee fix the time of payment, and find the amount due.

9. Write a negotiable demand note for $250, without interest, making yourself the payee and John Smith the maker.

Fix the time when payment is demanded. If the note is not paid until six months after demand, what amount is then due?

10. Write a negotiable note for $550.25, with interest at 8%, making yourself the payee and your teacher the maker. Indorse it in blank for transfer. What amount is due at maturity?

11. Write a note from the following data: Negotiable; face, $2500; maker, J. Smith; payee, yourself; maturity, 1 year, 6 months from date; interest, 5%. Find amount due at maturity.

12. A sixty-day note for $550, with simple interest at 10%, dated March 1, 1909, was paid at maturity. When was the note due? What was the amount due?

13. A thirty-day note for $1000, without interest, was paid three months after it matured. What was the amount due, the legal rate being 6%?

14. A sixty-day note for $875, with interest at 6%, was paid in ninety days. What was the amount paid, reckoning simple interest? Exact interest?

15. A two-months' note for $1500, with interest at 5%, dated April 1, 1908, was paid at maturity. Find the date of maturity and the amount due.

PROBLEMS IN INTEREST

Study Recitation

175. PROBLEM I. **Given the principal, interest, and time, to find the rate.**

1. At what rate will $100 yield $12 interest in 2 years?

The interest on a sum of money for a given time at a certain rate is $12. The interest on the same sum of money for the same time at 1% is $2.

What is the rate which gives $12 interest?

Give the rule for finding the rate when the principal, interest, and time are given.

PROBLEM II. **Given the interest, rate, and time, to find the principal.**

2. What principal will yield $12 interest in 2 years at 6%?

The interest on a certain sum of money for a given time and rate is $12.

The interest on $1 for the same time and rate is 12 cents.

What principal will gain $12 in the same time and at the same rate?

Give the rule for finding the principal when the interest, rate, and time are known.

PROBLEM III. **Given the principal, interest, and rate, to find the time.**

3. In what time will $100 gain $12 interest at 6%?

The interest on a given sum of money, $100, at 6% for a certain time is $12.

The interest on the same sum of money, $100, at 6% for 1 year is $6.

In how many years will the interest be $12?

Give the rule to find the time when the principal, interest, and rate are given.

PROBLEM IV. **Given the amount, time, and rate, to find the principal.**

4. A sum of money on interest 2 years at 6% amounts to $112. What is the principal?

The amount of a certain sum of money for a given time at a given rate, 6%, is $112.

The amount of $1 for the same time at the same rate is $1.12.

What number of dollars will amount to $112 under the same conditions?

Give the rule for finding the principal, when the amount, time, and rate are known.

In the first problem, the term required is the **rate**, and a *rate of one per cent* is used in finding the required rate.

In the second problem, the term required is the **principal**, and a *principal of one dollar* is used in finding the required principal.

In the third problem, the term required is the **time**, and a *time of one year* is used in finding the required time.

In the fourth problem, the term called for is the **principal**, and a *principal of one dollar* for the given time at the given rate is used in finding the principal.

PROBLEMS

ORAL AND WRITTEN

176. 1. At what rate of interest will $100 yield $14 in 2 years?

State the above problem in general terms as follows:

"Given the principal, interest, and time, to find the rate."

Solve the problem.

SOLUTION: The interest of $100 at 1% for 2 years is $2. If $100 in 2 years yields $14, the rate per cent equals $14 ÷ $2 = 7%.

Give the rule followed in solving this problem.

2. At what rate will \$200 yield \$36 in 3 years?

State the problem in general terms. Solve it and give analysis of its solution.

State the rule employed.

3. At what rate will \$426 gain \$53.25 in 2 years, 6 months?

State the problem in general terms. Solve it. State the rule employed in its solution.

4. The interest of \$560 for 2 years, 4 months, 16 days, is \$106.40. What is the rate?

5. A principal of \$260 gains \$29.90 in 2 years, 3 months, 18 days. What is the rate?

6. At 10% interest, what part of the principal is the interest of each year?

At the same rate of interest, what part of the principal is gained in 5 years?

7. In how many years at 10% will the interest equal the principal?

8. At what rate per cent will a principal gain $\frac{1}{20}$ of itself in 1 year?

At what rate per cent will it gain $\frac{1}{10}$ of itself in 1 year? In 2 years? In 5 years?

9. In how many years will a principal double itself at 10%? At 6%? At $12\frac{1}{2}$%? At $8\frac{1}{3}$%?

10. At what rate will a principal double itself in 5 years? 4 years? 20 years? 10 years? 7 years?

11. In how many years will a principal of \$600 gain \$30 at 6%?

State the problem in general terms. State the rule employed in solving it.

12. $134.50 is the interest at 8% on $586.40 for how many years?

13. At 7%, $600 amounts to $613.30 in how many years? State in general terms the problem given. By performing a single operation, make such a change as will enable you to state the problem in different terms.

14. The amount is $1040, the rate of interest is 6%, the principal is $800. How long was the principal on interest?

15. A man borrowed $1500 at 7% interest; he finally settled the note, principal and interest, for $3000. How long was it on interest?

16. $200 is on interest at 8%. In how many years will the interest amount to $32?

17. Use the answer of the last problem and such of the conditions given in it as are necessary to frame a problem falling under the general statement: "Given the interest, rate, and time, to find the principal." Solve the problem, and state the rule for the solution of this class of problems.

18. What principal at 5% will gain $45.625 in 2 years, 6 months?

19. A sum of money in 4 years, 7 months, at 7%, produces an interest of $95.35. What is the sum?

20. How much must I pay for a farm, which yields a rental of $500 annually, in order that I may make 10% on my investment?

21. A sum of money is put at interest Jan. 15, 1903, at 7%. When will the interest equal one half of the principal?

22. What sum of money on interest at 6% will amount to $1300 in 5 years? State the problem in general terms. Solve it and give rule for solution of this class of problems.

23. What sum on interest at 9% for 2 years, 3 months, will amount to $360.75?

24. A sum of money on interest for 3 years, 2 months, 12 days, at 8%, amounts to $376.80. What is the sum of money?

25. Given principal, rate, and time, to find the interest. Give an example coming under this general problem. Solve it.

Using the conditions given in this example and the answer, write out four different problems, each calling for a different term. State the general problem that each illustrates, and give the rule for its solution.

PARTIAL PAYMENTS

177. Sometimes the maker of a note is not able to pay it in full at maturity, or is allowed, if he chooses, to make part payment before the note is due. Such payment is then indorsed on the back of the note with the date of payment. Several such payments may be made before the entire amount is paid. Such payments are called **partial payments.**

The following problem is solved by the rule adopted by the United States Supreme Court, called the United States Rule. The United States Rule is based on the following principles:

(1) Payments are applied first to discharge or pay the interest due on the principal at the time the payment is made, and the balance, if any, is then applied to reduce the principal.

(2) Only the principal draws interest; hence, if the payment is less than the interest due when it is made, the unpaid interest must not be added to the principal and thus draw interest.

178. Operation when each payment exceeds the interest due at time payment is made.

Study Recitation

1. $1200.00 MILWAUKEE, WIS., Jan. 3, 1899.

Two years after date I promise to pay to H. H. Harvey Twelve Hundred Dollars, with interest at the rate of 6 per cent per annum. Value received.

JOHN C. ROE.

The following payments were indorsed: Paid June 12, 1899, $250. Oct. 15, 1900, $400. What is the amount due at the time of settlement, Jan. 3, 1901?

	yr.	mo.	da.
$1200, face of note.			
Date of settlement	1901 +	1 +	3
Date of 2d payment	1900 +	10 +	15
Date of 1st payment	1899 +	6 +	12
Date note was given	1899 +	1 +	3

	yr.	mo.	da.	Per Cents	Payments
Time from date of note to 1st payment		5 +	9	.0265	$250, 1st
Time from 1st payment to 2d payment	1 +	4 +	3	.0805	$400, 2d
Time from 2d payment to date of settlement		2 +	18	.013	

The per cents given above are obtained by finding the interest on $1 at 6% for each of the periods, as on p. 236, and expressing the per cent which this sum is of $1.

The product of any of these per cents by the principal gives the interest on the principal at 6% for the corresponding period of time.

If the rate is other than 6%, find the interest first for 6% and then for the given rate as directed under the 6% method of finding interest.

By arranging face of note, dates, differences between dates or interest period, per cents, and payments, as above, the work is shortened, and when the items are thus written, no further reference to the book is necessary for the completion of the work.

SOLUTION

1st step	$1200	
	.0265	
	6000	
	7200	
	2400	
	31.8000 *	Interest
2d step	250.00	1st payment
	218.20 *	Balance
3d step	1200.00	Principal
	981.80	New principal
4th step (same as 1st step)	.0805	
	490900	
	785440	
	79.0349 *	Interest
5th step (same as 2d step)	400.0000	2d payment
	320.9651 *	Balance
6th step (same as 3d step)	981.80	Principal
	660.8349	New principal
Last step	.013	
	198249	
	66083	
	8.59079	Interest
	660.83	Principal
	669.42	Amount

* These numbers are subtracted from the numbers below them.

UNITED STATES RULE FOR PARTIAL PAYMENTS

1st step. Find the interest due at the date of the first payment.

2d step. If the interest is less than the payment, subtract it from the payment.

3d step. Subtract the balance of the payment from the principal, and treat the remainder as a new principal.

4th step. Find the interest on the new principal from the date of the 1st payment to the date of the second payment.

5th and subsequent steps to the last. Proceed in the same manner as before until all the payments have been subtracted.

Last step. After the last payment has been subtracted, find the amount of the principal from the date of the last payment to the date of settlement.

NOTE. If any payment is less than the interest due at the time it is made, disregard the interest found, and compute interest from the same date from which this interest was found to the date of the next succeeding payment. If this interest is less than the sum of the two payments, subtract it from their sum, and subtract the balance of the payments from the principal and proceed as before. If the sum of the two payments does not exceed the interest, disregard this interest as before, and compute interest to a time when the sum of the payments equals or exceeds the interest. In case the sum of all the remaining payments does not exceed the interest due at date of last payment, compute interest to date of settlement, and subtract the sum of the remaining payments from the amount of the principal and interest.

2. A note for $5000 was given Aug. 2, 1899, bearing interest at 6%. The following payments were indorsed: Paid Sept. 9, 1899, $500; May 12, 1900, $350. What amount would settle the note July 2, 1900?

3. On a note dated June 30, 1907, for $650 at 6% were the following indorsements: Paid Dec. 15, 1907, $50; May 6, 1908, $75; Jan. 12, 1909, $100. What was due March 12, 1909?

4. Solve problem 2 above, using 8% as the rate.

5. Solve problem 3 above, using 5% as the rate.

179. Operation when a payment is less than the interest due at time payment is made.

Study Recitation

6. A note for $800 at 6% was given Aug. 1, 1907. On this note the following payments were made: $200, Feb. 16, 1908; $25, Dec. 28, 1908; $50, March 6, 1909. What was due June 30, 1909?

$800, face of note.	yr.	mo.	da.
Date of settlement	1909 +	6 +	30
Date of 3d payment	1909 +	3 +	6
Date of 2d payment	1908 +	12 +	28
Date of 1st payment	1908 +	2 +	16
Date note was given	1907 +	8 +	1

	mo.	da.	Per Cents	Payments
Time from date of note to 1st payment	6 +	15	.0325	$200
Time from 1st to 2d payment	10 +	12	.052	25
Time from 2d to 3d payment	2 +	8	.011⅓	50
Time from 3d payment to date of settlement	3 +	24	.019	

Application of the Rule

	$800	
	.0325	
Subtract the upper number	26.0000	Interest
from the one below it.	200.00	1st payment
	174	Balance
	800	Principal
	626	New principal
	.052	
	1252	
	3130	
	32.552	Interest more than payment

2d per cent	.052
3d per cent	.011⅓
Sum	.063⅓
Principal	626
	208⅔
	378
	126
	378
Interest	39.646
2d and 3d payments	75.000
Balance	35.354

When the interest is more than the payment, disregard the interest found, and find the interest on the same principal to the date of the next payment. This is best done by adding the per cents for the two periods as shown at the left, and multiplying the principal by the sum. This product is the interest on the principal from the date of the 1st payment

Principal	626.000
Balance	35.354
New principal	590.646
	.019
	5315814
	590646
Interest	11.222274
Principal	590.646
Amount	$601.87

to the date of the 3d payment. Add the two payments, $25 and $50, and as the sum, $75, is more than the interest, subtract the interest from the sum, and the balance from the principal, for a new principal. Find the amount of the new principal from the date of the last payment to the date when final settlement is made.

If in the final sum the number of mills is 5 or more, add 1 to the number of cents.

7. A demand note was given for $1000 at 6 % interest, dated Jan. 18, 1901. On this note the following amounts were paid: June 12, 1901, $150; Dec. 27, 1901, $20; April 6, 1902, $75. How much was due Dec. 18, 1902?

Write this note, make the proper indorsements on the back of the note, and then solve.

8. Solve the 7th problem, using 7 % as the rate.

180. When notes are fully paid within a year from date, the following rule, called the Merchants' Rule, is often used.

MERCHANTS' RULE FOR PARTIAL PAYMENTS: *Find the amount of the principal from the time it begins to draw interest to the time of final settlement.*

Find the amount of each payment from the time it is made to the date of settlement.

From the amount of the principal subtract the sum of the amounts of the payments; the difference is the balance due.

1. John Smith gives Henry Jones his note for $250, due in one year from Dec. 15, 1905, interest at 7 %.

On March 15, 1906, Smith makes a payment of $75, and on Sept. 15, 1906, he makes another payment of $50. How much is due at maturity?

By the Merchants' Rule, Mr. Jones is regarded as having loaned $250 to Mr. Smith, due in one year. When Mr. Smith pays Mr. Jones $75, he is regarded as having loaned Mr. Jones $75 from March 15 to Dec. 15, 1906, and when he pays the second amount, $50, he is regarded as having made a loan of $50 to Mr. Jones from Sept. 15, 1906, to Dec. 15, 1906.

Each man is entitled to interest on his loan from the date it was made to date of settlement.

$250
.07
17.50 Interest due to Jones on Smith's note for 1 year.
250.00
267.50 Amount due to Jones on Smith's note for 1 year.

From March 15, date of 1st payment by Smith, to date of settlement, Dec. 15, 1906, is 9 months, or ¾ of 1 year.

$75
.07
4)5.25 Interest on 1st payment for 1 year.
1.31¼
3
3.94 Interest on 1st payment for 9 months.
75.00
78.94 Amount of 1st payment for 9 months.

From September 15, date of 2d payment, to December 15, date of settlement, is 3 months.

$50
.07
4)3.50 Interest on 2d payment for 1 year.
.875 Interest on 2d payment for 3 months.
50.00
50.88 Amount of 2d payment for 3 months.
78.94
129.82 Sum of *amounts* of both payments at date of settlement.
267.50 Amount due Jones on Smith's note at date of settlement.
137.68 Balance due.

2. A man settled an account amounting to $150 by giving a demand note Aug. 15, 1905, for the amount, bearing interest at 6%. On this note he paid $25, Dec. 12, 1905, and $50, Jan. 12, 1906. He paid the balance March 15, 1906. What was the amount of the last payment?

3. On a 6% demand note for $1263, dated Jan. 1, 1895, the following payments were made: Feb. 1, 1895, $100; March 16, 1895, $150; April 30, 1895, $200. How much was due July 1, 1895?

BANK DISCOUNT

Study Recitation

181. Bank discount is a sum charged by a bank for payment of a note or draft before it becomes due.

This sum is the interest at the bank's rate of discount, on the *amount* of the note or draft at maturity from the date the note is discounted to the date it matures. In states where days of grace are allowed, the date of maturity is three days beyond the expiration of the time specified in the note.

When a bank loans money to the borrower on his note and takes bank discount, it deducts the interest from the face of the note at the time the note is given, and pays the maker of the note the difference. This difference between the face of the note and the interest is called the **proceeds** of the note.

If A borrows $100 from B and gives B a note for that sum, due in 4 months at 6%, A receives $100, and *4 months afterward* pays B $100 + the interest on $100 for 4 months at 6%, or $2, making $102. If, however, A gives his note for $100, due in 4 months, to a bank, and the bank discounts it at 6%, the interest on the face of the note for 4 months at 6%, or $2, is *at once* deducted from the face of the note and retained by the bank, and only $98 is paid to A, the maker of the note. At

maturity, A, the maker of the note, pays the bank $100. The note given to the bank does not specify interest until it matures. If not paid at maturity, it begins to draw interest. In the first case, A, the maker of the note, receives from B the amount specified in the note, and pays it with interest at maturity. In the second case, the interest on the amount of the note is paid by A in advance to the bank, and the maker, A, receives from the bank the proceeds, or the amount of the note less the interest.

A bank not only discounts notes made by its customers direct to the bank, but it also buys or discounts notes owned by its customers and payable to them. In such cases, the owner of the note indorses it, making it payable to the bank. Such notes may or may not bear interest.

When such a note does not bear interest, the holder receives the face of the note less discount on it, at the bank's rate, from the date it is discounted to date of maturity.

PROBLEMS

Study Recitation

182. 1. $100.00 MILWAUKEE, WIS., Mar. 3, 1903.

Three months from date, for value received, I promise to pay John Adams, or order, One Hundred Dollars.

PETER SMITH.

April 3, the payee, John Adams, needing money, took the note to the First National Bank to have it discounted. He indorsed the note, making it payable to the cashier of the bank. The bank, being asked to cash the note two months before it became due, charged interest on it at its rate of discount, 6%, for the two months. The interest was $1. This sum was deducted from the $100, the amount payable to the bank at maturity, two months later, by Peter Smith. John Adams received the proceeds, $99.

The transaction is the same as if John Adams gave his own note for $100 to the bank April 3, payable in two months, without interest, and the bank discounted it.

If to the above note the words "with interest at 6%" were added, the amount due at maturity would be $100 + $1.50, the interest for 3 months

at 6%, or $101.50. If John Adams had the note discounted April 3, as above, the bank would compute interest at its rate of discount, as before, on the amount due at maturity, $101.50. The discount on $101.50 at 6% for 2 months, the time from the date the note is discounted to maturity, is $1.02. This is the bank discount. John Adams would receive the proceeds, $101.50 − $1.02, or $100.48.

The bank's rate of discount may be different from the rate of interest specified in the note.

2. $500.00 MILWAUKEE, WIS., March 10, 1905.

Ninety days after date, for value received, I promise to pay to the order of Richard Roe, Five Hundred Dollars, with interest at 6%.

JOHN DOE.

This note was discounted at a bank April 2. The bank's rate of discount was 8%. What were the proceeds?

SOLUTION: This note at maturity would yield the bank $500 + the interest on that sum for 90 days at 6%. The interest = $7.50. $500 + $7.50 = $507.50.

When the note was brought to the bank for discount, 23 days had elapsed since the date the note was given, and it therefore had 67 days yet to run. The bank computed discount on the amount of the note, $507.50, for 67 days at 8%. The discount was $7.56. The proceeds of the note were $507.50 − $7.56 = $499.94.

Find the bank discount and proceeds of a note of:

3. $200, payable in 60 days, discounted at 6%.

4. $125, payable in 90 days, discounted at 7%.

5. $175, payable in 30 days, discounted at 8%.

6. $250, payable in 3 months, discounted at 9%.

7. $150, payable in 45 days, discounted at 6%.

8. A note for $200 was given Jan. 5, 1905, payable in 90 days. When did it mature? If days of grace were allowed, when did it mature?

9. A note for $450, bearing 6% interest, was given Nov. 3, 1901, payable in 3 months. When did it mature?

It was discounted at a bank December 10. The rate of discount was 7%. For what length of time was it discounted? What was the discount? What were the proceeds?

Find the discount and proceeds of each of the following interest-bearing notes:

	Face of Note	Date	Time	Rate of Interest	Date of Discount	Rate of Discount
10.	$750	Sept. 10	60 da.	6%	Oct. 15	8%
11.	$175	Aug. 12	90 da.	7%	Oct. 20	6%
12.	$125	June 20	3 mo.	8%	July 20	6%
13.	$225	June 1	4 mo.	6%	Aug. 1	9%
14.	$500	Jan. 15	60 da.	4%	Feb. 15	6%

15. What are the proceeds of $1 discounted at a bank for 90 days at 6%?

16. If the proceeds of a note for $1 discounted at a bank for 90 da. at 6% are $.985, what must be the face of a note that would yield $9850 when discounted for the same time at the same rate?

17. How much would a note for $1 yield when discounted at a bank for 60 days at 6%?

18. For how much must a 60-day note be given that, when discounted at a bank at 6%, yields $250?

19. For what sum must a 90-day note be drawn to yield $4925 if discounted at 6% on the day it is made?

20. How can you find the face of a note that, discounted for a certain time at a given rate, yields a certain sum as proceeds?

REVIEW PROBLEMS

Oral

183. 1. John had 36 marbles and lost $33\frac{1}{3}\%$ of them. How many had he left?

2. A knife costing \$.80 is sold at a profit of 25%. What is the selling price?

3. What per cent of 30¢ is 30¢?

4. I buy a watch for \$16 and sell it for \$24. Find the per cent of gain.

5. A merchant after deducting 10% from the list price of an article receives \$2.70 for it. Find the list price.

6. James sells an article for \$.96, thereby gaining $33\frac{1}{3}\%$ of the cost. What is the cost?

7. By selling apples at 4¢ a dozen more than cost, a boy gained 10%. How much did he pay per dozen?

8. How much is paid for a carriage listed at \$240, with discounts of 25% and 10%?

9. By selling 3 qt. of milk for the price of a gallon, I gain what per cent?

10. Mr. Brune buys stock at 20% discount from the list price, and sells it at the list price. Find the gain per cent.

11. What is the commission on the sale of \$850 worth of goods at 5%?

12. \$40 is 200% of what sum?

13. An agent buys a lot for \$1500 and charges 2% commission. Find the entire cost of the lot.

14. A building worth $4000 is insured for $\frac{7}{8}$ of its value at 80¢ a hundred. Find the premium.

15. A hardware merchant sold two knives for 90¢ each. On one he lost 25%, and on the other he gained 25%. Did he lose or gain in the transaction, and how much?

16. An insurance company charged $240 premium for insuring a store for $\frac{3}{4}$ of its value at $1 per hundred. Find the value of the store.

17. A school district having an assessed valuation of $90,000 wishes to raise a tax of $1500. What is the rate of taxation?

18. What is the tax of A, living in the school district mentioned, if his property is assessed at $5000?

19. What is the duty on 1200 yards of Brussels carpet, 27 in. wide, invoiced at $2.50 a yard, at 40% ad valorem, and a specific duty of 44¢ a square yard?

20. What is the interest on $500 at 5% for three years?

21. Find the amount due on a $600 note, bearing 6% interest, for 2 years and 6 months.

22. Find the bank discount of 8% on a $1000 note, due in 3 months, without interest.

WRITTEN

23. The face of a note is $1029.99, the interest at maturity is $70.01, the rate of discount is 8%, and the time for which the note is discounted is 6 months. Find the bank discount and the proceeds.

24. Mr. Brown sold his farm for $6400, thereby gaining 20%. What was his gain?

25. A horse costing $125 is sold for $140. Find the gain per cent.

26. 20 % of the cost is lost in selling an article for $3.60. Find the loss.

27. Find the commission at 2 % for selling 6 carloads of hogs, weighing in all 90,000 pounds, at 5¢ a pound.

28. An agent receives $160 as a 2 % commission for the purchase of a lot. Find the price paid for it.

29. A commission merchant receives $15,000, with orders to invest in cotton, and to send a bill for more money if necessary. He purchases $15,500 worth of cotton, charges $2\frac{1}{2}$ % commission, and sends a bill to his employer. How much is due to the merchant?

30. Find the number of days between Jan. 4, 1900, and August 10 of the same year.

31. Find the interest at $4\frac{1}{2}$ % on $450 for 2 years, 2 months.

32. Find a bank discount on a note amounting to $8020, discounted for 65 days at 8 %.

33. Mr. Smith sold his farm for $6360, thereby gaining 20 %. Would he have gained or lost, and what per cent, if he had sold it at $5500?

34. If 5 % is lost by selling an article for $2.47, what per cent is gained or lost by selling it at $2.99?

35. A stock of goods worth $16,400 is insured for $\frac{3}{4}$ of its value at $8\frac{1}{2}$ %. What is the premium? If the entire stock should burn, what would be the owner's loss?

36. A merchant buys through an agent 7300 yd. of carpeting at $1.25 a yard, paying $\frac{3}{4}$ % commission. The freight amounts to $7.37. At what price per yard must the carpeting be sold to realize a profit of 20 %?

37. An agent sold 13,125 bu. of wheat at \$.80 on 5% commission, and was instructed to invest part of the proceeds in 1200 barrels of flour at \$5 a barrel. If he charged 5% commission for buying, what was his entire commission, and how much did he remit to his employer?

38. If 24 bu. of corn are raised from 1 pk. of corn, what per cent is the increase? What is the increase?

39. A farmer buys 24 head of cattle at \$80 a head, and after losing 6 sells the remainder at \$105 a head. How much does he gain or lose per cent?

40. A grocer selling eggs at 20 cents a dozen clears $33\frac{1}{3}\%$ of his receipts. How much did the eggs cost him? Another grocer, selling at the same price, clears $33\frac{1}{3}\%$ on the cost. What was the cost?

41. A merchant bought silk at \$1.50 a yard. He marked it so as to give him a profit of 80%. As it did not prove a good seller, he offered it at a discount of $33\frac{1}{3}\%$ and 10% for "spot cash." What was his asking price? His selling price? His gain per cent?

42. Define bankrupt, insolvent, dividend, assets.

43. I receive \$30,423.75 as the net proceeds of a sale of goods at $\frac{1}{4}\%$ commission. Find the commission.

44. A merchant's prices are 12% above cost. He allows a discount of \$2.10 on a bill of \$33.60. What is his net per cent of profit?

45. If an article bought for one tenth less than its value is sold for one tenth more than its value, what is the per cent of gain?

46. A hotel is insured for $\frac{3}{5}$ of its value, at $1\frac{1}{2}\%$ premium, and the premium amounts to \$200. What is the value of the hotel?

47. I sold two machines for \$300 each. On one I gained 20% and on the other I lost 20%. Did I gain or lose on the transactions, and how much?

48. A merchant bought a bankrupt stock at 45 cents on the dollar, and sold it at 10% below the original price. What per cent did he gain?

49. An article lost 10% by wastage, and is sold for 30% above cost. What is the gain per cent?

50. What per cent of 30,000 bushels are 200 pecks?

51. A dealer bought an article at 10% below the price asked, and sold it for \$2400, gaining $33\frac{1}{3}\%$. What was the price asked?

52. A commission merchant receives 17 firkins of butter, each containing 43 lb.; 630 lb. of chickens; 68 cases of eggs, of 16 doz. each. He sells the butter at 26 cents a pound, the chickens at $12\frac{1}{2}$ cents per pound, and the eggs at 18 cents a dozen. His commission is 4%. Find the amount remitted.

53. If 3% more is gained by selling a horse for \$83.25 than by selling it for \$81, what is the cost of the horse?

54. If a boy gains 12% by selling 5 oranges for 14 cents, what per cent would he gain by selling them at 6 for 18 cents?

55. Mr. Jones insured his house, worth \$4800, for 3 years for $\frac{5}{6}$ of its value at 75 cents per hundred. Find the rate per cent charged, the premium, and the company's loss if the house should burn.

56. A boy on his first birthday receives a present of $600, which his father loans at 7% interest, and each year he loans out the interest at the same rate. What sum will be at the boy's disposal when he is 21 years old?

57. An invoice of 800 doz. linen handkerchiefs, valued in Berlin at $3.75 a dozen, calls for how much duty in New York at $.40 per dozen and 35% ad valorem?

58. A tax of $25,000 is assessed in a city upon a property valuation of $125,000,000. What is the rate of taxation, and what does Mr. S. Giles pay, whose store is assessed at $16,500, house at $2250, and personal property at $950?

59. What price must be asked for goods that cost $240 in order that there may be a clear profit of $16\frac{2}{3}\%$ after allowing a discount of $12\frac{1}{2}\%$ from the list prices?

60. What is the interest on $250 for 1 yr. 2 mo. at 5%?

61. A 4% note of $400 was given March 8 and paid December 15 of the same year. Find the amount.

62. Write a negotiable note for $300, bearing interest at the rate of 7%, drawn by G. J. Jones in favor of R. S. Brown, July 5, 1900, the time to be two years. Underscore the words that make it negotiable. What must R. S. Brown do if he sells the note to M. A. Star?

63. Find the proceeds of the foregoing if discounted at a bank Jan. 3, 1901, at 8%.

64. A 5% note of $1800 was given July 16, 1899. It was indorsed as follows: Sept. 6, 1899, $200; Feb. 2, 1900, $15; July 3, 1900, $300. Find the amount due Aug. 5, 1900.

65. A merchant is given the option of paying $500 in 6 months, or a 2% discount for cash. Which is the better offer, and by how much, if money is loaned at 6%?

REVIEW

184. 1. State the three general problems in percentage and the corresponding problems in common fractions and decimals. Illustrate each by an example.

2. What is profit? Loss? Cost? Selling price? List price? Discount? Net price? Upon what is the gain or loss per cent reckoned? Illustrate by an example.

3. What is commission? Upon what is the rate of commission reckoned? Illustrate by an example.

4. Define insurance policy; premium. How are premiums reckoned? Illustrate by an example.

5. What is a tax? The rate of taxation? How is the tax a person is required to pay on his property computed?

6. What are duties or customs? What is an invoice? An ad valorem duty? A specific duty?

7. Define simple interest; compound interest. What is meant by legal rate? By usury?

8. Write a negotiable promissory note. Who is the maker of the note? The payee? When does the note mature? If A buys this note from the payee before it is due, what must he do to collect the amount due at maturity?

9. Make and solve problems giving (1) principal, interest, and time, to find the rate; (2) interest, rate, and time, to find the principal; (3) principal, interest, and rate, to find the time; (4) amount, time, and rate, to find the principal. Give reason for each step in each solution.

10. Give the United States Rule for Partial Payments. The Merchants' Rule.

11. What is bank discount? How is it found?

PART III

RATIO AND PROPORTION

RATIO

Study Recitation

185. The **ratio** of one number to another of the same kind is the quotient arising from the division of that number by the other.

The ratio of 4 to 2 is $4 \div 2$, or 2; the ratio of 10 to 2 is $10 \div 2$, or 5; the ratio of 5 to 3 is $5 \div 3$, or $\frac{5}{3}$; the ratio of 7 to 9 is $7 \div 9$, or $\frac{7}{9}$; the ratio of $\frac{5}{6}$ to $\frac{2}{3}$ is $\frac{5}{6} \div \frac{2}{3}$, or $\frac{5}{4}$.

The ratio of two numbers may be expressed by writing a colon between them; as,

$6 : 9$, read, the ratio of 6 to 9.

$4 : 2$, read, the ratio of 4 to 2.

$\frac{1}{3} : \frac{3}{4}$, read, the ratio of $\frac{1}{3}$ to $\frac{3}{4}$.

The first term of a ratio is called the **antecedent.**

The second term of a ratio is called the **consequent.**

A ratio may always be expressed as a fraction, the numerator being the antecedent, and the denominator, the consequent.

The ratio of $6 : 7 = \frac{6}{7}$ Antecedent. Consequent.

A fraction is the ratio of the numerator to the denominator. Since every ratio may be regarded as a fraction, the laws governing the changes of one or both terms of a fraction by multiplication or division apply to changes in the terms of a ratio.

Multiplying the (numerator) of a (fraction) multiplies the (fraction). Dividing the (numerator) of a (fraction) divides the (fraction). Multiplying the (denominator) of a (fraction) divides the (fraction). Dividing the (denominator) of a (fraction) multiplies the (fraction).

Multiplying or dividing both (numerator) and (denominator) of a (fraction) by the same number does not change the value of the (fraction).

In the foregoing statements, substitute the word "antecedent" for the word "numerator," the word "consequent" for the word "denominator," and the word "ratio" for the word "fraction," and give each statement as thus changed.

Ratios are **simple** or **compound.**

A **compound ratio** is the product of several simple ratios term by term.

$\left.\begin{matrix} 2:3 \\ 4:5 \end{matrix}\right\}$ or $2 \times 4 : 3 \times 5$ is a compound ratio.

PROBLEMS

ORAL

186. **1.** John has 8 cents and James has 4 cents. What is the ratio of John's money to James's money?

The question, "What is the ratio of John's money to James's money?" means the same as the question, "John's money is how many times James's money?" *Ans.* 8 : 4, or $\frac{8}{4}$, or 2.

2. A can do a piece of work in 3 days. B can do the same piece of work in 5 days. What is the ratio of the time it takes A to the time it takes B?

Ans. $\frac{3}{5}$. The ratio expressed by a fraction is always abstract. It tells how many times or what part of a time the antecedent contains the consequent. The antecedent is what is measured; the consequent is the measuring unit. The result is the quotient, or the ratio.

3. In problem 2, what is the ratio of the time it takes B to the time it takes A? How else may the question be asked?

4. In one bin there are 40 bushels of wheat; in a second bin there are 20 bushels. What is the ratio of the quantity of wheat in the first bin to the quantity in the second bin?

5. If a third bin contains 48 bushels, how many bushels must the fourth bin contain in order that the ratio of the quantity of wheat in the third bin to the quantity in the fourth bin shall be the same as the ratio of the quantity in the first bin to the quantity in the second bin?

6. What is the ratio of \$3 to 6 days?

There can be no ratio between dollars and days, because dollars cannot be measured by days. The unit of the first number is \$1; the unit of the second number is 1 day. Like numbers have the same unit; as, \$3 and \$6; 4 days and 6 days; 8 hours and 4 hours.

A ratio cannot exist except between two numbers having the same unit.

7. What is the ratio of 12 days to 3 days?

8. The ratio of \$16 to some other number of dollars is the same as the ratio of 12 days to 3 days. What is the second term of the ratio \$16 : ?

9. What is the ratio of 18 : 2?

10. The ratio of 6 to some other number is $\frac{1}{3}$ the ratio of 18 to 2. What is the other number? Express the ratio.

11. The ratio of 18 to 24 is what?

12. The ratio of 8 to some other number is $\frac{1}{3}$ the ratio of 18 to 24. Write the ratio of which 8 is the antecedent.

13. What is the ratio of 4 : 4? What is the ratio of any number to itself?

14. Is the ratio of a number to a larger number greater or less than 1? Illustrate.

Is the ratio of a number to a smaller number greater or less than 1? Illustrate.

Two ratios are equal when each ratio is 1.

$$4 : 4 = 7 : 7. \qquad a : a = b : b.$$

Two ratios cannot be equal when one of them is greater than 1, and the other less than 1, or equal to 1.

Express the following ratios as fractions, and tell which are greater than 1; which are less than 1:

15. 3 : 4, 8 : 5, 12 : 35, 13 : 14

16. 256 : 185, 244 : 18, 127 : 130

17. $\frac{2}{3} : \frac{4}{5}$, $\frac{1}{2} : \frac{1}{4}$, $2\frac{1}{2} : 1\frac{2}{3}$, $\frac{3}{5} : 2\frac{3}{8}$

SIMPLE PROPORTION

Study Recitation

187. A **proportion** is an expression of equality between ratios; it may be written in three ways:

12 : 6 = 8 : 4, read, the ratio of 12 to 6 equals the ratio of 8 to 4.

12 : 6 :: 8 : 4, read, 12 is to 6 as 8 is to 4.

$\frac{12}{6} = \frac{8}{4}$, read, twelve sixths equals eight fourths.

The first and second terms of a proportion are the **first couplet**, or first ratio; the third and fourth terms of a proportion are the **second couplet**, or second ratio.

The first and fourth terms of a proportion are the **extremes**.

The second and third terms of a proportion are the **means**.

The first and third terms of a proportion are the **antecedents**.

The second and fourth terms of a proportion are the **consequents**.

The fourth term of a proportion is a **fourth proportional** to the other three terms.

When the second and third terms of a proportion are the same number, this number is a **mean proportional** to the other two. In $4:8::8:16$, 8 is a mean proportional to 4 and 16.

In every proportion, the product of the means equals the product of the extremes.

This is the principle upon which the solution of problems in proportion depends.

If three terms of a proportion are given, either two extremes and a mean, or two means and an extreme are given.

The product of the extremes divided by the given mean equals the other mean.

The product of the means divided by the given extreme equals the other extreme.

A **simple proportion** is an equality of two simple ratios; as, $8:10::12:15$.

A **compound proportion** is an equality of ratios, one or more of which are compound; as,

$$\begin{matrix} 4:6 \\ 13:5 \end{matrix} :: 2:5 \qquad \begin{matrix} 4:8 \\ 2:10 \end{matrix} :: \begin{matrix} 3:6 \\ 5:25 \end{matrix}$$

ORAL

188. Find the missing term in each of the following proportions:

1. $3:8::6:?$ Given the means and one extreme, to find the other extreme.

2. $?:9::18:54$. Given the means and one extreme, to find the other extreme.

3. $3:?::4:16$. Given the extremes and one mean, to find the other mean.

4. $2:5::?:20$. Given the extremes and one mean, to find the other mean.

5. $3:7::8:?$ Given? How solved?

6. $\frac{1}{2}:4::8:?$ Given? How solved?

7. $\frac{3}{2}:\frac{3}{4}::\frac{5}{8}:?$ Given? How solved?

WRITTEN

Cancellation may be used to shorten the operation. Indicate the operations and cancel:

8. $3:8::6:?$ $\qquad \frac{8 \times \overset{2}{\cancel{6}}}{\cancel{3}} = 16.$

9. $?:9::18:54$ $\qquad \frac{\cancel{9} \times \overset{3}{\cancel{18}}}{\underset{\cancel{6}}{\cancel{54}}} = 3.$

10. $\frac{1}{2}:4::8:?$ $\qquad 4 \times 8 \times \frac{2}{1} = 64.$

11 $\frac{3}{2}:\frac{3}{4}::\frac{5}{8}:?$ $\qquad \frac{\cancel{3}}{\underset{2}{\cancel{4}}} \times \frac{5}{8} \times \frac{\cancel{2}}{\cancel{3}} = \frac{5}{16}.$

12. $\frac{5}{8}:\frac{2}{5}::\frac{5}{6}:?$ $\qquad \frac{\cancel{2}}{5} \times \frac{\cancel{5}}{\underset{3}{\cancel{6}}} \times \frac{8}{\cancel{5}} = \frac{8}{15}.$

13. $7:9::3:?$

14. $5:?::4:12$

15. $?:15::3:18$

16. $10:12::?:15$

17. $3\frac{1}{2}:2\frac{2}{3}::\frac{1}{2}:?$

18. $1\frac{3}{8}:6::?:3$

19. In a proportion 4 is the antecedent in the first ratio and 6 is the antecedent in the second. Supply the consequents and prove that they are correct.

20. $3:?::?\ 12$. Supply the means.

Study Recitation

189. In making the statement of a proportion, when three of the terms or numbers are given to find the fourth, it is best to write the third term of the proportion first. This term is always the number which has the same unit as the required number, or answer. Then, with the conditions given in the problem, determine whether the answer must be greater or less than the third term. If the answer is to be *greater* than the third term, the *greater* of the other two numbers must be the *second* term, and the less, the first. If the answer is to be *less* than the third term, then the *less* of the other two numbers must be the *second* term, and the greater, the first.

1. If a man earns $12 in 6 days, how many dollars can he earn in 28 days?

In this problem three numbers are given, to find a fourth. The unit of two of these numbers is days, and the third number is dollars. What is asked for in the problem? *Ans.* Dollars.

Q. Which of the three numbers is the third term of the proportion? *Ans.* $12, because its unit, dollars, is the same as the unit of the required term.

Q. If a man earns $12 in 6 days, can he earn a greater or less number of dollars in 28 days? *Ans.* A greater number. Therefore 28, the larger of the two numbers, is the second term of the proportion and 6 is the first term.

The proportion is $6 : 28 :: 12 : ?$

In this proportion two means and one extreme are given to find the other extreme. Multiplying 12 by 28 and dividing by 6 gives the fourth term, 56.

$$\frac{28 \times \overset{2}{\cancel{12}}}{\cancel{6}} = 56.$$

NOTE. In solving a proportion the numbers must be considered as being abstract. In the above operation we do *not* multiply 28 da. by $12 and divide the product by 6 da.

2. If the interest on a note for 7 months is $34, what would be the interest on the same note for 15 months?

Q. In this problem what is asked for? *Ans.* Interest.

Q. Which of three numbers represents the interest? *Ans.* $34. Make $34 the third term of the proportion.

Q. If the interest on a note for 7 months is $34, is the interest on the same note for 15 months greater or less than $34? *Ans.* Greater. Therefore, place 15, the greater of the other two numbers, for the second term and 7 for the first. Solve as before.

3. If a staff 10 ft. long casts a shadow 8 ft. long, what is the height of a steeple casting a shadow 150 ft. long at the same time of day?

Q. In this problem what is asked for? *Ans.* Height of an object casting a shadow.

Q. Which of the three numbers represents the height of such an object? *Ans.* 10. Place this number for the third term of the proportion.

Q. If an object 10 ft. long casts an 8-ft. shadow, must the steeple that casts a 150-ft. shadow be greater or less in length? *Ans.* Greater. Therefore, place 150 for the second term and 8 for the first term.

4. How many days will it take 24 men to do a piece of work that can be done by 9 men in 4 days?

Q. What is asked for in this problem? *Ans.* Days.

Q. Which of the three numbers represents days? *Ans.* 4. Place this number for the third term.

Q. If a piece of work can be done by 9 men in 4 days, will it take 24 men a greater or less number of days? *Ans.* A less number. Therefore, place 9, the smaller of the other two numbers for the second term, and 24 for the first.

5. A man sold $\frac{8}{16}$ of his farm for $1850. What is the value of $\frac{2}{5}$ of the farm at the same rate?

Q. Which is the greater fraction, $\frac{3}{16}$ or $\frac{2}{5}$? *Ans.* $\frac{2}{5}$.

Q. What is asked for in this problem? *Ans.* Dollars, or value of the farm.

Q. Which of the numbers represents value? *Ans.* 1850. Place this number for the third term.

Q. If $\frac{5}{16}$ of a farm is worth $1850, is the value of $\frac{2}{3}$ of the farm greater or less than $1850? *Ans.* Greater. Therefore, place $\frac{2}{3}$ for the second term and $\frac{5}{16}$ for the first term of the proportion.

SOLUTION: $\frac{5}{16} : \frac{2}{3} :: \$1850 : ? = \$1850 \times \frac{2}{3} \div \frac{5}{16}$.

By cancellation: $\overset{370}{\cancel{1850}} \times \frac{2}{\cancel{3}} \times \frac{16}{3} = 3946\frac{2}{3}$.

Since the product of 1850 by $\frac{2}{3}$ is to be divided by $\frac{5}{16}$, we invert the divisor $\frac{5}{16}$, and indicate the operation as above.

6. If 25 pairs of boots cost as much as 40 pairs of shoes, how many pairs of shoes will cost as much as 60 pairs of boots?

Q. What is asked for? *Ans.* Number of pairs of shoes.

Q. Which of the three numbers represents pairs of shoes? *Ans.* 40. Place this number for the third term.

Q. If 25 pairs of boots cost as much as 40 pairs of shoes, will it take a greater or less number of pairs of shoes to cost as much as 60 pairs of boots? *Ans.* Greater. Therefore, place 60 for the second term and 25 for the first.

7. If a man's pulse beat 70 times a minute, how many times will it beat in $2\frac{1}{2}$ hours?

Before making the statement, reduce the hours to minutes.

Q. What is asked for in this problem?

Q. Which of the three numbers represents what is asked for? Place this number for the third term.

Q. What is the next question? Its answer?

Q. Which of the other two numbers is the second term?

8. If $\frac{7}{8}$ of a yard of broad cloth costs $2.80, how much will $12\frac{1}{2}$ yd. cost?

What is asked for? What is the third term? What is the next question and its answer? Which of the other two numbers is the second term? Which the first? Write for solution by cancellation.

PROBLEMS

WRITTEN

190. 1. How many bushels of wheat can be bought for $51.06, if 12 bushels can be bought for $6.66?

2. A garrison of 800 men has provisions for 40 days. How many men must leave to make the provisions hold out 10 days longer?

Before making the statement, the form of the problem should be changed to: A garrison of 800 men has provisions for 40 days. How many men will the same provisions feed for 50 days? When this number is found, it will be a mere matter of subtraction to find how many men must leave.

3. Thirty men can do a piece of work in 6 days. At the same rate, how long will it take 18 men to do one half the same work?

First find how long will it take 18 men to do the same work.

4. If 5 men can do a piece of work in 20 days, how many men must be added to the number to do the work in $12\frac{1}{2}$ days?

Find first how many men will be required to do the work in $12\frac{1}{2}$ days, then find what is asked for.

5. A piece of lead pipe $5\frac{1}{2}$ ft. long weighs 11 pounds. What is the weight of a piece 17 ft. in length?

6. How much wheat can be bought for $150 if 20 bushels can be bought for $16?

7. A contractor requires 80 days to build a house when his men work 10 hours a day. How many days will he require when his men work 8 hours a day?

8. A freight train runs 20 miles an hour, and a passenger train on the same road runs 35 miles an hour. If the

passenger train makes a trip between two points in four hours, how many hours will it require for the freight train to make the same trip?

9. The rates of two steamboats are as 9 : 12. If the faster boat makes a trip in 6 hours, how long will it take the slower boat to make the same trip?

10. A contractor agrees to do a piece of work in 40 days, and employs 20 men upon it. The work is half finished at the end of 30 days. How many additional workmen must he employ in order to finish the work on time?

Before beginning the solution, change the statement of the problem: At the end of 30 days as much work remains to be done as has already been done.

The problem may then be stated: If 20 men do a piece of work in 30 days, how many men will it require to do an equal piece of work in 10 days?

11. If a loaf of bread costs 4¢ when flour is $6 a barrel, how much should it cost when flour is $7½ a barrel?

12. If 4 men or 6 boys can do a piece of work in 12 days, in how many days will 6 men and 3 boys do the same piece of work?

ANALYSIS

Study Recitation

191. After solving the foregoing problems by proportion, solve them by analysis.

1. How many bushels of wheat can be bought for $51.06, if 12 bushels can be bought for $6.66?

Analysis by question and answer:

If 12 bushels can be bought for $6.66, how many bushels can be bought for $1?

If —— bushels of wheat can be bought for $1, how many bushels can be bought for $51.06?

This problem may also be analyzed in the following manner:

If 12 bushels can be bought for $6.66, what is the cost of 1 bushel?

If 1 bushel costs ——, how many bushels can be bought for $51.06?

It is better, however, in analyzing problems by the question and answer method, to ask each time for the same kind of unit as the unit required in the answer.

NOTE TO TEACHER. If pupils are trained to frame each question so as to ask for the same kind of unit as is required in the answer, they will be much less likely to become confused in their reasoning. The advantage of this plan will be very evident in dealing with problems in compound proportion.

Analysis should be required for two purposes chiefly. One is for the reasoning it involves; the other, for the training in accuracy and completeness of statement.

It is not necessary to require pupils to perform the indicated operations in analytical work, unless they can be readily performed mentally.

In the foregoing problem, the first question in the analysis is: If 12 bushels of wheat can be bought for $6.66, how many bushels can be bought for $1? The answer may be $\frac{12}{6.66}$, or, without even indicating the operation, the answer may be given as x bushels, and the next question may be framed in either of two ways: If $\frac{12}{6.66}$ bushels of wheat can be bought for $1, how many bushels can be bought for $51.06? *Ans.* $51.06 \times \frac{12}{6.66}$. Or the question may be stated in this way: If x bushels of wheat can be bought for $1, how many bushels can be bought for $51.06? *Ans.* $51.06 \times x$ bushels.

While the answer is not obtained, all the reasoning involved in the problem is required as well as the necessary statement. Bear in mind that the operation in each case is simply a matter of multiplying, dividing, adding, or subtracting.

The analysis is the reasoning, or a determination of relations and an orderly statement of these relations. Much time is wasted in arithmetical work in requiring pupils to do mere mechanical operations which they ought to be able to perform readily and accurately, when what they need at a given stage is training in reasoning and in concise statement.

After pupils have mastered the analysis of problems by finding at each step a number whose unit is the same as the unit of the answer, they may then be allowed to vary this process as in the second analysis of the problem given above.

2. A garrison of 800 men has provisions for 40 days. How many men must leave to make the provisions hold out 10 days longer?

Analysis by question and answer:

If a garrison of 800 men has provisions for 40 days, how many men would it feed for 1 day?

If a garrison of —— men has provisions for 1 day, how many men can be fed for 50 days?

If 800 men have provisions for 40 days, and the same provisions will feed —— men for 50 days, how many men must withdraw from the garrison?

3. Thirty men can do a piece of work in six days. At the same rate, how long will it take eighteen men to do one half the same work?

Q. If it takes 30 men 6 days to do a piece of work, how long will it take one man to do the same piece of work?

If it takes 1 man —— days to do a piece of work, how many days will it take 18 men to do the piece of work?

If it takes 18 men —— days to do a piece of work, how many days will it take them to do one half the work?

PROPORTIONAL PARTS

192. **1.** Divide the number 12 into two parts whose ratio is equal to the ratio of 2 to 4.

Analysis by question and answer:

Q. If the ratio of the smaller part is to the larger part as 2 : 4, for every two units in the smaller part, how many units are there in the larger part? *Ans.* 4.

Q. For every 2 units in the smaller part, how many units are there in both parts? *Ans.* $2 + 4$, or 6.

Q. If for every 2 units in the smaller part there are 6 units in both parts, or 6 units of the whole number, how many times 2 units are there in the smaller part? *Ans.* As many times as 12 units is times 6 units, or 2.

Q. If there are 2×2 units in the smaller part, what is the smaller part? *Ans.* 4.

Q. If there are 2×4 units in the larger part, how many units are there in the larger part? *Ans.* 8.

The sum of the two parts 4 and 8 is 12. The ratio of 4 to 8 equals the ratio of 2 to 4.

2. Divide the number 24 into two parts whose ratio equals the ratio of 3 to 5.

Solve in the same way as Ex. 1.

3. Separate 36 into two parts whose ratio equals the ratio of 5 to 7.

4. Separate 15 into two parts whose ratio equals the ratio of $\frac{1}{2}$ to $\frac{1}{3}$.

Before beginning the analysis, reduce $\frac{1}{2}$ and $\frac{1}{3}$ to 6ths. The ratio then becomes $\frac{3}{6} : \frac{2}{6}$, which is the same ratio as 3 : 2, and the requirement is to separate 15 into two parts whose ratio equals the ratio of 3 : 2.

5. 24 marbles are divided between James and John so that the number James has is to the number John has as 2 : 3. How many has each?

6. Divide 24 marbles between James and John so that the number James has is $\frac{2}{3}$ the number John has.

7. If John earns $\frac{3}{4}$ as much per day as Henry, what is the ratio of John's earnings to Henry's earnings? What is the ratio of Henry's earnings to John's earnings?

8. Two men are 20 miles apart and are traveling toward each other. One travels 2 miles an hour, and the other 3 miles an hour. How far will each have traveled when they meet? How many hours before they will meet?

9. A garden 18 ft. long is divided into 3 beds whose lengths are to each other as 2, 3, and 4. What is the length of each bed?

10. A farm of 360 acres is divided among three children whose shares are to one another as $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{10}$. How many acres does each have?

11. Two boys decide to commence the business of selling newspapers. One boy furnishes 25¢ and the other 40¢ to start the business. At the end of a week they have made $6.50. What is each boy's share of the profit?

12. Two men engage in business, one putting in $2500 and the other $4000. At the end of a year they have gained $3250. What is each one's share of the gain?

NOTE. The following problems are given to prepare for work with similar surfaces.

13. Draw a line 2 inches long, and mark its extremities with the letters *A*, *B*. If the length of this line is to the length of another line as 2 : 3, what is the length of the other line? Draw it. If the length of the first line drawn is to a third line as 4 : 8, what is the length of the third line? If the length of the first line drawn is to a fourth line as 3 : 5, what part is it of the fourth line? What is the length of the fourth line?

14. Draw a line 5 in. long. Its proportion to a second line is as 4 : 5. What part of the second line is it? What is the length of the second line?

15. Draw a rectangle 2 in. by 4 in. Draw another rectangle at the right of the first one whose corresponding sides shall be 4 in. and 8 in. What is the ratio of the width of the first rectangle to the width of the second rectangle? What is the ratio of the length of the first rectangle to the length of the second rectangle? Is the ratio the same in both cases? If so, the corresponding sides of the two rectangles are proportional.

16. The width of a rectangle is 4 inches. Its width is to its length as 2 : 5. What is the length of the rectangle?

Draw the figure. The sides of a second rectangle are in proportion to the corresponding sides of the first rectangle. Give the dimensions of such a second rectangle.

Rectangles whose corresponding sides are proportional are **similar rectangles.**

17. Draw a triangle whose sides are respectively 2 inches, 3 inches, and 4 inches. Mark the 2-inch side a, the 3-inch side b, the 4-inch side c. The sides of a similar triangle are in proportion to the corresponding sides of the first triangle as 4 : 2. What is the length of side a? Of side b, and of side c in the second triangle?

In similar surfaces the corresponding sides are proportional, and the corresponding angles are equal.

18. One side of a triangle is 10. If the sides of this triangle are to the sides of a similar triangle as 5 : 8, what is the length of the corresponding side of the second triangle? The side of the first triangle is what part of the corresponding side of the second triangle? The side of the second triangle is what fraction of the corresponding side of the first triangle?

19. The diameter of a circle is 6 inches. The diameter of a second circle is to the diameter of the first as 2 : 3. What is the diameter of the second circle?

20. The radius of a circle is 4 inches. What is its diameter? What is the radius of a second circle whose diameter is to the diameter of the first circle as 2 : 5?

REVIEW

193 1. Express the ratio of 6 to 3; of 3 to 6; of $\frac{2}{3}$ to $\frac{5}{7}$.

2. Name the terms of each ratio in example 1. Each term of a fraction corresponds to what term of a ratio? Illustrate.

3. State the principles governing the changes in a fraction resulting from the multiplication, and the division of one, and of both of its terms. Show how these principles apply to the terms of a ratio.

4. If a ratio is greater than 1, which of its terms is the larger? Illustrate.

5. What is a proportion? Give the different names of the terms of a proportion. What is a mean proportion? Illustrate. What is a fourth proportional? Illustrate.

6. Upon what principle does the solution of problems in proportion depend? In problems in simple proportion, what three terms are given to find the fourth?

7. If 3 cords of wood are worth \$15, how much are 4 cords worth? Of the three terms given, which two constitute one ratio? What terms constitute the other ratio?

If 3 cords of wood are worth \$15, are 4 cords worth more or less than \$15? The answer *more* indicates that the required term is greater than \$15. If \$15 is made the antecedent of the first ratio, the required term must be the consequent of this ratio. \$15 : (?) :: : . If the consequent of the first ratio is greater than the antecedent, which of the other two numbers is the consequent of the second ratio? Complete the proportion and prove its correctness.

8. Make \$15 the consequent of the second ratio. Complete the proportion. Prove its correctness. Make \$15 the consequent of the first ratio. Complete the proportion. Prove its correctness.

POWERS AND ROOTS

Study Recitation

194. A **power** is the product arising from using the same number two or more times as a factor.

$3 \times 3 = 3^2 = 9$.

9 is the product of 3 used 2 times as a factor.

In 3^2 the figure 2 shows how many times 3 is to be used as a factor.

If a number is used twice as a factor, the product is the **square**, or the **second power**, of the number. See Factoring, p. 73.

9 is the square of 3, or 9 is the second power of 3.

If a number is used 3 times as a factor, the product is the **cube**, or the **third power**, of the number.

$4 \times 4 \times 4 = 4^3 = 64$. The 3 in 4^3 shows that 64 is the product of 4 used 3 times as a factor. 64 is called the cube of 4, or the third power of 4.

A number written at the right of, and a little above, another number is called an **exponent**, and shows how many times the other number is to be used as a factor.

In the expressions 5^2, 7^3, 8^4, 9^6, 2, 3, 4, and 6 are exponents. The expressions are read, 5 square, or the second power of 5, or 5 to the second power; 7 cube, or the third power of 7, or 7 to the third power; the fourth power of 8, or 8 to the fourth power; the sixth power of 9, or 9 to the sixth power.

What does each of these expressions mean? Write the equivalent of each expression in two ways.

To the Teacher. A number is sometimes called the 1st power of itself; as, 2 is the 1st power of 2; but as there must be two factors in order to have a product, and as a power is the product of equal factors and 2 is not such a product, it cannot properly be called a power of itself. In the higher mathematics, however, the expression *first power* is used.

Read the following expressions; learn and give the value of each:

1. 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 20^2, 50^2

Find the powers indicated in the expressions below:

2. $24^2 = ?$ $25^2 = ?$ $48^2 = ?$ $55^2 = ?$ $75^2 = ?$ $100^2 = ?$

3. $8^3 = ?$ $7^3 = ?$ $9^3 = ?$ $10^3 = ?$ $100^3 = ?$ $110^3 = ?$

4. $(\frac{1}{2})^2 = ?$ $(\frac{2}{3})^2 = ?$ $(\frac{4}{5})^2 = ?$ $(\frac{3}{8})^2 = ?$ $(\frac{5}{6})^2 = ?$ $(\frac{9}{12})^2 = ?$

To find a power of a fraction, reduce it to its lowest terms, and find the required power of each of its terms.

5. $.1^2 = ?$ $.5^2 = ?$ $.01^2 = ?$ $.15^2 = ?$ $.001^2 = ?$ $.99^2 = ?$

The operation of finding the product of equal factors is called **involution**, or raising the number to a power.

The number raised to a power, is a **root** of the power.

A **root** of a number is one of the *equal* factors of the number. If it is one of two equal factors of a number, it is the **square root** of the number. $6^2 = 36$. 36 is the square of 6, or the second power of 6. 6 is one of the two equal factors of 36, and is therefore the square root of 36.

$5^3 = 125$. 125 is the cube of 5, or the 3d power of 5. 5 is one of the three *equal* factors of 125, and is therefore the cube root of 125.

In the expression $5^3 = 125$, 125 is the power, 5 is the root of the power, and 3, the exponent, shows to what power 5 is to be raised.

The operation of finding the equal factors, or **extracting the roots** of a power, is called **evolution**.

When a power is given, to find the root which was one of the equal factors used in producing the power, the root is

indicated by the **radical sign** $\sqrt{\ }$, which is placed over the number whose root is to be found. $\sqrt{25}$ indicates that the square root of 25 is to be found. The figure 2 is rarely written with the sign; thus, $\sqrt[2]{25}$. A figure so placed before the sign is called the **index**, and tells what root is to be found. $\sqrt[3]{125}$ indicates that the cube root of 125 is required. $\sqrt[4]{81}$ indicates that the fourth root of 81 is to be found.

SQUARE ROOT

195. To extract the square root.

Study Recitation

1. Extract the square root of 625.

$$\begin{array}{r|l} & 6'25(25 \\ & 4 \\ \hline 45 & 2\,25 \\ & 2\,25 \\ \hline \end{array}$$

Steps:

1. Separate the number (625) into periods of two figures each, beginning with units.
2. Find the greatest square (4) equal to or less than the left-hand period (6) and write its square root (2) for the first figure in the root, and subtract the square (4) from the left-hand period (6).
3. To the remainder (2) annex the next period (25) for a dividend.
4. Double the root (2) already found, and write the product (4) at the left of the remainder for a trial divisor.
5. Divide the new dividend (225), exclusive of the right-hand figure (5) by the trial divisor (4); write the quotient (5) for the next figure of the root, and also at the right of the trial divisor, to make the complete divisor (45).
6. Multiply the complete divisor (45) by the last figure (5) of the root, and subtract the product (225) from the dividend.
7. If there are other periods remaining, proceed in the same way.

The seven steps should be *thoroughly memorized*, omitting the numbers as they apply to this example only.

These statements of the different steps form the rule for extracting the square root of numbers.

2. Extract the square root of 1296.

Follow the directions on p. 287, without regarding the numbers.

	12′96(36 (1st, 2d)	Step 1. State it. What is the first period at the left?
	9	
	3	Step 2. State it. What is the greatest square less than 12?
	3 96	Step 3. State it. What is the new dividend?
6	3 96	Step 4. State it. What is the trial divisor?
66	3 96	Step 5. State it. What is the new dividend, exclusive of the right-hand figure? 6 is contained in 39 how many times? Where is the quotient placed?
	3 96	Step 6. State it.

Bringing the work together (as in example 1) to shorten the operation, we have the following:

	12′96(36
	9
66	3 96
	3 96

Extract the square root of the following numbers as in example 1:

3. 2304
4. 2025
5. 256
6. 4225
7. 7225
8. 4096
9. 729
10. 1156
11. 5184

12.

	6′05′16(246
	4
44	2 05
	1 76
486	29 16
	29 16

In the fifth step, on dividing 20 by the trial divisor 4, we get 5; but when 5 is annexed to the trial divisor and the complete divisor 45 is multiplied by 5, the product is found to be larger than the dividend. This shows that the quotient 5 is too large, and must be decreased by 1, making it 4.

Bring down the next period and proceed as before, doubling the number already found in the root, and dividing 291 by 48, etc. If there were still other periods, the process would be continued in the same way.

13. 133225
14. 104976
15. 522729
16. 98596
17. 390625
19. 492804
20. 256036
21. 62500
22. 490000

18.
$$\begin{array}{r|l} 9'42'49 & 307 \\ 9 & \\ \hline 607 & 4249 \\ & 4249 \\ \hline \end{array}$$

The trial divisor 6 is not contained in 4. In such cases, place a zero in the root and at the right of the trial divisor, and bring down the next period. Divide 424 by 60; place the quotient 7 in the root and at the right of the trial divisor.

If, after annexing a zero as above and bringing down the next period, the trial divisor is not contained in the dividend, exclusive of the right-hand figure, annex another zero, bring down another period, etc.

In extracting the square root of decimals, begin at the decimal point and point off to the right into periods of two places, annexing zeros if necessary.

Point off .47856
.47'85'60

Begin at the decimal point and point off to the right, annexing one zero, so that each period shall have two places; thus, .47'85'60.

In pointing off mixed decimals, point from the decimal point to the left in the integral part and to the right in the decimal part, annexing a zero if necessary to make the last period in the decimal two places.

23. 45.1584
24. 3003.04
25. 27.5625
26. 39.8161
27. 50.8369
28. 66.2596

When the number whose root is to be extracted is not a perfect square, the root can be found approximately, but not exactly, by annexing periods of zeros after the decimal point, carrying the work as far as desired. It is usually sufficient to carry the work out two decimal places.

29. Extract the square root of 295 to two decimal places. (See work on next page.)

30. Extract the square root of 6 to three decimal places. (See work on next page.)

29. $2'95.'00'00)17.17+$

	1
27	195
	189
341	600
	341
3427	25900
	23989

30. $6.'00'00'00)2.449$

	4
44	200
	176
484	2400
	1936
4889	46400
	44001

31. 75

32. $\frac{9}{16}$

33. $\frac{3}{8}$

34. $\frac{2}{7}$

When both terms of a fraction whose square root is to be extracted are squares, extract the square root of each; otherwise, reduce the fraction to a decimal, carrying on the division until the decimal has twice as many places as there are figures desired in the root, then extract the square root of the decimal.

PROBLEMS

WRITTEN

196. **1.** A square piece of land contains 1440 A. How wide is it?

2. A man has a rectangular board 32 in. wide and 10 ft. 8 in. long, from which he makes the top of a square table as large as possible. Find its dimensions.

3. What is the cost of fencing a square farm of 160 A. at the rate of \$4 per rod?

4. At 60¢ a rod, what is the cost of fencing a square farm whose area is 262 A., 105 sq. rd.? How much additional will it cost to divide it into 3 equal parts by building cross fences?

5. Formulate a problem involving an application of square root. Solve it.

NOTE. For further applications of square root, see work in similar figures, right-angled triangles, and circles.

EXPLANATION OF THE PROCESS OF EXTRACTING THE SQUARE ROOT

197. Relation of the number and places of figures in a root to the number and places of figures in its square.

$1^2 = 1$ The square of the smallest number of one figure contains one figure.

$9^2 = 81$ The square of the largest number of one figure contains two figures.

The square of any number between 1 and 9 contains one, or at most two figures.

$10^2 = 100$
$90^2 = 8100$ Annex a zero to 1, making 10, and to 9, making 90, and square these numbers. It will be seen that annexing a zero to the smallest and to the largest number of one figure increases the number of figures in each square by two.

$19^2 = 361$
$99^2 = 9801$ Annex 9, the largest number of one figure, to 1, making 19, and to 9, making 99. It will be seen that annexing the figure 9 to the smallest, and to the largest number of one figure, increases the number of figures in each square by two.

It follows that annexing a figure of any value to any one-figure number increases the number of figures in the square by two.

$100^2 = 10000$
$990^2 = 980100$ Annex a zero to 10, making 100, and to 99, making 990, and square these numbers. It will be seen that annexing a zero to the smallest, and to the largest number of two figures, increases the number of figures in each square by two.

$109^2 = 11881$
$999^2 = 998001$ Annex 9, the largest number of one figure, to 10, making 109, and to 99, making 999, and square each. Annexing the figure 9 to the smallest and to the largest number of two figures, increases the number of figures in the square by two.

$1000^2 = 1000000$
$9990^2 = 99800100$ Compare the square of 100 with that of 1000 and the square of 999 with that of 9990.

$1009^2 = 1018081$
$9999^2 = 99980001$ Compare the square of 100 with the square of 1009 and the square of 999 with the square of 9999.

$10^2 = 1'00$
$100^2 = 1'00'00$
$900^2 = 81'00'00$
$990^2 = 98'01'00$
$999^2 = 99'80'01$

The square of the first figure of the root is found in the first period of the square.

The square of the first two figures of the root is found in the first two periods of the square, and so on.

From the above, it follows that the square of any number of one figure contains one or two figures; and that if any figure is annexed to any one-figure number, the number of figures in its square is increased by two; that the number of figures in any square is twice the number, or one less than twice the number of figures in its square root; that pointing off a square into periods of two places each, beginning with units, will show the first period at the left, whether of one or two figures; that the left-hand or first period contains the square of the first figure in the root, and that the first two periods contain the square of the first two figures in the root, and so on.

Composition of the Square of a Number

A number is squared by multiplying it by itself.

$25^2 = 25$ or $25^2 = (20 + 5)^2 = 20^2 + 2$ times $20 \times 5 + 5^2$

$\underline{25}$

$25 = 5 \times 5$ $\qquad 20 + 5$

Indicating the operations.

$100 = 5 \times 20$ $\qquad \underline{20 + 5}$

$20^2 = 400$

$100 = 20 \times 5$ $\qquad 20 \times 5 + 5^2$ $\qquad 2 \times 20 \times 5 = 200$

$\underline{20^2 + 20 \times 5}$ $\qquad 5^2 = 25$

$400 = 20 \times 20$

$\overline{625} = 25 \times 25$ $\qquad 20^2 + 2 \times 20 \times 5 + 5^2$ $\qquad \overline{625}$

In the number 25, the figure 5 represents units, and the figure 2 represents tens, or 20 units.

Regard 20 as representing the value of 2 tens, in units, and 5 as representing the value of 5, in units; then the square of 25, or of $20 + 5$, equals the square of the tens, or 400, + 2 times the product of the tens by the units, or 200, + the square of the units, or 25. Represent the tens, or 20, by t, and the units, or 5, by u; then the square of $t + u$, or $(t + u)^2 = t^2 + 2 \times t \times u + u^2$. Since any number of two or more figures

may be regarded as made up of tens and units, as 275 = 27 tens, or 270 + 5 units, the square of any number of two or more figures always equals:

The square of the tens plus two times the tens multiplied by the units plus the square of the units.

1. Extract the square root of 625.

```
 2  5
6'25
 4 00
40 2 25
   2 00
     25
     25
```

Beginning at units to point off into periods of two figures, we find that the first period consists of one figure, 6.

It has been shown that the square of the first figure of the root is found in the first period. The greatest square equal to or less than 6 is 4. Its square root is 2, the first figure of the root. The two periods show that there are two figures in the root, hence the value of its first figure in units is 20. The square of 2 tens, or 20, is 4 hundreds, or 400. To show clearly what is done, the two zeros are annexed to 4, the square of the first figure of the root, and the subtraction is then made. The result, 225, is the same as though the zeros had been omitted (as is ordinarily done), the 4 subtracted from the first period and the next period annexed to the remainder.

We have now taken from the square one of the three parts of which it is composed; namely, the square of the tens. The remainder is made up of the other two parts; namely, 2 times the product of the tens by the units + the square of the units.

We have found the tens' figure of the root, but have yet to find the units' figure. Of the remaining two parts, the second part, the square of the units is much the smaller, and in finding the trial units' figure may be ignored. Leaving it out of account, the 225 may be regarded as made up of 2 times the product of the tens, or 20, by the units. Here we have the product of three factors, and two of them known, to find the other factor. The two known factors in 2 times the product of 20 by the units are 2 and 20. To find the trial units' figure in the root, divide 225, the supposed product of the three factors, by 40, the product of the known factors, 2 and 20, and we have 5 for the quotient, or the other factor, the units' figure of the root.

Multiplying 40 by 5, we have 200, or 2 times the product of the tens by the units, or the second part of the square. Subtracting, we have left 25, or the square of the units, the third part of the square. If the trial units' figure is correct, its square equals 25, which is true; hence, 5 is the correct units' figure.

$$\begin{array}{r} 2\ 5 \\ \overline{625} \\ 4 \\ \hline 45\,\big|\,225 \\ \quad\ \ 225 \end{array}$$

In actual work all the zeros are omitted. The trial divisor, 4, is used to divide the dividend, 225, exclusive of the right-hand figure, or to divide 22, as this gives the same result as though 40 and 225 were used. When the trial units' figure is found, it is annexed to 4, the trial divisor, making 45. 45 is equal to 2 times the tens + 5. This being multiplied by 5, the units' figure, gives 2 times the product of the tens by the units + the square of the units in one operation.

2. Extract the square root of 3249, explaining the process. Solve the example, answering each question in order.

1. What is the first step?
2. Give the reason for this step.
3. How is the first figure of the root found? Why? What is it?
4. The square of the first figure of the root, considered as tens, is what part of the number whose root is to be found?
5. What is done with this square?
6. What remains contains what parts of the entire number?
7. What part of the entire number is it regarded as being, for the purpose of finding the second figure of the root?
8. What factors does this part contain?
9. Which of these factors are known?
10. When the product of these factors and two of the factors are known, how can you find the other factor?
11. What is done in this case to find the trial second figure of the root?
12. What is done with this trial figure? Why?
13. The result contains what parts of the entire number?
14. What is done with this result?
15. What is the required root?

The square subtracted from the first period, and the product of the second figure of the root by the complete divisor, together equal the entire number. If there were three figures in the root, these two numbers together would equal the square of the first two figures of the root considered as tens, and the units' figure would remain to be found in the same way as the second figure of the root was found, and so on, if the root contained more than three figures.

REVIEW

198. 1. Define power; square; cube; exponent; involution.

2. What is meant by evolution in arithmetic? By square root?

3. What is the difference between a root and a factor of a number?

4. What does the radical sign indicate? What is the index of a root?

5. What is the relation of the number and the places of figures in a root to the number and the places of figures in its square?

6. Square the number 34 by separating it into its tens and units $(30+4)\times(30+4)$. In the resulting equation $(30+4)^2=30^2+2\times30\times4+4^2$, represent the tens, or 30, by t and the units, or 4, by u. What formula results? Translate this formula into a general statement expressing the value of the square of any integer composed of tens and units.

7. What is a trial divisor? How is it found? On what principle?

8. Extract the square root of 841 and explain each step in the operation.

9. Make a rule for the extraction of any square root.

10. How is the square root of a common fraction extracted? The square root of a decimal?

11. When a number is not a perfect square, how may its approximate square root be found?

12. Make a problem demanding for its solution the extraction of a square root. Solve it.

APPENDIX

GREATEST COMMON DIVISOR

199. Another method of finding the greatest common divisor of two or more numbers.

1. Find the G. C. D. of 24, 36, 60.

$$\begin{array}{r|l} 2 & 24,\ 36,\ 60 \\ \hline 2 & 12,\ 18,\ 30 \\ \hline 3 & 6,\ 9,\ 15 \\ \hline & 2,\ 3,\ 5 \end{array}$$

$2^2 \times 3 = 12$, G. C. D.

The numbers given are exactly divisible by 2, and their quotients again by 2. Therefore, 2×2 is a factor of the greatest common divisor. The quotients obtained after dividing the second time by 2 are all divisible by 3; hence, 3 is a factor of the greatest common divisor. No number will exactly divide each of the last set of quotients; hence, there is no other factor common to all the numbers. Therefore, 2, 2, and 3 are the only exact divisors of the given numbers, and their product, 12, is the greatest common divisor.

Find the G. C. D. of:

2. 18, 24, 60
3. 64, 105, 168
4. 60, 45, 120
5. 144, 264, 360, 384

A divisor of any two numbers is a divisor of their sum and of their difference.

A divisor of a number is a divisor of any multiple of it.

In example 2, the G. C. D. cannot be greater than 6 because it must divide the difference between 18 and 24, or 6. If 6 will divide each of the numbers, it is the G. C. D.

In example 3, the G. C. D. must divide the sum of 64 and 105, or 169. It must also divide the difference between this sum and the third number, 168. Therefore it cannot be greater than 1.

LEAST COMMON MULTIPLE

200. Another method of finding the least common multiple.

1. Find the least common multiple of 24, 30, 45.

2	24, 30, 45
3	12, 15, 45
5	4, 5, 15
	4, 1, 3

$2 \times 3^2 \times 4 \times 5 = 360$, L. C. M.

The numbers are arranged as for greatest common divisor, and the prime number that will divide two or more of the quotients or undivided numbers is selected. The process is continued until the quotients and undivided numbers are prime to each other. The least common multiple is then obtained by taking the continued product of the several divisors, the last quotients, and the undivided numbers, if any.

Find the least common multiple of:

2. 16 and 24
3. 30, 45, and 70
4. 84, 96, and 108
5. 75, 150, and 190

ANNUAL INTEREST

201. In some states, when a note reads, "with interest payable annually," if any year's interest is not paid when due, simple interest may be collected not only on the principal but also on each year's interest that remains unpaid. This is called **annual interest.**

1. Find the amount due on a note for $500 given Jan. 10, 1904, with interest payable annually at 6%, and paid June 20, 1908.

$$\begin{array}{r} 1908 + 6 + 20 \\ 1904 + 1 + 10 \\ \hline 4 + 5 + 10 \end{array}$$

At the end of each year from the date of the note, 1 year's interest was due, but was not paid until June 20, 1908, and so should bear interest from the date it was due until the date of settlement.

	yr.	mo.	da.	
The first year's interest was unpaid and bore interest for	3	5	10	
The second year's interest was unpaid and bore interest for	2	5	10	
The third year's interest was unpaid and bore interest for	1	5	10	
The fourth year's interest was unpaid and bore interest for		5	10	
Therefore, 1 year's interest, or $30, bore interest for	7	9	10	
The interest on the unpaid interest due annually =				$ 14.00
The interest on the principal, $500, for 4 yr. 5 mo. 10 da. at 6% =				133.33
The entire interest due =				$147.33
The entire amount due =				$647.33

Find the amount of the following with annual interest:

2. $1500 for 3 yr. 6 mo. at 8%.

3. $1250 for 4 yr. 5 mo. 18 da. at 6%.

4. $1020 for 3 yr. 7 mo. 20 da. at 6%.

5. $250 for 4 yr. 8 mo. 15 da. at 8%.

6. $175 for 6 yr. 4 mo. 14 da. at 5%.

7. What is the amount, annual interest being unpaid, on a debt of $500 due in 6 years at 7%?

COMPOUND INTEREST

202. Compound interest is interest computed on the amount of the principal and interest, if the interest is not paid when due. In most states it cannot be collected by law.

Interest is compounded annually unless otherwise stipulated. It may be compounded semiannually, quarterly, etc., by agreement.

To compute compound interest:

RULE: *Find the amount of the principal for the first period of time. With this amount as a new principal, find the amount for the second period of time, and so continue for all the full periods of time for which interest is compounded If, at the expiration of the last*

full period, there is still some time for the note to run, treat the last amount found as a new principal, and find the amount for the remaining time.

The final amount is the sum due. The difference between this amount and the first principal is the compound interest.

1. What is the amount due on a note for $700, paid 3 years and 4 months from date, interest compounded annually at 7 %? What is the compound interest?

SOLUTION

$700	First principal.
.07	
49.00	
700.00	
749.00	Amount for first period of time. New principal.
.07	
52.43	
749.00	
801.43	Amount for second period of time. New principal.
.07	
56.1001	
801.43	
857.53	Amount for last full period of time. New principal.
.07	
3)60.0271	
20.00	Interest for remaining time, 4 months.
857.53	
877.53	Amount for remaining time, 4 months. Amount due.
700.00	
$177.53	Compound interest.

2. What is the amount of $300 for 4 years at 6 %, interest compounded annually? What is the compound interest?

3. What is the compound interest of $250 for 3 yr. 6 mo. 18 da. at 6 %, interest compounded annually?

Compound interest tables are used in computing compound interest. The table on page 301 shows the amount of $1 at compound interest for any number of years up to 20, and for any rate per cent up to 7.

Written Exercises, using the Table

203. 1. Find the amount of $500 for 20 years at 7%, compound interest.

Solution

The amount of $1 for 20 years at 7%, as shown by the table (p. 301), equals $3.869684.

Hence, the amount of $500 = 500 × $3.869684 = $1934.84. *Ans.*

2. Find the amount of $800 at 6% for 12 years, compound interest.

3. What is the compound interest at 7% on $2000 for 15 years?

4. Find the amount of $1200 for 9 years at 4%, compounded semiannually.

Note. When interest is compounded semiannually, one half the given rate and twice the number of periods for which interest is compounded are used. Thus, in problem 4, find the interest at 2% for 18 years.

5. What is the amount and the compound interest of $400 for 2 yr. 3 mo. at 6%, interest compounded semiannually?

Note. Compute the interest for 4 yr. at 3%, and then for the remaining time at 6%.

6. Find the compound interest on $5000 for 10 years at 7%, compounded semiannually.

7. What is the difference between the compound and simple interest on $6000 for 6 years at $4\frac{1}{2}$%?

8. Formulate and solve a problem in compound interest.

COMPOUND INTEREST TABLE

Amount of $1, *at various rates, compound interest,* 1 *to* 20 *years*

Years	1 Per Cent	1¼ Per Cent	1½ Per Cent	2 Per Cent	2½ Per Cent	3 Per Cent
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848
18	1.196148	1.250477	1.307341	1.428246	1.559659	1.702433
19	1.208109	1.266108	1.326951	1.456811	1.598650	1.753506
20	1.220190	1.281935	1.346855	1.485947	1.638616	1.806111

Years	3½ Per Cent	4 Per Cent	4½ Per Cent	5 Per Cent	6 Per Cent	7 Per Cent
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815
18	1.857489	2.025817	2.208479	2.406619	2.854339	3.379932
19	1.922501	2.106849	2.307860	2.526950	3.025600	3.616528
20	1.989789	2.191123	2.411714	2.653298	3.207136	3.869684

PRESENT WORTH AND TRUE DISCOUNT

204. The **present worth** of a debt payable at a future time, without interest, is such a sum as, placed on interest for the time at a certain rate, will amount to the debt. Thus, the amount of $1 for 3 yr., at 6 %, is $1.18. Therefore, the present worth of $1.18 due in 3 yr., without interest, is $1.

The **true discount** is the difference between the amount of the debt and the present worth.

NOTE. The true discount is the interest on the sum representing the present worth. The *present worth* is the principal, the *true discount* the interest, and the *debt* the amount. Problems coming under this head are solved, therefore, like corresponding problems in interest.

PROBLEMS

WRITTEN

205. 1. What is the present worth of a debt of $163.50, due in 1 year 6 months, at 6 %? What is the true discount?

SOLUTION

1 yr. 6 mo. = 1½ yr.
Int. of $1 for 1½ yr. at 6 % = 1½ × $.06 = $.09.
$1 + $.09 = $1.09, amount of $1 for the given time.
$163.50 ÷ $1.09 = $150, present worth.
$163.50 − $150 = $13.50, true discount.

2. How much must be paid now to cancel a debt of $99.45, due in 1 year 9 months, at 6 %?

3. What is the present worth of $12,060, due in 5 years 8 months, without interest, money being worth 6 %?

4. I owe $871.50, payable in 3 years 9 months, without interest, but wish to pay it immediately. How much should be paid, money being worth 7 %?

5. I bought two lots for $2541 on 3 years' time, without interest. What is the cash value, money being worth 7 %?

6. Mr. Brown gives his note for $ 88, payable in 2 years 8 months, without interest. How much should he pay instead, at the end of 8 months, money being worth 5%?

7. A owes $ 6000, of which one third is to be paid in one year, and the remainder in two years. What is its present cash value, money being worth 6%?

8. I bought a store for $ 1500 cash, and sold it for $ 2000, one fourth to be paid in cash, the remainder in 2 yr., without interest. What was my gain, money being worth 6%?

9. A merchant purchased $ 900 worth of hardware at a discount of 25%. He sold the goods at the list price on 9 months' credit. What was his gain, money being worth 6%?

10. With money at 8%, which is more advantageous, to buy goods for $ 425 on 6 months' credit, or for $ 450 on 9 months' credit?

DOMESTIC EXCHANGE

206. Exchange is the method of paying debts or collecting credits in distant places without transmission of money.

Exchange between two places in the same country is called **domestic exchange.**

Payments in the same country may be made by *postal money order*, by *express money order*, by *telegraphic money order*, by *bank check*, by *bank draft*, or by a commercial draft of a creditor on a debtor.

A **postal money order** is a written order by the postmaster in one place, to the postmaster in another place, to pay a specified sum of money to the person named in the order.

An **express money order** issued by an agent of an express company to another agent is similar to a postal money order. Both are negotiable.

Postal and express money orders, in addition to the sum specified on the face, cost as follows:

Orders for sums not exceeding	$2.50	3¢
Over $2.50 and not exceeding	5.00	5¢
Over 5.00 and not exceeding	10.00	8¢
Over 10.00 and not exceeding	20.00	10¢
Over 20.00 and not exceeding	30.00	12¢
Over 30.00 and not exceeding	40.00	15¢
Over 40.00 and not exceeding	50.00	18¢
Over 50.00 and not exceeding	60.00	20¢
Over 60.00 and not exceeding	75.00	25¢
Over 75.00 and not exceeding	100.00	30¢

NOTE. Postal orders are not issued for sums larger than $100.

A **telegraphic money order** is similar to the two orders mentioned. The rates are:

For $25 or less, double the cost of a ten-word message plus 25 cents.

For amounts over $25, double the cost of a ten-word message plus 1% of the amount of the order.

A **bank check** is an order on a bank, given by one of its depositors, to pay a specified sum to the order of a certain person.

STUB

No. 209
Date Feb. 2, '09.
Payable to
H. W. Brown
For ---Labor---
Am't. $31.75

CHECK

No. 209 Milwaukee, Wis., Feb. 2, 1909

First National Bank
OF MILWAUKEE

Pay to the order of H. W. Brown _____

Thirty-one and 75/100 ~~~~~~~~~~ Dollars.

$31.75 James T. Arnold.

For what amount is the check issued? To whom? On what bank? If Mr. Brown wishes to dispose of the check to Henry Smith, what must he do?

A check like the foregoing may be sent by mail. It is cashed in some bank, and this bank collects it from the one on which it is drawn. As a rule a small fee, called **exchange**, is charged for collecting a check in any town outside the location of the bank on which the check is drawn.

A **bank draft**, or **bill of exchange**, is a check drawn by one bank upon another.

BANK DRAFT

No. 769

Third National Bank

Chicago Ill., June 8, 1909.

Pay to the order of Robert Saunders $700.00

Seven hundred $\frac{no}{100}$ ~~~

Collections may also be made by **commercial draft**. If Henry Smith of Chicago owes William Brown of Milwaukee $3000, which Mr. Brown wishes to collect at once, he may draw on Henry Smith as follows:

SIGHT COMMERCIAL DRAFT

$3000 $\frac{no}{100}$ *Milwaukee, Wis., April 10, 1909.*

At sight pay to the order of Traders' Bank of Milwaukee

Three thousand $\frac{no}{100}$ *Dollars.*

Value received, and charge to account of

To Henry Smith, *William Brown.*

533 Wabash Ave., Chicago, Ill.

The Milwaukee bank sends this draft to some Chicago bank, which collects the amount of Henry Smith and remits it to the Milwaukee bank, which in turn pays the amount, less a small fee for collection, to William Brown.

The following draft due "Ninety days after sight," is called a **time draft**.

TIME COMMERCIAL DRAFT

$3000 $\frac{00}{000}$ *Milwaukee, Wis., April 10, 1909.*

Ninety days after sight Pay to

the order of Traders' Bank of Milwaukee

Three thousand $\frac{no}{100}$ *Dollars.*

Value received, and charge to account of

To Henry Smith, *William Brown.*

533 Wabash Ave., Chicago, Ill.

If Mr. Smith intends to pay the draft when presented to him, he writes across the face in red ink "Accepted," and then adds the date and his signature. This draft has now the force of a promissory note and may be discounted, for the time it has to run, after date of acceptance.

Formerly, if trade between two trade centers like Chicago and New Orleans was equal, exchange was at par. If Chicago owed New Orleans, the demand in Chicago for drafts on New Orleans was greater than the demand in New Orleans for Chicago drafts. Hence, the drafts were at a premium in Chicago. Under reversed conditions, they were at a discount. For a long time, however, domestic exchange has been at par.

PROBLEMS

WRITTEN

207. **1.** Find the cost of a postal or an express money order for \$2.35; \$2.55; \$8; \$55.75; \$98.60.

2. Find the cost of sending a telegraphic money order for \$2345 at 1 % plus \$.75 for the message.

3. What must be paid in New Orleans for a draft on Chicago for \$800, at $\frac{1}{10}$ % exchange?

SOLUTION: $\frac{1}{10}$ % of \$800 = \$.80, cost of exchange.
\$800 + \$.80 = \$800.80, cost of draft for \$800.

4. A merchant in Chicago owes a merchant in Milwaukee \$475. How much must be paid in Chicago for a draft to settle the account, at $\frac{1}{4}$ % exchange?

5. How much must be paid for a draft of \$354 at $\frac{1}{5}$ % exchange?

6. Find the proceeds of a draft for \$5000, payable 30 days after sight, if discounted at 6 % for the full term.

SOLUTION: The bank discount on \$1 for 30 da. at 6 % = \$.005.
5000 × \$.005 = \$25, bank discount on \$5000.
\$5000 − \$25 = \$4975, proceeds.

7. Find the proceeds of a draft for $21,000, for ninety days after date, if discounted at 5% for the full term.

8. T. C. Arnold & Co. of New York drew a sight draft for $2000 on R. J. Steele of Superior, Wis. This draft was sold at a bank at $\frac{1}{2}$% discount. Find the proceeds.

FOREIGN EXCHANGE

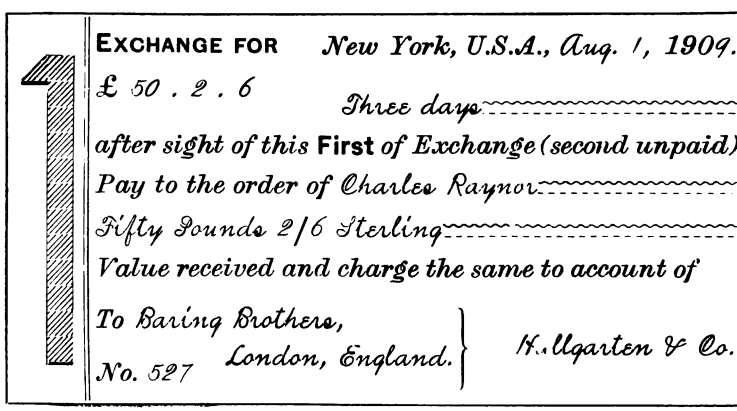
208. Exchange between places in different countries is called **foreign exchange.**

Foreign exchanges are usually transacted by *international money orders, letters of credit, travelers' checks,* and *bills of exchange.*

A **letter of credit** is a letter from a bank in one country to one or more of their correspondent banks in another, directing them to pay to the holder any sum not exceeding a certain amount specified in the letter.

Travelers' checks are guaranteed checks issued by express companies or banks in denominations of $10 and multiples thereof, at a fixed rate of $\frac{1}{2}$% of the face value.

A **foreign bill of exchange** is a draft of a bank in one country upon a bank in another. It is payable in the money of the country on which it is drawn.


EXCHANGE FOR New York, U.S.A., Aug. 1, 1909.
£ 50 . 2 . 6
Three days
after sight of this First of Exchange (second unpaid)
Pay to the order of Charles Raynor
Fifty Pounds 2/6 Sterling
Value received and charge the same to account of
To Baring Brothers,
No. 527 London, England.
K. Ugarten & Co.

To guard against loss, foreign bills of exchange are usually issued in duplicate. The duplicate reads "Second of Exchange (first unpaid)" and the number 2 appears instead of 1 in the left margin.

Problems in foreign exchange are solved in the same manner as in domestic exchange, except that the currency of one country must be reduced to that of another.

Exchange on England is quoted by giving the cost of a bill for £1. Thus, 4.86 means that a bill of exchange for £1 costs $4.86; German exchange is quoted by giving the number of cents that 4 marks cost. Thus, a quotation of 95⅞ means that 4 marks cost 95⅞¢; French exchange is quoted by giving the number of francs that can be bought for $1. Thus, 5.17 means that 5.17 francs can be bought for $1.

PROBLEMS

WRITTEN

209. 1. What is the cost of a draft on London for £380 15*s*., exchange at $4.8665?

SOLUTION: £380 15*s*. = £380.75; 380.75 × $4.8665 = $1852.92, cost of draft.

2. Find the cost of a bill of exchange on Berlin for 1750 marks, exchange being at 96¼.

SOLUTION: Since 4 marks cost $.96¼, 1750 marks will cost (1750 ÷ 4) × $.96¼, or $421.09, cost of draft.

3. Find the cost of a bill of exchange on Paris for 2060 francs when exchange is quoted at 5.15.

SOLUTION: Since 5.15 francs cost $1, 2060 francs cost (2060 ÷ 5.15) × $1, or $400, cost of draft.

Find, to the nearest cent, the cost of exchange for:

4. £200 @ $4.87.
5. 5000 francs @ 5.17.
6. 10,000 M. @ 95⅛.
7. £500 12*s*. @ 4.88.

8. How much must a merchant in Canton pay for a draft of $1134, if 1 tael costs $.642?

STOCKS AND BONDS

210. A **company** is an association of individuals for the transaction of business.

A **corporation** is a company governed in its operations by a general law or a special charter.

A **charter** is the legal act of incorporation. It states the powers and duties of the corporation.

The **stock** of a company is the money invested by it to carry on its business. The owners of stocks are called **stockholders.**

A **share** is one of the equal parts into which the stock has been divided. In bank, insurance, and railway companies it is usually $100.

The **face,** or **par value,** is the sum for which the share is issued. This is usually $100.

The **market value** is the sum for which a share can be sold.

A **dividend** is a sum paid to the stockholders out of the gains of the company. It is computed as a per cent of the face or par value of the stock.

An **assessment** is a sum required of the stockholders, according to the amount of stock held by each, to meet losses or expenses of the company.

Bonds are promissory notes issued by corporations and secured by mortgages on some property owned by the corporation. The usual face value of a bond is $100.

Coupon bonds have interest certificates attached to them, which are redeemed as the interest comes due.

Registered bonds are recorded in the name of the owner in the books of the company issuing them.

Par and market value in bonds have the same meaning as in stocks.

Stocks and bonds are **at par** when they sell for the face value; **above par**, or at a premium, when they sell for more than the face value; **below par**, or at a discount, when they sell for less than the face value.

Bonds bear interest at the rate specified on the face. It is always computed on the face value. A 4% bond is one that bears $4 interest per $100 share in one year. U. S. 5-20's are United States bonds bearing 5% interest, and running for 20 years.

A **broker** is a person who buys and sells stocks and bonds for another.

Brokerage is the compensation of the broker. It is computed on the par value. Brokerage $\frac{1}{8}$ means that the broker receives $\frac{1}{8}$ of a dollar for each $100 share bought or sold; that is, $\frac{1}{8}$% of the par value.

STOCK CERTIFICATE

1. What is the name of the corporation that issued this stock certificate? Where is it located? What is the amount of the capital stock? What is the par value of each share?

Who is the owner of this certificate? How many shares of the stock are represented by it? What is its face value?

2. If the company declares a 5 % dividend, what is Mr. Fluker's income from the stock?

3. If he sells it at a premium of $12\frac{1}{2}$ %, how much will he receive for it?

The following are some stock quotations copied from a daily paper:

	Open.	High.	Close Nov. Low.	14.		Open.	High.	Close Nov. Low.	14.
Allis-Chalmers.	18¾	18¾	18¾	18¾	Col. Southern	46¼	46⅝	45½	46
Am. Sugar	136	136⅞	135⅞	135⅞	Con. Gas	150½	151⅝	149⅞	149¼
Am. Loco.	56½	56¾	56⅛	56⅜	Erie	35⅝	35⅝	34⅞	34⅞
Am. Car. & F.	47½	47½	46¾	47¼	Erie, 1st pfd.	49	49⅞	48⅞	48⅞
Am. Smelter	96¾	96⅞	95¾	95⅞	Ill. Central	147	149	147	147
Amal. Copper	86⅝	87	85⅞	85⅞	K. C. Southern	30⅞	30⅞	30½	30½
Am. Ice	25¾	25¾	25⅝	25⅝	Louis. & Nash.	116¼	116¼	115⅞	116
Am. Tob.	93	93	93	93	Mo. Pac.	63⅞	63½	62⅝	62⅝
Am. Wool	31¼	31⅞	31¼	31¾	Nat. Lead.	85¼	85¼	84¼	84½
Anaconda	52⅝	52⅞	52⅞	52⅞	Northern Pac.	153½	154⅛	153⅝	153⅞
Atchison	96¼	96¼	95⅞	95⅞	Nor. & West.	83¾	83¾	83	83
Atchison, pfd.	98¼	98⅝	98¼	98½	N. Y. Central	115½	115½	114½	114¾
B. R. T.	54⅝	56⅛	54⅝	56	People's Gas	100	100⅛	100	100¼
B. & O.	106⅝	107	106½	106⅞	Penna. R. R.	130¼	130⅝	129⅝	129⅞
Biscuit	89¼	89¼	89	89¼	Reading	140¼	140⅝	138⅝	138½
Can. Pacific	178	179½	178	178¼	So. Pac.	117⅝	117⅞	116⅝	116½
Cent. Leather	28½	28⅞	28	28¼	Union Pacific	181½	181½	180½	180⅝
Ches. & Ohio	46½	46½	46	46	U. S. Steel	57⅞	58⅛	57¼	57½
C. M. & St. Paul.	149½	149½	148¼	149	Wabash	15⅞	15⅞	15	15
C. & N. W.	171½	171½	171¼	171½	Wis. Central	30¼	30¼	29¼	29¼

NOTE. Stock quoted at 93 sells at $93 for each share of $100 face value.

4. What was the highest price commanded by the C. M. & St. Paul railway stock in above list? The lowest? At what premium did it sell at highest price?

5. Answer the foregoing questions for U. S. Steel stock.

6. What was the highest price paid for Wabash railway stock? The lowest? At what discount did it sell at lowest price?

7. Why do some stocks sell at a discount while others sell at a premium?

PROBLEMS

WRITTEN

211. 1. What is the cost of 45 shares of railway stock at $5\frac{3}{8}\%$ premium, brokerage $\frac{1}{8}$?

SOLUTION

$\$100 + \$5\frac{3}{8} = \$105\frac{3}{8}$, market value of one share.
$\$105\frac{3}{8} + \$\frac{1}{8} = \$105.50$, cost of one share.
$45 \times \$105.50 = \$4747.50$. *Ans.*

2. B sold 46 shares of creamery stock at a discount of 3%. What was the market value and the discount?

3. What is the cost of 65 shares of Mo. Pacific railway stock at $62\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

4. Find the cost of 75 shares of bank stock at $103\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

5. A speculator bought 350 shares of People's Gas at $3\frac{1}{4}\%$ discount and sold it at a premium of $1\frac{1}{2}\%$. Find his gain.

6. I bought Amal. Copper stock at $86\frac{5}{8}$ and sold at 75, brokerage $\frac{1}{8}$ each way. How much did I lose on 1000 shares?

7. I bought stock at par and sold at $107\frac{1}{4}$, thereby gaining $\$174$. How many shares did I buy?

8. Mr. Brown exchanged 45 shares of bank stock at a discount of $4\frac{1}{5}\%$ for 70 shares of canning stock, face value $\$50$, selling at 5% premium. What amount of cash did he receive for the difference?

9. How many shares of stock must be bought at a discount of $3\frac{1}{2}\%$ and sold at a premium of $2\frac{1}{4}\%$, to gain $\$258.75$?

10. A broker exchanged 700 shares of railway stock at 5% discount, for bank stock at 5% premium, paying $\$70$ cash. How many shares did he buy?

11. Mr. Green paid $258,750 for bonds at a premium of $3\frac{1}{2}\%$. What was the face value of the bonds?

12. I bought 96 shares of mining stock, face value $50, for $4476. Find the rate of discount.

13. I paid my broker $125 brokerage for buying N. Y. Central. How many shares did he buy, brokerage being $\frac{1}{8}\%$?

14. I bought some railway stock at 73, received $94.50 as a $2\frac{1}{4}\%$ dividend, then sold my stock at $86\frac{1}{4}$. How many shares had I, and what was my gain?

15. The earnings of a company, capitalized at $40,000, are $8650, and the expenses are $1250. If a surplus of $1200 is reserved, what dividend can be declared?

16. What is the annual income from $160,500 invested in B. & O. R. R. Co. 6's at 107?

17. What income can be secured by investing $684 in 6% stock at 95?

18. What sum must be invested in 5% stock, at 105, to produce an income of $10,010?

19. I sold $8320 of 6's at $104\frac{1}{8}$ and invested the proceeds in 5's at $79\frac{7}{8}$, brokerage $\frac{1}{8}$ in each case. How much did I gain or lose annually?

20. How much must I invest in Milwaukee 4% Park bonds at $102\frac{7}{8}$, brokerage $\frac{1}{8}$, in order to have an annual income of $600?

SOLUTION

A 4% bond earns $4 per share.
$600 ÷ $4 = 150, number of shares required.
$102\frac{7}{8} + \$\frac{1}{8} = \$103$, cost of one bond.
150 × $103 = $15,450. *Ans.*

21. How much must be invested in St. Paul 7's at $149\frac{7}{8}$, brokerage $\frac{1}{8}$, to produce an annual income of $420?

22. What sum must I invest in 5-20's at $112\frac{3}{8}$, brokerage $\frac{1}{8}$, to insure an annual income of $500?

23. I wish to leave an annual income of $1200 to my daughter. Which is the better offer, 3% bonds at $87\frac{3}{8}$, or 4% bonds at $102\frac{7}{8}$, brokerage in each case being $\frac{1}{8}$?

24. A exchanges 250 shares of 6% stock at 70 for 8% stock at 125. What is the difference in his income?

25. What per cent of income is realized by buying a 7% bond at 95?

SOLUTION: A 7% bond nets $7 a year.
$\frac{7}{95}$ of 100% = $7\frac{7}{19}$%, rate of income.

26. What is the rate of income on U. S. 6's bought at 106?

27. On bank stock bought at 73, a dividend of $4\frac{1}{2}$% was declared. What was the rate of income?

28. Which is the better investment, to purchase 3's at 90, or 4's at 112?

29. Wishing to secure a permanent investment, I have the option of taking 5's at 75, or 7's at 90. Which is the better?

30. What is the rate of income on 4% bonds bought at 85?

31. How much must be paid for 4% stock to yield 5% income?

SOLUTION: 4% is $\frac{4}{5}$ of 5%, hence 4% stock yields $\frac{4}{5}$ of desired income. Since 4% stock yields only $\frac{4}{5}$ of desired income, it must be bought for $\frac{4}{5}$ of its face value; $\frac{4}{5}$ of $100 = $80, a share. *Ans.*

32. How much may be paid for 3% stock to yield 4% income?

33. At what rate of discount must 5% bonds be purchased so as to net an income of $6\frac{1}{4}$% on the investment?

34. At what price must 6% bonds be bought to yield 5% income?

COMPOUND PROPORTION

212. **To the Teacher.** **Several solutions are given in order to drill pupils in accuracy and readiness in the various forms of questions and statements required in different problems. The use of correct and concise forms of expression is of more importance here than the getting of the answer.**

Study Recitation

1. If 10 men can build 50 rods of wall in a certain time, how many men can build 80 rods of wall in the same time?

STATEMENT: $50 : 80 :: 10 : ?$

2. If 10 men can build a wall in 12 days, how many men can build it in 16 days?

STATEMENT: $16 : 12 :: 10 : ?$

The above problems are problems in simple proportion. Each statement is the statement of a simple proportion. Combining the two problems into one, we have the following problem:

3. If 10 men can build 50 rods of wall in 12 days, how many men can build 80 rods of wall in 16 days?

This problem is a problem in compound proportion. It will be noticed that in each of the two statements given above, the third term is 10. The first ratio in the first statement differs from the first ratio in the second statement. Bringing these statements together as the statement of a compound proportion, we have the third term 10, and the first and second terms, a compound ratio.

$$\begin{matrix} 50 : 80 \\ 16 : 12 \end{matrix} :: 10 : ?$$

Multiplying the two ratios together, term by term, we have:

$$(50 \times 16) : (80 \times 12) :: 10 : ?$$

Multiplying the means together, and dividing by the extremes, we have:

$$\frac{80 \times 12 \times 10}{50 \times 16}.$$

It will thus be seen that problems in compound proportion result from combining two or more problems in simple proportion.

In solving such problems, it is best to use the fractional form of solution. This gives us the last expression above, without the preceding steps.

Notice that in the above problem there are two numbers representing rods, two numbers representing days, and one number representing men, and that what is asked for is the number of men. In getting the statement for multiplication, proceed as in simple proportion, using the third term first with the two numbers representing rods.

Q. If 10 men can build 50 rods of wall, will it take a greater or a less number to build 80 rods of wall in the same time? *Ans.* A greater number, or $\frac{80}{50} \times 10$ men.

Write the expression $\frac{80}{50} \times 10$ men at the right as follows: $\frac{80}{50} \times 10.$

Next deal with the men and days. Drop out everything else.

Q. If 10 men can build a wall in 12 days, will it take more or less men to build it in 16 days? *Ans.* Less men, or $\frac{12}{16} \times 10$ men.

Write this fraction, $\frac{12}{16}$, at the left of the expression above, so that it now stands:

$$\frac{12}{16} \times \frac{80}{50} \times 10.$$

Canceling and performing the indicated operations, we secure the result, 12.

4. If 18 men build 75 rods of fence in 10 days of 9 hours each, how many hours a day must 24 men work to build 144 rods in 12 days?

In this problem, hours a day are called for. We have two numbers, 18 and 24, representing men; two numbers, 75 and 144, representing rods; two numbers, 10 and 12, representing days; and one number, 9, representing hours. Each set of two like numbers constitutes a ratio, and must be compared with the number representing hours.

Q. If 18 men build a fence working 9 hours a day, must 24 men work more or less hours a day to build it in the same number of days?

Ans. Less, or $\frac{18}{24}$ as many hours. $\frac{18}{24} \times 9.$

Q. If a number of men build 75 rods of fence working 9 hours a day, must they work a greater or less number of hours a day to build 144 rods in the same number of days? *Ans.* A greater number of hours a day, or $\frac{144}{75}$ as many hours. Write $\frac{144}{75}$ at the left of the expression above.

$$\frac{144}{75} \times \frac{18}{24} \times 9.$$

Q. If it requires a number of men 10 days working 9 hours a day to build a fence, will it take a greater or a less number of hours a day if they work 12 days? *Ans.* A less number, or $\frac{10}{12}$ as many hours. Prefix this fraction to the others with the multiplication sign.

$$\frac{10}{12} \times \frac{144}{75} \times \frac{18}{24} \times 9.$$

5. If 37 horses in 5 days eat 75¢ worth of oats, how much will it cost to feed 60 horses on oats for 8 days?

Q. In this problem what is asked for? *Ans.* Cents.

In the solution of these problems begin the question with the statement given in the problem.

Q. If 37 horses eat 75¢ worth of oats in a certain time, will it cost more or less to feed 60 horses the same time? *Ans.* More, $\frac{60}{37}$ as much. Write this fraction at the left of 75, with the sign of multiplication between them.

$$\frac{60}{37} \times 75.$$

Q. If a number of horses eat 75¢ worth of oats in 5 days, will it cost more or less to feed the same number of horses 8 days? *Ans.* More, $\frac{8}{5}$ as much. Write $\frac{8}{5}$ at the left of the other numbers.

$$\frac{8}{5} \times \frac{60}{37} \times 75.$$

6. If 30 masons in 15 days can build a wall 387.5 ft. long, what is the length of a wall that can be built by 48 masons in 20 days?

What is asked for in this problem? What is the number to be multiplied by the fractions to be found?

If 30 masons can build a wall 387.5 ft. long in a given time, will 48 masons build a greater or less length of wall in the same time? What is the resulting fraction?

If a number of masons can build a wall 387.5 ft. long in 15 days, can they build a greater or less length of wall in 20 days? What is the resulting fraction?

7. If it takes 60 men 12 days, working 10 hours a day, to make a concrete pavement 160 feet long and 40 feet wide, how long will it take 54 men, working 9 hours a day, to make a similar pavement 180 ft. long and 48 ft. wide?

8. If 180 men in 6 days of $6\frac{2}{3}$ hours each can dig a ditch 600 yd. long, 3 yd. wide, 2 yd. deep, in how many days can 100 men working $5\frac{1}{3}$ hours a day dig a trench 540 yd. long, 4 yd. wide, and 3 yd. deep?

9. 36 men are engaged to do a piece of work in 48 days. After 12 days, 12 men stop. In how many days can the rest finish the work?

CUBE ROOT

213. The cubes of the numbers from 1 to 9 inclusive must be learned.

$1^3 = 1$	$4^3 = 64$	$7^3 = 343$
$2^3 = 8$	$5^3 = 125$	$8^3 = 512$
$3^3 = 27$	$6^3 = 216$	$9^3 = 729$

1. Extract the cube root of 12,167.

$$
\begin{array}{rrr}
 & & 2\ \ 3 \\
 & & \overline{12'167} \\
 & & 8 \\
3 \times 20^2 & = 1200 & 4167 \\
3 \times 20 \times 3 = & 180 & \\
3^2 & = \quad 9 & 4167 \\
 & \overline{1389} &
\end{array}
$$

Steps:

1. Separate the number (12,167) into periods of three figures each, beginning with units.

2. Find the greatest cube (8) equal to or less than the left-hand period (12); write its cube root (2) for the first figure of the root, and subtract the cube (8) from the left-hand period (12).

3. To the remainder (4) annex the next period (167) for a dividend.

4. Write three times the square of the first figure (2) of the root, considered as tens (or 20), as trial divisor (1200), at the left of the dividend (4167).

5. Divide the dividend (4167) by the trial divisor (1200) and write the quotient for the next figure of the root.

6. Add to the trial divisor three times the first part of the root (2) considered as tens, (or 20), multiplied by the last figure (3), (180); and also the square of the last figure (3) of the root (9), to make the complete divisor (1389).

7. Multiply the complete divisor (1389) by the last figure (3) of the root, and subtract the product from the dividend (4167).

8. Repeat steps 3–7 if there are more than two figures in the root, substituting the proper numbers for those given.

NOTE 1. If at any time the trial divisor is not contained in the dividend, place a zero in the root, annex two zeros to the trial divisor, and annex the next period to the dividend and then divide.

NOTE 2. If at any time the product of the complete divisor by the last figure of the root found is greater than the dividend, the root figure is too large. In that case reduce the root figure by 1, make the new additions to the trial divisor, and proceed as before.

NOTE 3. In extracting the cube root of decimals, point off from the decimal point to the right; if necessary, annex zeros, so that each period may have three places.

NOTE 4. In finding the cube root of a fraction whose terms are not perfect cubes, reduce the fraction to a decimal with three times as many places as the number of figures wanted in the root, and extract the root.

TO THE TEACHER. The statement of the steps 1–8, omitting the numbers, is the rule for extracting the cube root. Each step should be thoroughly learned. Pupils should be able to state the entire rule without hesitating, and to apply each step in order.

After going through the rule with pupils to see that they understand and can apply its directions, give drills in applying each step to a number of examples before taking up the next step. Before this drill is begun on any step, have the exact wording of the step learned and then apply it without referring to the book.

Point off the following numbers for the extraction of their cube roots, keeping the work on each number on a separate sheet of paper:

2.	3.	4.	5.	6.
32768;	79507;	614125;	373.448;	.004913

State the first step.

Give the cubes of the numbers from 1 to 10.

State the second step, and apply it in each case.

State the third step, and apply it to each remainder.

In the same way, state and apply the succeeding steps with each of the results thus far obtained.

EXPLANATION OF THE PROCESS OF EXTRACTING THE CUBE ROOT

214. Relation of the number and places of figures in a root to the number and places of figures in its cube.

$1^3 = 1$ The cube of the smallest number of one figure consists of one figure.

$9^3 = 729$ The cube of the largest number of one figure consists of three figures.

The cube of any number between 1 and 9 contains one, two, or three figures.

$10^3 = 1000$
$90^3 = 729{,}000$

Annex a zero to 1, making 10, and to 9, making 90, and cube these numbers. Note that annexing a zero to the smallest and to the largest number of one figure increases the number of figures in the cube by three.

$19^3 = 6859$
$99^3 = 970{,}299$

Annex 9, the largest number of one figure, to 1, making 19, and to 9, making 99. It will be seen that annexing the figure 9 to the smallest and to the largest number of one figure increases the number of figures in each cube by three. Hence, annexing a figure of any value to any one-figure number increases the number of figures in its cube by three.

In a manner similar to that employed in dealing with the squares of numbers, in the explanation of the process of extracting the square root, it may be shown that annexing a figure to any number increases the number of figures in its cube by three.

$10^3 = 1\,000$
$100^3 = 1\,000\,000$
$900^3 = 729\,000\,000$
$990^3 = 970\,299\,000$
$999^3 = 997\,002\,999$

The cube of the first figure of the root is found in the first period of the cube; of the first two figures of the root, in the first two periods of the cube; of the first three figures of the root, in the first three periods of the cube, etc.

From the preceding, it appears that the cube of any number of one figure contains one, two, or at most three figures, and that if any figure is annexed to any one-figure number, the number of figures in its cube is increased by three; that for every figure after the first in the root there are three figures in its cube, and that for the first figure in the root there may be one, two, or three figures in the cube; that pointing off the number into periods of three places each, beginning with units, will show the first period, whether of one, two, or three figures, and the number of figures in the root.

Composition of the Cube of a Number

A number is cubed by multiplying its square by the number. $25^2 = 20^2 + 2(20 \times 5) + 5^2$.

Multiply $(20 + 5)^2$ by $20 + 5$, indicating the operations. Remember that multiplying any one of several factors of a product multiplies the product.

$$\begin{array}{r} 20^2 + 2(20 \times 5) + 5^2 \\ \hline 20 + 5 \\ \hline 20^2 \times 5 + 2(20 \times 5^2) + 5^3 \\ 20^3 + 2(20^2 \times 5) + (20 \times 5^2) \\ \hline 20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3 \end{array}$$

In the partial products we have 2 times 20×5^2 and once 20×5^2; the sum is 3 times 20×5^2.

We also have once $20^2 \times 5$ and 2 times $20^2 \times 5$; the sum is 3 times $20^2 \times 5$. $25^3 = (20 + 5)^3 = 20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3$.

Substituting t for the tens, or 20, and u for the units, or 5, we have:

$(t + u)^3 = t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3$.

$t^3 = 20^3 = 8000$ = Cube of the tens.

$3(t^2 \times u) = 3(20^2 \times 5) = 6000$ = Three times the square of the tens by the units.

$3(t \times u^2) = 3(20 \times 5^2) = 1500$ = Three times the tens by the square of the units.

$u^3 = 5^3 = 125$ = Cube of the units.

$t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3 =$
$20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3 = 15{,}625.$

As any number of two or more figures may be regarded as made up of a certain number of *tens* + a certain number of units, it follows that the cube of any number of two or more figures equals:

The cube of the tens plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

$$75^3 = (70 + 5)^3 = 70^3 + 3(70^2 \times 5) + 3(70 \times 5^2) + 5^3 = ?$$

$$215^3 = (210 + 5)^3 = 210^3 + 3(210^2 \times 5) + 3(210 \times 5^2) + 5^3 = ?$$

1. Extract the cube root of 15,625.

We have found that the cube of 25 is made up of four parts:

(1) The cube of the tens. (2) Three times the square of the tens multiplied by the units. (3) Three times the tens multiplied by the square of the units. (4) The cube of the units.

We have also found that the cube of the first figure of the root is found in the first period of the cube.

$$t^3 = \begin{array}{r|l} & 2\ \ 5 \\ \hline 15'625 & \\ 8 & \\ \hline 7\,625 & \end{array}$$

The first period is 15. The largest cube equal to or less than 15 is 8; its cube root is 2, the first figure of the required root.

As there are two periods, and therefore two figures in the root, the first figure, 2, is 2 tens, or 20 units. As the cube 8 is in thousands' place, we need not write the three ciphers.

$$\left.\begin{array}{r} 3(t^2 \times u) \\ 3(t \times u^2) \\ u^3 \end{array}\right\} \text{A}$$

In subtracting 8 thousand from 15,625, we take away the cube of the tens, and have left the other three parts of the cube, as shown at A.

Using first trial units' figure, 6:

$$\left.\begin{array}{r} 3(20^2 \times 6) = 7200 \\ 3(20 \times 6^2) = 2160 \\ (6^3) = \ \ 216 \\ \hline 9576 \end{array}\right\} \text{B}$$

The first of these parts, $3(t^2 \times u)$, is the largest of the three, and as it contains all the different kinds of factors found in the other two parts, that is, 3, t, and u, we may disregard the other two parts and use only the first of these parts in finding the trial second figure of the root.

Using 2d trial units' figure, 5:		
$3(20^2 \times 5)$	$= 6000$	
$3(20 \times 5^2)$	$= 1500$	
5^3	$= 125$	
		7625
Trial divisor, or		
$3 \times t^2$	1200	
$3 \times t \times u$	300	
u^2	25	7625
Complete divisor	1525	

This is called a trial figure because as we have disregarded the other two parts of the cube, and considered 7625 as made up of only one of the three parts, our trial divisor may be found to be too large when we come to consider the two parts we disregarded for the purpose of finding the units' figures.

Regarding 7625, then, as made up of 3 times the square of the tens multiplied by the units, we see that two of these factors are known, namely: 3, and the tens which we have found to be 2, or 20 units. The square of the tens is 20 × 20, or 400. 3 times the square of the tens is 1200. The units' figure is not known, but may be found approximately by dividing 7625 by 1200 as a trial divisor, or, what is the same thing, ignoring the zeros and dividing 76 by 12. The quotient is 6, our trial units' figure.

Before writing the 6 in the root, we should test to see whether it is the proper figure. Writing out the value of each of the three parts as shown at B, p. 323, we find their sum is 9576, or more than 7625. We therefore conclude that our trial units' figure, 6, is too large. Substituting for it the next smaller number, 5, and writing out the value of the three parts as before, we find their sum is 7625, showing that 5 is the correct units' figure, which we place in the root over the second period.

In the process we have employed in extracting the cube root of numbers, we have shortened the above operation as follows:

After dividing 7625 by 1200 as a trial divisor, and finding the trial units' figure, 6, we may complete the divisor by adding to it $3(20 \times 6)$, or 360, and 6^2, or 36, making $1200 + 360 + 36$, or 1596, which equals $3 \times t^2 + 3(t \times u) + u^2$. Each of these three parts lacks the same factor, the units. On multiplying their sum, 1596, called the complete divisor, by 6, the product, 9576, is the same as we found the sum of the three full parts to be, in testing to determine whether 6 was the correct units' figure.

In testing 5 for the units' figure, we proceed in the same way: Adding to 1200, $3(20 \times 5)$, or 300, and 5^2, or 25, we have 1525, the complete divisor, which, when multiplied by the units' figure, 5, gives 7625, the sum of the three parts of the cube remaining after the cube of the tens, 8000, has been subtracted.

The cube subtracted from the first period, and the product of the complete divisor by the second figure of the root, together equal the entire number or cube. If there were three figures in the root, these two numbers together would equal the cube of the first two figures of the root considered as tens, and the units' figure would be found in the same way as the tens' figure was found, and so on, if the root had more than three figures.

2. Extract the cube root of 97,336, explaining the process. Solve the example, answering each question in order.

The information necessary for answering these questions may be obtained by a careful study of the explanation of the process of extracting the cube root, pp. 321–325.

1. Why do you point off the number into periods of three figures each, beginning at units' place?
2. How is the first figure of the root found? Why? What is it?
3. The number subtracted from the first period is what as related to the first figure of the root?
4. The number formed by annexing the next period to the remainder contains what three parts of the entire cube?
5. What part of the entire cube is it regarded as being for the purpose of finding the second figure of the root?
6. What factors does this part contain?
7. Which of these factors are known?
8. When the product of these factors and two of the factors are known, how can you find the other factor?
9. What is done in this case to find the trial second figure of the root? What is this figure?
10. What is the trial divisor? Of what factors is it the product? What two additions are made to it to form the complete divisor? Give the numbers added, and state in general terms of what factors each number is the product.
11. State the terms of which the complete divisor is composed.
12. The product of the complete divisor by the trial units' figure of the root is composed of what terms of the cube?
13. What is done with this product? What does this operation show?

3. Extract the cube root of 148,877, explaining each step in the operation.

COMMON MEASURES

TROY WEIGHT

215. Troy weight is used in weighing gold, silver, and jewels.

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces, or 5760 grains	= 1 pound (lb.)

NOTES. In indicating the *fineness* or purity of gold, it is considered as composed of 24 parts, called carats, and the number of carats specified is the number of 24ths of pure gold that it contains. The difference between this number and 24 is the number of parts of alloy, or less valuable metal, combined with the gold. An eighteen-carat ring is $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

The weight of diamonds is usually given in carats, each carat weighing $3\frac{1}{4}$ grains (nearly). A six-carat diamond weighs 20 grains Troy, nearly.

APOTHECARIES' WEIGHT

216. Apothecaries' weight is used by physicians in prescribing, and by apothecaries in compounding medicines.

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound, or 5760 grains

1 lb. Avoirdupois weight	= 7000 grains
1 lb. Troy weight	= 5760 grains
1 lb. Apothecaries' weight	= 5760 grains
1 oz. Avoirdupois weight	= 437½ grains
1 oz. Troy weight	= 480 grains
1 oz. Apothecaries' weight	= 480 grains

Drugs are sometimes bought by druggists in bulk by Avoirdupois weight, but are sold by Apothecaries' weight, as smaller subdivisions than ounces and drams are frequently needed in compounding medicines to fill physicians' prescriptions.

The pound Avoirdupois is heavier than either the Troy or Apothecaries' pound.

The ounce Avoirdupois is lighter than either the Troy or Apothecaries' ounce.

What unit is common to the three weights?

Avoirdupois pounds may be reduced to grains by multiplying 7000, the number of grains in 1 lb., by the number of pounds.

Avoirdupois ounces may be reduced to grains by multiplying 437½, the number of grains in 1 oz., by the number of ounces.

Any number of grains may be reduced to higher denominations of Apothecaries' or Troy weight by the process of Reduction Ascending.

Any number of grains may be reduced to pounds and fractions of a pound, Avoirdupois, by dividing the number of grains by 7000; to ounces, Avoirdupois, by dividing the number of grains by 437.5.

Written

1. How many grains heavier is a pound Avoirdupois than a pound Troy?

2. Which is the heavier, a pound of feathers or a pound of gold, and how much heavier?

3. Express 2 lb. Avoirdupois in the denominations of Apothecaries' weight.

4. Express 3 lb. 6 oz. Avoirdupois in the denominations of Apothecaries' weight.

5. How many ounces are there in 32 ʒ? In 25 ʒ?

6. How many drams are there in 12 ℈? In 15 ℈?

7. How many grains are there in 5 ℥ and 3 ʒ?

8. Reduce 2465 gr. to higher denominations, Apothecaries' weight.

9. What part of a pound Avoirdupois is a pound Troy?

10. What part of an ounce Troy is an ounce Avoirdupois?

11. Reduce 2465 gr. to higher denominations, Troy weight.

United States gold and silver coins are .9 pure gold or silver and .1 copper.

The weight of a silver dollar is 412½ grains.

The weight of the 10-dollar gold piece is 258 grains.

12. What is the weight of the pure silver in 100 silver dollars?

13. What is the weight of the pure gold in a 10-dollar gold piece?

14. If 100 pounds of pure gold are to be coined into eagles, how much copper is needed as alloy?

15. When the necessary amount of copper has been added to 100 pounds of pure gold, what is the value of the metal in dollars?

SURVEYORS' LINEAR MEASURE

217. This measure is used by surveyors in measuring distances.

7.92 inches (in.)	= 1 link (l.)
25 links	= 1 rod (rd.)
100 links, or 4 rods	= 1 chain (ch.)
80 chains	= 1 mile (mi.)

The chain referred to in the foregoing is called Gunter's chain; its length is 66 ft., or 792 inches. Its divisions are decimal, so that chains and links may be written as chains and decimals of chains; thus, 15 links equals .15 of a chain; 25 chains 15 links is written 25.15 chains.

ORAL

1. What part of a chain is 1 link? 5 links? 18 links?

2. What part of a chain are 10 links? What part of 1 rod?

3. How many chains are there in ½ mile? In ¾ mile?

4. How many feet are there in 25 links? In 50 links?

Written

5. How many links are there in 76 chains, 80 links? How many rods?

6. Reduce .725 mi. to chains and decimals of a chain.

7. How many miles are there in 2750.8 chains?

8. Change 24 miles, 120 rods to chains.

SURVEYORS' SQUARE MEASURE

218. This measure is used by surveyors in computing the area of land.

625 square links (sq. l.)	= 1 square rod (sq. rd.)
16 square rods	= 1 square chain (sq. ch.)
10 square chains	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Oral

1. How many links are there on each side of a square rod?

2. How can you find the number of square links in a square rod?

3. How many rods are there on each side of a square chain?

4. How can you find the number of square rods in a square chain?

5. How many square rods are there in an acre?

6. How can you find the number of square chains in an acre?

7. How many square chains are there in 6 chains square?

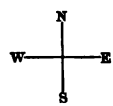
8. How many acres are there in a rectangular field 40 chains, 5 links long, and 20 chains, 25 links wide?

LAND SURVEY

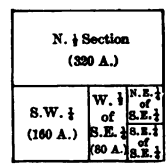
219. In most states the land is divided by survey into townships 6 miles square, and these are divided into 36 equal sections of a square mile, or 640 acres each. The sections are subdivided into half-sections and quarter-sections, and are numbered and designated as in the following diagrams.

A TOWNSHIP

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36



A SECTION



The townships are numbered and designated east or west from a definite north and south line, called the **principal meridian**, and north or south from a definite east and west line called the **base line.**

ORAL

1. How many acres are there in the N.E. $\frac{1}{4}$ of S.E. $\frac{1}{4}$ of section 18?

2. How many acres are there in the N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of section 34? In the W. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of section 30?

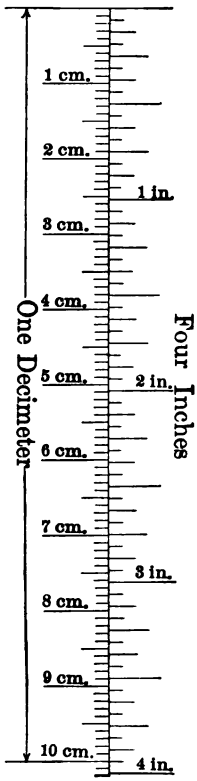
3. Find the cost of the S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of section 9 at \$40 an acre.

4. How many rods of fence are required to inclose the land mentioned in problem 3?

5. Find the cost of the N. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of section 36 at \$50 an acre.

6. At \$100 an acre, find the cost of the S. $\frac{1}{2}$ of section 3 and the N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of the same section.

METRIC MEASURES



220. The system of weights and measures in our country is irregular and somewhat difficult to learn, as well as inconvenient to apply. The same held true with the old systems of other nations. In order to overcome these defects, France in 1799 adopted the metric system of weights and measures, based upon the decimal system of notation. This system is now in use in all civilized nations, England and the United States being the only exceptions. Congress in 1866 legalized its use in our country, but it is used here at the present time in scientific work only.

Multiples of the primary units are represented by the Greek prefixes, **deka** (10), **hekto** (100), **kilo** (1000), **myria** (10,000); and decimal divisions, by the Latin prefixes, **deci** ($\frac{1}{10}$), **centi** ($\frac{1}{100}$), **milli** $\frac{1}{1000}$.

LINEAR MEASURE

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer** (Km.)
10 kilometers	= 1 myriameter (Mm.)

The units most commonly used are those printed in black type, namely, the millimeter, centimeter, meter, and kilometer.

A decimeter, or 10 centimeters, as shown in the cut, equals a little less than 4 in. The meter is equal to 39.37 in., or 1.09 yd.

SQUARE MEASURE

100 sq. millimeters (sq. mm.) = 1 sq. centimeter (sq. cm.)
100 sq. centimeters = 1 sq. decimeter (sq. dm.)
100 sq. decimeters = 1 **sq. meter** (sq. m.)
100 sq. meters = 1 sq. dekameter (sq. Dm.)
100 sq. dekameters = 1 sq. hektometer (sq. Hm.)
100 sq. hektometers = 1 **sq. kilometer** (sq. Km.)

The square meter is equal to 1.196 sq. yd.

LAND MEASURE

100 centares (ca.) or square meters = 1 **are** (a.), or 1 sq. Dm.
100 ares = 1 **hektare** (Ha.)

A hektare is equal to 2.471 acres.

MEASURES OF VOLUME

1000 cubic millimeters (cu. mm.) = 1 **cubic centimeter** (cu. cm.)
1000 cubic centimeters = 1 **cubic decimeter** (cu. dm.)
1000 cubic decimeters = 1 **cubic meter** (cu. m.), or **stere**

The cubic meter is equal to 1.308 cubic yards.

MEASURES OF WEIGHT

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 **gram** (g.)
10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 hektogram (Hg.)
10 hektograms = 1 **kilogram** (Kg.)

The gram is the unit of weight. It is equal to the weight of a cubic centimeter of pure water at 4° C., and equals 15.432 grains.

MEASURES OF CAPACITY

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 liter (l.)

The liter equals one cubic decimeter, and is equivalent to .908 qt. dry, or 1.05 liquid, measure. A **hektoliter** (100 l.) equals 2.8 bu.

MENSURATION

221. **Mensuration** treats of the measurements of lines, angles, surfaces, and solids.

A **plane surface** or a **plane figure** is a surface such that a straight line joining any two parts of it lies wholly in the surface.

QUADRILATERALS

222. A **quadrilateral** is a plane figure having four sides.

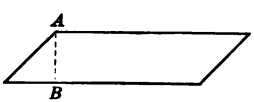
A **parallelogram** is a quadrilateral whose opposite sides are parallel.

A **rectangle** is a quadrilateral whose angles are all right angles.

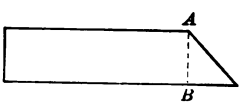
Is a rectangle a parallelogram? Why?

A **trapezoid** is a quadrilateral having only two sides parallel.

Any line (AB) perpendicular to and connecting the parallel sides of a parallelogram or of a trapezoid is called the **altitude**.



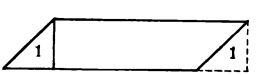
PARALLELOGRAM



TRAPEZOID

TO THE TEACHER. Teach the definitions by means of blackboard drawings, figures cut from pasteboard, etc. When the definitions have been taught, test the pupil's knowledge thereof by requesting him to draw the figures you designate by name.

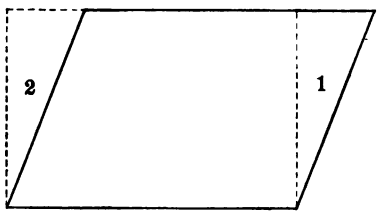
Prove by means of cardboard figures similar to the accompanying one the rules for finding the areas of plane figures, and the reasons for the rules.



Use actual type solids in teaching the definitions relating to solids.

ORAL

223. Define square, quadrilateral, parallelogram, rectangle. What is the difference between a square and a parallelogram? Between a quadrilateral and a parallelogram?



Cut from paper or pasteboard a rectangular parallelogram 6 inches long and 4 inches high. Cut another parallelogram of the same size, similar in shape to the accompanying figure. What is the area of the rectangular parallelogram? Cut off the projecting part marked "1," and put it at the other end of the figure marked "2." Compare the area of this parallelogram with the preceding one. What is its area?

Any parallelogram has the same area as a rectangle having the same base and altitude.

To find the area of a parallelogram.

RULE: *Multiply the base by the altitude.*

NOTE. In this and similar rules, by *base* and *altitude*, or other dimensions, we mean the numbers representing them, when expressed in *like units*.

PROBLEMS

ORAL

224. 1. What is the area of a parallelogram 10 inches wide and 15 inches long?

2. How many acres are there in a piece of land 80 rods long and 60 rods wide?

3. A field is 40 rods wide and 80 rods long. How long a fence will be required to divide the field into two equal squares? How many acres are there in each square?

WRITTEN

4. What is the area of a parallelogram having a base of 15 in. and an altitude of $9\frac{1}{2}$ in.?

5. A strip of land in the shape of an oblique parallelogram is 150 rd. long and $60\frac{2}{3}$ rd. wide. How many acres does it contain?

6. What is the difference in area of two strips of land in the shape of parallelograms, one having a 40-rd. base and 165-ft. alt., and the other a 150-yd. base and 181.5-ft. alt.?

7. The area of a parallelogram is 650 sq. ft., and the base is 40 ft. Find the altitude.

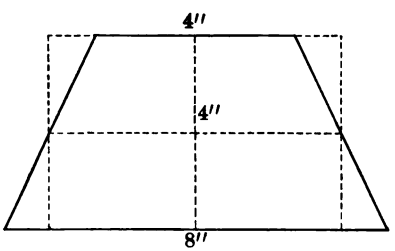
8. The altitude of a parallelogram that has an area of 36 sq. yd., is 8 ft. What is the length of the base?

9. Given the area of a parallelogram and either the base or the altitude; how can you find the other dimension?

ORAL

225. 1. What is a trapezoid?

Construct a trapezoid 4 in. high, with an 8-in. base and a 4-in. top. Halfway between the bases draw a line parallel to them. Cut off the corners of the trapezoid and place them as shown in the illustration. What kind of figure have you? What are its dimensions? What is its area? Solve the example by the rule for finding the area of a trapezoid, and compare the result with the preceding answer.



To find the area of a trapezoid.

RULE: *Multiply one half the sum of the parallel sides by the altitude.*

WRITTEN

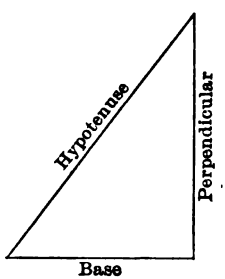
2. Find the area of a trapezoid whose sides are 12 in. and 18 in., the perpendicular distance being 10 in.

3. Find the area of a trapezoid, one side of which is 125 in. and the other 96 in., the altitude being 48 in.

4. What is the area of a plank $16\frac{1}{2}$ in. on one end, $8\frac{3}{4}$ in. on the other, and 16 ft. long?

5. A has a field in the form of a trapezoid. The parallel sides are 75 and 95 rd., and the perpendicular distance between them is 66 rd. How many acres are there in the field?

TRIANGLES



226. A triangle is a plane figure having three sides.

A triangle having a right angle is called a **right-angled triangle.**

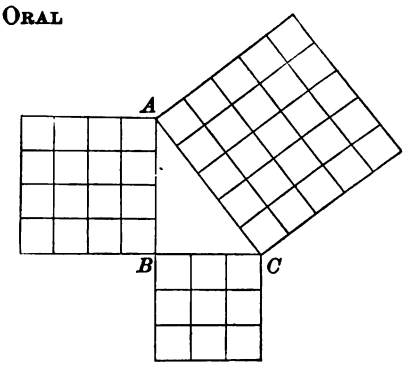
The **hypotenuse** is the side opposite the right angle. The other two sides are the **base** and the **perpendicular.**

ORAL

227. Right-angled triangles.

1. How many squares are in the large square on side AB of the triangle? On side BC? On side AC?

2. Compare the number of squares on the hypotenuse with the sum of the squares on the other two sides.



To find one side of a right-angled triangle when the other sides are given:

RULE: *Square the two sides. If the hypotenuse is given, extract the square root of the difference of the squares, to find the required side; if the hypotenuse is not given, extract the square root of the sum of the two squares to find the hypotenuse.*

WRITTEN

3. The two sides of a right-angled triangle are 51 ft. and 68 ft. respectively. What is the length of the hypotenuse?

SOLUTION: $\sqrt{51^2 + 68^2} = \sqrt{7225} = 85$, number of feet.

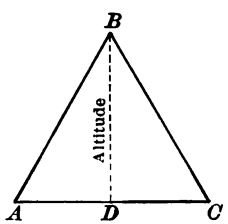
4. The hypotenuse of a right-angled triangle is 115 in., and the base is 92 in. What is the perpendicular?

5. A ladder 65 ft. long is placed against a house on level ground so that it reaches a window 60 ft. from the ground. How far from the house is the foot of the ladder?

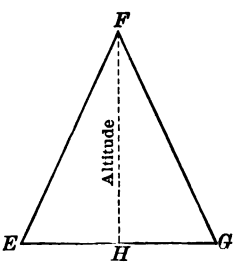
6. One vessel sails due north from a port at the rate of 3 miles an hour. Another, sailing at the same time, goes east at the rate of 4 miles an hour. How far are they apart in 2 days?

228. Other triangles.

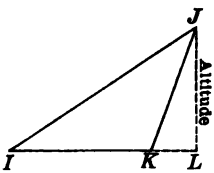
A triangle with three equal sides is an **equilateral triangle.** The angle opposite the base of a triangle is the **vertex.**



EQUILATERAL TRIANGLE



ISOSCELES TRIANGLE



SCALENE TRIANGLE

The **altitude** of a triangle is the length of a perpendicular line from the vertex to the base, or to the base extended.

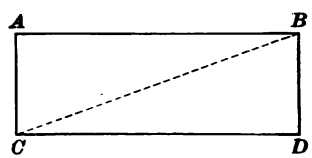
A triangle having two sides equal is an **isosceles triangle.**

A triangle having no two sides equal is a **scalene triangle.**

ORAL

229. 1. What is the difference between a right-angled triangle and an equilateral triangle? An equilateral triangle and an isosceles triangle? A scalene triangle and an isosceles triangle?

2. Construct a parallelogram 6 in. long, 4 in. high. What is its area?



3. Cut the parallelogram in two on a line joining two opposite corners. Compare the two triangles produced. What is the area of each? What is the length of each? The height?

4. State the rule for finding the area of a triangle.

5. What is the area of a triangle with a 10-in. base and 12 in. high? Of another triangle 6 in. at the base and 8 in. high?

6. How many acres are there in a triangular piece having a 320-rod base and a perpendicular altitude of 60 rods?

To find the area of a triangle.

RULE: *Multiply the base by one half of the altitude.*

WRITTEN

7. What is the area of a triangle whose base is 75 rd. and altitude 81 rd.?

8. A triangle has a $17\frac{1}{2}$-in. base and a 20-in. altitude. What is its area?

9. What is the area of a scalene triangle with a 30-inch base and a $1\frac{1}{2}$-yd. altitude?

10. The base of a triangle having an area of 322 sq. in. is 42 in. Find the altitude.

REVIEW

Oral

230. 1. What is the unit used by contractors in estimating cost of plastering? Of papering? Of painting? Of macadamizing roads? What is the area of the cover of this book? Of the top of your desk?

2. What measure is used in stating the area of a lot? The area of a block? The area of a farm? By the geographer in stating the area of the county? Of the state?

3. What is the area of the floor of a room 10 ft. wide, 12 ft. long? What is the area of a blackboard 4 ft. wide and 16 ft. long? What is the unit of measurement?

4. What is the cost of painting a solid board fence 10 ft. high and 150 ft. long at a cost of 50¢ per square of one hundred square feet? Work in two ways. Which way is the better?

5. A room is 9 ft. high, 12 ft. wide, and 15 ft. long. How many square yards are there in the walls? In the ceiling?

Written

6. Reduce 420 A. 60 sq. rd. to square rods.

7. Reduce 8 sq. mi. 24 A. to acres.

8. Reduce 12 sq. ft. 120 sq. in. to square inches.

9. Reduce 102,400 sq. rd. to higher denominations.

10. Reduce 8424 sq. in. to higher denominations.

11. What is the cost of painting a floor 16 ft. wide and 18 ft. long at 10¢ per square yard?

12. How many square yards of plastering are there in a hall 20 ft. high, 30 ft. wide, and 90 ft. long, no allowance being made for doors and windows?

SOLUTION

30 ft. × 2 + 90 ft. × 2 = 240 ft., length of walls.

$\frac{240 \times 20}{9} = 533\frac{1}{3}$, number square yards in walls.

$\frac{30 \times 90}{9} = 300$, number square yards in ceiling.

$533\frac{1}{3} + 300 = 833\frac{1}{3}$, number square yards of plaster in hall.

13. What is the cost of lathing and plastering a store 22 ft. high, 50 ft. wide, 120 ft. long, at $33\frac{1}{3}$ cents per square yard, an allowance of ten square yards being made for doors and windows?

14. What is the cost of plastering a room 10 ft. 6 in. high, 16 ft. 3 in. wide, 18 ft. 6 in. long, at 15¢ per square yard, allowance being made for one half of the space occupied by three doors and two windows, each 6 ft. 9 in. in height and 3 ft. 6 in. in width?

15. A roll of paper usually covers 33 sq. ft. What will be the cost of papering the walls in the preceding problem at 20 cents per roll, and border at 3 cents per foot?

16. A dining room in a hotel 28 ft. wide, 50 ft. long, and 19 ft. high is being renovated, — the floor oiled and polished at 5 cents per square foot, the ceiling papered with paper at 27 cents a roll, the walls frescoed at $37\frac{1}{2}$ cents per square yard. Write a receipted bill for the contractor making the repairs.

17. A room is 16 ft. wide and 18 ft. long. What is the cost of carpeting it at 75 cents a yard, the strips to run crosswise and the carpet being a yard wide? The strips to run lengthwise?

NOTE. In laying the carpet lengthwise, five strips of the carpeting 1 yd. wide would cover only 15 ft., leaving a strip of the floor 1 ft. wide and 18 ft. long still to be covered. To insure proper matching of figures in the carpet, another strip full width and 18 ft. long must be bought, and the needed strip must be cut from one side.

18. What is the cost of carpeting a room 15 ft. wide, 16 ft. 6 in. long, with 27-inch Brussels carpet, at $1.25 per yard, strips to run lengthwise, making an allowance of 8 inches per strip after the first strip is laid for matching designs? What will be the cost when the strips run crosswise?

19. The floor of a hall 50 ft. 6 in. wide, 121 ft. 9 in. long, is being covered with 30-inch carpet, at $1.50 per yard. Nine inches being allowed on each strip but the first for matching designs, what is the cost of the carpet?

20. What is the value of a piece of land 40 rd. wide, 120 rd. long, at $55 an acre?

21. A map is drawn to the scale of $\frac{1}{4}$ inch to the mile. How many acres are represented by 1 sq. ft. of the map?

22. A road is ordinarily 4 rd. wide. How many miles of it will make 20 acres?

23. What will be the cost of vitrified bricks for paving a street 4 rd. wide and $2\frac{1}{2}$ mi. in length, at $15 per thousand, the bricks being 4 in. wide, 2 in. thick, and 8 in. long?

NOTE. The bricks are laid on edge.

24. Which is cheaper, and how much, — macadamizing a yard 80 ft. wide, 125 ft. long, at 50 ¢ a square yard, or paving it with cut stones 8 in. wide, 10 in. long, at $20 per M?

CIRCLES

231. A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.

The **circumference** is the curved line that bounds the circle.

The **diameter** of a circle is a straight line passing through the center, with both ends terminating in the circumference.

The **radius** of a circle is a straight line from the center to the circumference. It is equal to half the diameter.

Oral

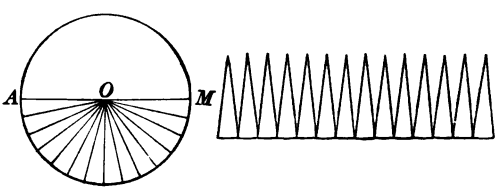
232. Define circle; diameter; circumference; radius.

By means of a piece of paper wrapped about a cylinder whose diameter is known, find the circumference and determine how many times the diameter it is. It will be found to be a trifle more than $3\frac{1}{7}$ times the diameter.

The circumference of a circle is more exactly 3.1416 times the length of its diameter. 3.1416 is often represented by the symbol π, called pi.

What is the circumference of a circle having a 12-inch diameter? A 5-inch radius?

Construct a circle of fair size on pasteboard and cut it out. Divide each half of the circle as shown in the figure.



Cut the circle into two equal parts along the line AM. From O cut along the lines indicated toward the circumference, but do not separate the parts entirely. Spread the

resulting figures out and combine them as indicated in the figure on p. 342.

If the sections cut out are small enough, each may be regarded as a triangle whose base is a straight line, and a small part of the circumference.

1. What is the length of the combined bases of the triangles as compared with the circumference of the circle? Of the height of the triangles as compared with the radius? What is the area of the figure? How is it found? What is the rule for finding the area of a circle?

2. What is the area of a circle 10 feet in diameter? What is the area of a circle having a 20-foot radius?

3. Over how many square feet of surface may a horse graze if he is fastened to a post with a rope that allows him to graze 100 feet from the post?

To find the area of a circle.

RULE: *Multiply the circumference by one half of the radius; or, multiply the square of the radius by* 3.1416.

WRITTEN

4. What is the area of a circle having a 12-ft. radius?

5. What is the area of a circle having a diameter of 50 ft.?

6. What is the area of a circle whose circumference is 157.08 ft.?

7. The area of a circle is 1017.8784 sq. in. Find the radius and circumference.

8. The area of a circular garden is 2578.23 sq. rd. Find the diameter and circumference.

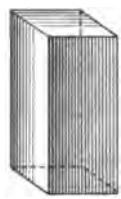
SOLIDS

233. A **solid** or body has length, breadth, and thickness.

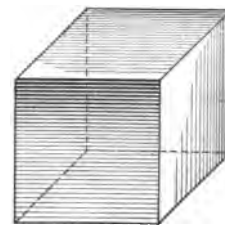
A **prism** is a solid whose sides are parallelograms, and whose two ends are equal polygons parallel to each other.

Prisms, according to the form of their bases, are triangular, quadrangular, pentagonal, etc.

TRIANGULAR PRISM



QUADRANGULAR PRISM



CUBE

A **triangular** prism is one that has triangles for its bases.

A **quadrangular** prism is one that has quadrilaterals for its bases. Quadrangular prisms are also called **parallelopipeds.**

A **cube** is a solid with six equal square faces.

CYLINDER

A **cylinder** is a solid with a curved surface and two equal parallel circular bases.

The **convex surface** of a cylinder is the area of its curved surface.

The **axis** of a cylinder is the line joining the center of the two bases.

The **altitude** of a prism or a cylinder is the perpendicular distance between its bases.

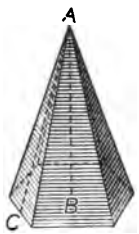
A **pyramid** is a solid whose base is a polygon, and whose sides are triangles meeting at the vertex.

The altitude of a pyramid is the distance from the vertex to the center of the base; as AB in the figure.

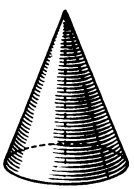
The **slant height** of a pyramid is the distance from the vertex to the middle of the base of any of its triangular sides; as AC in the figure.

A **cone** may be regarded as a pyramid whose base is a circle.

A **sphere** is a solid with a curved surface, every point of which is equally distant from a point within called the center.



PYRAMID


CONE

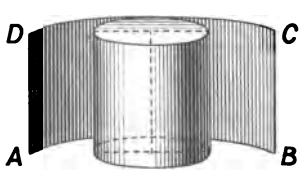
SPHERE

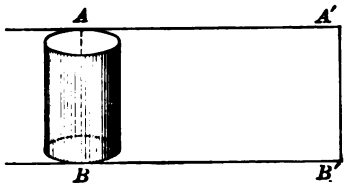
The **diameter** of a sphere is a straight line passing through the center of the sphere and terminating in its surface. One half the diameter is the **radius** of the sphere. The greatest distance around the sphere is its **circumference.**

All the surface of a solid except its base or bases is called the **lateral surface.** The entire surface is the sum of the lateral surface and of the bases.

NOTE TO THE TEACHER: Derive the rules for finding the area of the surface of different solids in manner similar to the one that follows:

Cover exactly the lateral surface of any cylinder with a sheet of paper of the same height. Unroll the paper. Find its area. What is the circumference of the cylinder? Multiply the circumference of the cylinder by its height. How does the product compare with the area of the paper? State the rule for finding the lateral surface of a cylinder.





The cylinder in the illustration is represented as lying on a plane touching it along the line AB. Conceive it rolled to the right. When A reaches the plane at A', B is exactly at the point B', and the cylinder has made one rotation. The path described by it is a rectangle equal to the entire lateral surface of the cylinder.

ORAL

234. 1. What is the lateral surface of a cylinder 8 inches in circumference, 14 inches high? Of a cylinder 10 inches in circumference, 10 inches high?

To find the surface of solids.

RULES: I. *The lateral surface of a prism or a cylinder equals the product of the perimeter or circumference of the base by the altitude.*

II. *The lateral surface of a pyramid or a cone equals one half the product of the perimeter of the base by the slant height.*

III. *The convex surface of a sphere equals the product of the diameter by the circumference, or it equals the area of four times the circle having the diameter of the sphere.*

WRITTEN

2. What is the lateral surface of a triangular prism whose sides are 3 in., 4 in., and 5 in., and whose slant is 20 in.? What is the entire surface of this prism?

3. What is the lateral surface of a square prism whose base is 4 in. and altitude 9 in.? What is the entire surface?

4. What is the lateral surface of a cylinder with an altitude of 16 in. and a diameter of 12 in.? What is the entire surface?

5. What is the lateral surface of a cylinder having an altitude of 20 in. and a radius of 8 in.? What is the entire surface?

6. What is the lateral surface of a square pyramid with a 12-in. base and 20-in. slant height? The entire surface?

7. What is the lateral surface of a pyramid whose base is an equilateral triangle having 12-in. sides, and whose slant height is 18 in.?

8. What is the lateral surface of a cone 20 in. in diameter with a slant height of 40 in.? The total surface?

9. What is the total surface of a conical tower 40 ft. high, whose radius is 30 ft.?

10. What is the surface of a 10-in. sphere?

11. What is the difference between the surfaces of a 20-in. sphere, a 20-in. cube, and a circle 20 in. in diameter?

REVIEW

WRITTEN

235. 1. The length of the base of a parallelogram is 60 ft., its altitude is 39 ft. Find its area.

2. How many acres are there in a farm in the form of a parallelogram 45 rods wide and 68 rods long?

3. What is the area of a trapezoid whose parallel sides are 42 rods and 68 rods, the perpendicular distance between them being $37\frac{1}{2}$ rods?

4. The area of a trapezoidal field contains $6\frac{1}{2}$ acres; the bases are 300 ft. and 140 ft. respectively. Find the perpendicular distance between the sides.

5. The area of a trapezoid is 32,600 sq. in.; the perpendicular distance between the two bases is 76 in. If the length of one of the parallel sides is 120 in., what will be the length of the other parallel side?

6. What is the area of a triangle whose base is 67 ft. and altitude 19 ft.?

7. What is the area of a triangular farm with a base of 62 rods and a perpendicular distance of 95 rods from the vertex to the base?

8. What is the cost of a triangular piece of land with a base of 2112 ft. and an altitude of 1650 ft., at $50 per acre?

9. A triangular farm containing 50 acres has a base of 60 rods. Find its altitude.

10. What is the area of a circle whose diameter is 8 in.? Of a circle whose radius is 7 ft. 6 in.? Find the area of a circle whose circumference is 62.832 ft.

11. How much cloth is wasted in cutting the largest circle possible from a piece of cloth 16 in. square?

12. What is the difference in size between an 8-inch circle and a 16-inch circle?

13. Find the total atmospheric pressure, at 15 pounds per square inch, upon the top of a circular glass plate 32 in. in diameter.

14. Two circles, 5 in. and 12 in. respectively in diameter, have the same center. What is the area of the ring between their circumferences?

15. The area of a circle is one square inch. What is its diameter?

16. How long a rope will it take to fasten a horse to a post so that he may graze over one half acre of grass and no more, providing he can graze 2 ft. beyond the end of the rope?

17. Find the entire surface of a 6-inch cube; of a 12-inch cube. Find the diagonal of each of these cubes.

18. Find the entire surface of a parallelopiped 8 in. high, 6 in. wide, and 10 in. long.

19. How many square inches of cloth are required for covering the top and sides of a cabinet 7 in. high, 8 in. wide, and 14 in. long?

20. How many square yards of cement plastering are required for the sides and bottom of a cistern 7 ft. 6 in. wide, 10 ft. 3 in. long, and 6 ft. deep?

21. What is the lateral surface of a prism 16 in. high, having a triangular base with sides of 4 in., 5 in., and 6 in. respectively?

22. What is the lateral surface of a cylinder 15 in. high, 10 in. in diameter? Its entire surface?

23. How many square yards of cement plastering are required for a cylindrical cistern 8 ft. deep and 8 ft. in diameter?

24. How many sides of tin cans 8 in. high and 4 in. in diameter may be cut from a piece of metal 24 in. wide and 126 in. long?

25. What is the lateral surface of a pyramid having a slant height of 360 ft. and a square base 240 ft. long?

26. What is the lateral surface of a triangular prism having a height of 12 in., and having for the base an equilateral triangle the entire length of whose sides is 36 inches?

27. What is the lateral surface of a cone having a slant height of 16 inches and a base with an 8-in. diameter? What is its entire surface?

28. How many square inches of metal are required in tinning the entire surface of a cone having a slant height of 21 in. and a base 12 in. in diameter?

29. What is the area of the surface of a 1-inch sphere? Of a 10-inch sphere?

30. At $1.50 per square yard, what is the cost of gilding a 3-foot globe?

31. What is the cost of frescoing the inside of a semi-spherical dome 50 ft. in diameter, at $1.20 per square yard?

32. What is the difference in area between a circle 20 in. in diameter and the surface of a sphere 10 in. in diameter?

MEASURES OF VOLUME

236. To find the volume of solids.

RULES: I. *The volume of a rectangular solid equals the product of its three dimensions, or it is equal to the product of the number representing the area of the base by the altitude.*

II. *The volume of a cylinder equals the product of the number representing the area of the base by the altitude.*

III. *The volume of a pyramid or a cone equals the product of the number representing the area of the base by one third of the altitude.*

IV. *The volume of a sphere equals the product of the number representing the area of the surface by one third of the radius.*

ORAL

237. 1. Define cube; parallelopiped; prism; pyramid; cone; sphere.

2. Draw or construct a cube one inch on each side. How many cubic inches does it contain?

3. Draw or construct a parallelopiped having a one-inch base and two inches long. How many cubic inches does this solid contain? Construct it three inches long. How many cubic inches does it contain? How many would it contain if it were five inches long? Twelve inches long?

4. Draw or construct a prism or a parallelopiped one inch thick, two inches wide, three inches long. How many cubic inches does it contain? How many would it contain if it were three inches wide?

5. How many cubic inches are there in a solid 6 inches wide, 8 inches thick, and 10 inches high?

6. How many cubic feet of air are there in a room 15 ft. wide, 20 ft. long, 10 ft. high?

7. How many cords of wood are there in a pile 32 ft. long, 4 ft. wide, and 4 ft. high?

8. How many cubic inches are there in a cylinder 10 inches in diameter and 10 inches high?

9. Construct a square prism of soft wood or turnip. Ascertain its weight. By cutting the edges with a knife, transform the prism into a pyramid having the same base and same height. Ascertain its weight. Compare the weight of the pyramid with that of the prism.

10. A prism of a certain size weighs 10 ounces. What will be the weight of the largest pyramid that can be constructed from it? State the rule for finding the volume of a pyramid or a cone.

11. A square prism is 12 inches wide and 30 inches high. What is its volume?

12. A cone is 10 inches in diameter and 10 inches high. Find its volume.

13. How many quarter-inch cubes can be cut from a one-inch cube?

14. How many two-inch cubes can be cut from a four-inch cube?

WRITTEN

15. How many cubic inches are there in an 8-inch cube? In a 16-inch cube?

16. How many cubic feet of water are there in a tank 16 ft. long, 4 ft. 6 in. wide, and 18 in. deep?

17. What is the cost of excavating a cellar 60 ft. long, 30 ft. wide, 6 ft. deep, at 10¢ per cubic yard?

SOLUTION: $\frac{60 \times 30 \times 6}{27} = 400$, number of cubic yards.

$400 \times 10¢ = \$40$. *Ans.*

18. Find the cost of excavating for the basement of a store 80 ft. wide, 150 ft. long, and 9 ft. deep, at $8\frac{1}{3}$ cents per cubic yard.

19. How many cubic yards of soil are removed in digging a sewer on a level street two miles long, the ditch being 2 ft. wide and 6 ft. deep?

20. How many cubic yards of soil are removed in digging a ditch for the foundation of a house 30 ft. × 64 ft., the foundation to be 2 ft. 6 in. thick, and to extend 2 ft. 6 in. beneath the surface of the ground?

Illustrate surface of the ditch by a drawing.

21. How many barrels of water will a cistern 11 ft. long, 9 ft. wide, and 7 ft. deep hold?

SOLUTION: $\frac{11 \times 9 \times 7 \times 1728 \times \frac{3}{4}}{231} = 164\frac{4}{7}$, number of barrels.

State the reason for the employment of the different factors in the operation.

22. How many barrels of water are there in a reservoir 44 ft. long, 36 ft. wide, 14 ft. deep?

23. How many barrels of water are there in a rectangular tank 21 ft. long, 3 ft. 6 in. wide, 1 ft. 6 in. deep?

24. How many bushels of grain are there in a bin 12 ft. long, 10 ft. wide, 8 ft. deep?

25. How many cubic inches are there in a cylindrical vessel 8 inches in diameter, 12 inches high?

26. What is the volume of the largest cone that can be constructed from a cylindrical piece of wood 12 inches in diameter, 21 inches high? The volume of the largest sphere?

27. What is the weight of water in a cylindrical bucket 14 inches in diameter, 18 inches high, a cubic foot of water weighing $62\frac{1}{2}$ lb.?

28. What is the weight of an iron sphere 8 inches in diameter, the iron being $7\frac{2}{10}$ times as heavy as an equal bulk of water?

29. How many cubic inches of water will a cone-shaped funnel 8 inches in diameter, 12 inches high, hold?

30. 100 one-half inch glass globes are placed in a cone 10 in. in diameter and 15 in. high. How many cubic inches of water may now be poured into the cone?

31. A heap of wheat in the corner of a bin is 5 ft. high, and extends 5 ft. each side from the corner. If its surface is spherical, find the number of bushels in the heap.

32. A 12-inch sphere is reduced to 10 inches. How many cubic inches are there in the shell removed?

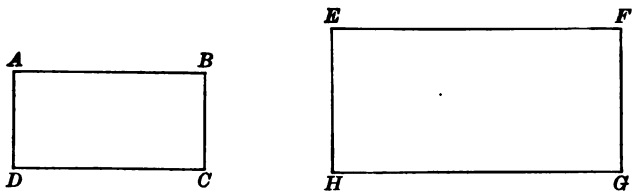
SIMILAR SURFACES AND SOLIDS

238. Similar surfaces are such as have the same form, differing from one another only in size. Their corresponding sides are proportional, and their corresponding angles are equal.

ILLUSTRATIONS.

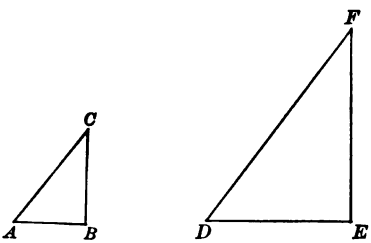
In the rectangle $ABCD$, suppose the ends are 2 inches long and the sides are 4 inches long. As in all rectangles, the angles are all right angles. They are, therefore, equal.

Another rectangle is similar to the rectangle $ABCD$ if the corresponding sides are proportional. If we construct the rectangle $EFGH$ with



the ends 3 inches long, then they are each, in length, $\frac{3}{4}$ of the ends of the rectangle $ABCD$. In order that the rectangles may be similar, the sides of the rectangle $EFGH$ must be in length $\frac{3}{4}$ of the sides of the rectangle $ABCD$, or 6 inches.

Suppose the side AB of the triangle ABC is 3 inches long, the side BC is 4 inches long, the side AC is 5 inches long, and the angle B is a right angle.



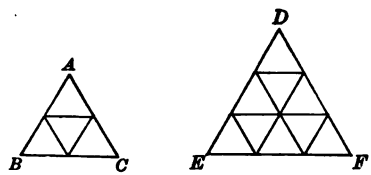
To construct a similar triangle with the base, DE, 6 inches long, how long must the perpendicular EF be? How long must the hypotenuse DE be? The angle E equals what angle in the triangle ABC? The angle D equals what angle in the triangle ABC? The angle F equals what angle in the triangle ABC?

In the triangle ABC, on p. 355, suppose each side is 2 inches long, and in the triangle DEF, each side is 3 inches long, and the angles D, E, and F are respectively equal to the angles A, B, C. The two triangles are similar.

Their areas are to each other as the squares of their like dimensions; that is, area of $\triangle ABC$: area of $\triangle DEF$: : $2^2 : 3^2$.

The area of the $\triangle ABC$ is made up of the four small triangles, while

the area of $\triangle DEF$ is made up of nine small triangles of equal size. 4 may represent the area of $\triangle ABC$ and 9, the area of $\triangle DEF$; therefore, we have $4 : 9 :: 2^2 : 3^2$. The product of the means equals the product of the extremes.



If the sides of the triangle DEF were 4 inches instead of 3 inches, the number of small triangles in its area would be 16, and we should have the proportion, 4, the area of $\triangle ABC$: ?, the area of $\triangle DEF :: 2^2 : 4^2$. Solving the proportion by dividing the product of the extremes by the given mean, we have 16 for the other mean, thus reaching the same result as though we had divided the triangle DEF into 16 equal small triangles.

Similar solids are such as have the same form, differing from one another only in volume. Their corresponding measurements are proportional, and their corresponding angles are equal.

The areas of similar surfaces are to each other as the squares of their like dimensions.

The volumes of similar solids are to each other as the cubes of their like dimensions.

PROBLEMS

WRITTEN

239. **1.** A triangle containing 90 sq. in. has a base of 12 in. What is the area of a similar triangle having a base of 16 in.?

SOLUTION: $12^2 : 16^2 :: 90 : x$. $\frac{16 \times 16 \times 90}{12 \times 12} = 160$, number of sq. in. *Ans.*

2. A circle 20 in. in diameter contains 314.16 sq. in. How many square inches are there in a circle 5 in. in diameter? 40 in. in diameter? 10 in. in diameter?

area 1st circle	:	area 2d circle	::	diam.^{2} 1st circle	:	diam.^{2} 2d circle
314.16	:	x	::	20^2	:	5^2

3. The areas of two similar rectangles are, respectively, 108 sq. in. and 192 sq. in. The base of the smaller is 12 inches. What is the base of the other?

$$108 : 192 :: 12^2 : (x)^2$$

4. The radius of a circle is 3 ft. Find the diameter of another circle containing 4 times the area of the first.

Represent the area of the first circle by 1.

5. Three farmers, A, B, and C, purchase a grindstone 32 in. in diameter, each paying $\frac{1}{3}$ of its cost. Each uses the stone in rotation until he has used up a portion equivalent to the part of the cost he pays. How large is the grindstone when A sends it to B? When B sends it to C?

The thickness of the grindstone is immaterial; therefore, solve as a problem in similar surfaces.

6. If a ball 4 in. in diameter weighs 36 lb., find the weight of another ball of the same material, 6 in. in diameter.

SOLUTION: $4^3 : 6^3 :: 36 : x.$

$$\frac{6 \times 6 \times 6 \times 36}{4 \times 4 \times 4} = 121\tfrac{1}{2},$$ number of lb. *Ans.*

7. A stack of hay 16 ft. high weighs 2 tons. What is the weight of a similar stack 24 ft. high?

8. If the altitude of a cone that weighs 1000 lb. is 6 ft., what is the altitude of a similar cone that weighs 500 lb.?

9. A rectangular cistern 5 ft. in depth holds 40 bbl. of water. How many barrels will a similar rectangular cistern 8 ft. in depth hold?

10. Three ladies, A, B, and C, agree to furnish a ball of yarn 9 inches in diameter, each furnishing an equal proportion of it. A does her share and hands the ball to B. What is its diameter? B puts on her share and sends it to C. What is its diameter now?

PROBLEMS IN AGRICULTURE

HAY IN MOWS AND IN STACKS

240. The only accurate method of determining the number of tons of hay in a stack or mow is by weighing. The amount, however, may be determined, approximately, by measurement.

In high stacks and deep mows:

Wild hay requires about 400 cu. ft. per ton.
Timothy hay requires about 500 cu. ft. per ton.
Clover hay requires about 600 cu. ft. per ton.

For stacks or mows less than 10 ft. high, add about 100 cu. ft. to each of the foregoing.

WRITTEN

1. How many tons of clover are there in a mow 36 ft. by 40 ft. and 24 ft. deep?

2. How many tons of wild hay are there in a mow 6 ft. narrower and 2 ft. deeper than the foregoing?

3. A load of clover is 11 ft. wide, 16 ft. long, and 10 ft. high. What is its value at $8.50 per ton?

4. How many tons of timothy are there in a cylindrical stack 40 ft. in diameter and 26 ft. high?

5. A cone-shaped stack of wild hay 50 ft. in diameter and 30 ft. high is worth how much at $6.50 a ton?

6. How many tons of wild hay are there in a stack 40 ft. long, 20 ft. wide, and 44 ft. high?

7. Find the cost of a cone-shaped stack of clover hay 50 ft. high, with a base 60 ft. in circumference.

8. A mow is 24 ft. by 30 ft., and contains 35 tons of clover. How high is it?

9. A cone-shaped stack of wild hay contains 50 tons. If its height is 30 ft., what is its diameter?

10. The space under a gable roof 38 ft. wide, 16 ft. high, and 50 ft. long is filled with timothy hay. What is its value at $9.25 a ton?

11. A haymow designed to hold 150 tons of clover is 60 ft. long and 24 ft. high. How wide must it be?

12. Which is more valuable and by how much: a mow of timothy 80 ft. by 40 ft. and 20 ft. high at $9.50 a ton, or a cylindrical stack of wild hay 60 ft. in diameter and 32 ft. high at $7.25 a ton?

FENCING

241. Barbed wire averages about one pound per rod. It is sold in spools containing from 95 to 115 pounds.

Written

1. What is the cost of fencing a square 40-acre tract with a 4-barbed-wire fence at $2.85 per hundred?

2. A field 80 rods long and 40 rods wide is fenced with woven wire 32 in. wide at 30¢ a rod, and 2 barbed wires at $3 per hundred. The posts are one rod apart and cost $12\frac{1}{2}$¢ apiece. What is the price of the material?

3. An 80-acre farm of two adjoining square forties is to be surrounded by a fence, and to be separated into 20-acre fields by cross fences. What is the cost of the material if the same kind is used as given in problem 2?

4. What is the cost of material required for surrounding the S.W. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of a section of land with a 4-wire fence, at $2.75 per hundred, posts one rod apart at 15¢ apiece?

5. If a man can drive 80 posts per day and string 4 wires

across a forty during the same time, what will be the cost of labor at $1.50 a day to build the fence mentioned in problem 4?

6. With the kind of material and at the prices mentioned in problems 4 and 5, what will be the difference in the cost of fencing a quarter-section in the form of a square, and another in the form of an oblong 80 rods wide?

SILOS

242. A **silo** is a structure used for preserving in its green state finely cut corn stalks, sugar cane, alfalfa, etc. This finely cut vegetable material is called **silage**, or **ensilage**. Silos are built of wood, stone, or cement. The circular silo is the most economical form. Why?

The average weight of silage in silos from 25 to 36 ft. high is about 40 pounds a cubic foot. Forty pounds is also the amount of silage ordinarily fed to a cow per day.

Written

1. 10 A. of good corn should provide silage for 20 cows for 180 da. Reckoning 40 lb. a day for each cow, how many tons of silage should be furnished by 10 A.?

2. On this basis, how many acres of good corn are required to fill a silo 16 ft. in diameter and 36 ft. high?

3. How many head of stock will this supply for 200 days?

4. A circular silo 20 ft. in diameter contains 200 tons of ensilage. If this averages 40 lb. per cu. ft., find the depth of the silo.

5. If a silo is filled at the rate of 8 tons per day, how long will it take to fill a silo 16 ft. in diameter and 40 ft. high?

6. How many cows can be supplied from it for 190 days?

CREAMERIES

243. Many creameries are owned by the patrons who furnish the milk. Officials elected by the patrons supervise the management of the factory, the sale of the butter, etc. At the close of each month a statement is formulated, showing the amounts received from the sale of the butter, the expenses, and the balance that is to be divided among the patrons, in proportion to the amount of cream furnished by each.

WRITTEN

1. The totals of a creamery of 10 patrons were as follows for the month of March:

NAME	POUNDS MILK	AVERAGE TEST	POUNDS BUTTER FAT
		Per Cent	
1. John Smith	6450	4.02	259.29
2. Captain Doe	4326	3.35	
3. James Dean	7610	3.39	
4. Henry King	3064	4.05	
5. Peter Rule	5675	4.25	
6. P. G. Riley	6940	4.20	
7. Roy Gimlin	5015	3.85	
8. Fred Johns	2480	3.96	
9. John Lund	4965	4.15	
10. W. Bell	2983	3.83	

The butter for the month was sold for $634.85, and the expenses amounted to $75.97. Find the amount that was due each patron.

NOTE. First find the amount of butter fat furnished by each patron. (4.02% = 4.02 lb. per hundred pounds of milk.) Divide the net proceeds by the total number of pounds of butter fat. This gives the price of a pound of butter. Multiply the number of pounds furnished by the different patrons by the price of one pound.

2. About 80 lb. of skimmed milk is returned for every 100 lb. of milk furnished. This is worth about 20¢ a hundred. What is the total gross value of the milk handled in this creamery?

3. L. M. Carpenter's milk sheet for the month of February showed the following entries:

1. 245 lb.	8. 240 lb.	15. 209 lb.	22. 250 lb.
2. 256	9. 237	16. 242	23. 245
3. 199	10. 225	17. 219	24. 256
4. 244	11. 210	18. 229	25. 260
5. 263	12. 209	19. 224	26. 262
6. 213	13. 230	20. 218	27. 260
7. 245	14. 217	21. 232	28. 275

Three tests were taken during the month giving these results: 3.95%, 3.99%, and 4.30%. If the net price of butter was 25¢ a pound, find the amount due Mr. Carpenter.

4. What is the total value of his milk industry for the month?

5. Formulate a monthly report for a creamery of 20 patrons, and solve the problem involved therein.

LIVE AND DRESSED WEIGHT OF ANIMALS

244. Hogs and poultry lose about 20% of the live weight in dressing, while sheep and cattle lose from $33\frac{1}{3}$% to 50%.

Written

1. What is the dressed weight of a crate of chickens weighing 240 lb.?

2. A 380-lb. live hog, purchased at $4.75 a hundred, is retailed at an average of 10¢ a pound. What is the profit?

3. Experiments at agricultural stations have shown that 500 lb. of corn fed to hogs increases their weight about 100 lb. If hogs are worth $7.50 per hundred dressed, does a farmer gain or lose by feeding them 360 bu. of corn worth 40¢ a bushel? (1 bu. corn weighs 56 lb.) How much?

4. I sell 5 steers weighing 4950 lb., and some cows weighing 3460 lb. What is the total of the dressed beef produced by them, allowing a loss of 40 % in dressing?

5. When pork sells at $5 per hundred live weight, how much ought corn to bring in order that neither loss nor gain may result in feeding it to hogs?

6. Hens average about 150 eggs each per year. Allowing 1080 eggs for hatching, what is the total income from a yard of 120 hens, when eggs average 20¢ a dozen, and when 1850 lb. of young chickens are sold at 15¢ a pound, dressed weight?

WATER IN PLANT GROWTH

245. Experiments conducted by the Department of Agriculture have proved that about 300 lb. of water are required to mature 1 lb. of corn. Oats need 500 lb., while clover, barley, and wheat need about 400 lb. each to mature 1 lb. of the product.

Written

1. How much water is required to mature 40 acres of oats averaging 45 bu. per acre? (1 bu. oats weighs 32 lb.)

2. To what depth would it cover the field if applied at one time? (1 cu. ft. of water weighs 62.5 lb.)

3. A 10-acre field of corn receives 6 in. of rain. If the rest is supplied by irrigation, how many cubic feet of water are required, if no allowance is made for evaporation, figuring 100 bu. per acre?

4. Experiments have proved that from a given strip of land with a firm surface, 5.34 inches of water evaporated during a given time. From an adjacent strip of like size and soil, but whose surface was kept well tilled to a depth of 3 inches, only .88 inch evaporated in the same time. How many tons of water were saved the soil by tilling 80 acres?

5. How much less rainfall is needed on a 40-acre piece of corn kept well tilled than on a field of the same size that is allowed to remain hard and firm? What will be the weight in tons?

6. Find the difference between the amount of water required to mature a 10-acre field of corn yielding 90 bu. per acre and a field of wheat of the same size yielding 32 bu. per acre. Make no allowance for evaporation. (1 bu. wheat weighs 60 lb.)

7. What is the average rainfall in your state per year? How much would the total fall for one year on an acre weigh?

8. A bushel of potatoes contains about 75% water. If an acre yields 175 bu., determine the number of pounds of water they contain. How many barrels? (1 bu. potatoes weighs 60 lb.)

9. During a summer 12 in. of rain fell on a field of wheat that gave an average yield of 24 bu. per acre. What per cent of the rain was used to mature the crop?

VALUE OF FERTILIZERS

246. Protein contains the nitrogen, and the carbohydrates consist mostly of starches and sugars. The carbon is secured by the plants from the air, but the other substances are furnished by the soil. For every 1000 lb. of wheat raised, 79 lb. of nitrogenous compounds, 7 lb. of phosphoric acid, and 4 lb. of potash are withdrawn from the soil. When these substances are exhausted from the land, fertilizers which restore them to the soil become a necessity.

The leading artificial fertilizers are wood ashes, rock phosphate, ground bones, ammonium sulphate, and potassium sulphate. Barnyard manure contains all the substances mentioned in sufficient quantities for restoring fertility to the soil.

AMOUNT OF LEADING COMPONENTS IN 100 LB.

	PROTEIN	CARBOHYDRATES	PHOSPHORIC ACID	POTASH
Corn	7.9 lb.	66.7 lb.	.7 lb.	.4 lb.
Wheat	10.2	69.2	.8	.5
Rye	9.9	67.6	.8	.5
Barley	8.7	63.6	.7	.5
Oats	9.2	47.3	.8	.6
Silage Corn	.9	11.3	.5	.9
Fodder Corn, cured	2.5	34.6	.5	.9
Potatoes	.9	16.3	.1	.5
Timothy Hay	2.8	43.4	.3	.6
Red Clover	6.8	35.8	.1	.5

ORAL

247. 1. How many pounds of the leading substances are withdrawn from 10 A. of corn averaging 100 bu. per acre?

2. How many pounds of the leading substances are absorbed by 2 A. of potatoes averaging 300 bu. per acre?

3. What crop may profitably follow corn? Show by means of the table.

WRITTEN

4. It has been estimated that the value of the barnyard manure of a horse is $27 per year; the manure of a cow is estimated at $18; and the manure of a hog at $12. If a farmer has 4 horses, 22 cows, and 40 hogs, what is the annual value of the manure?

5. At the Minnesota Agricultural Experiment Station it was shown that timothy grown on manured land contained 8.75% of protein, while on unmanured land only 6.45% was obtained. Corn fodder gave 8.85% and 6.32% respectively. A 40-acre tract was thoroughly manured and planted with fodder corn. How much more protein was secured from it than from an equal but unmanured tract, reckoning 10 tons to an acre? What would be the difference in the feeding value at 4¢ a pound?

6. Experiments have shown that from 1 A. of firm soil 159.46 lb. of nitrates, 4.59 lb. of phosphates, and 333.17 lb. of sulphates were washed in a given time by rains, while from an acre of soil essentially similar to the other, but whose surface was kept soft and porous by tillage to a depth of 3 inches, only 67 lb. of nitrates, 2.69 lb. of phosphates, and 31.94 lb. of sulphates were washed at the same time. How much of these valuable components are saved for plant growth by keeping a 30-acre tract of corn well tilled? What is the value of the amount saved, at an average price of $\frac{5}{8}$¢ a pound?

7. A ton of barnyard manure increases the yield of shelled corn by about 70 lb. What is the gain on a 20-acre tract if 10 tons are applied per acre, corn selling at 55¢ a bushel?

8. Formulate a problem based upon the value of fertilizers, and have the class solve it.

9. If 1000 pounds of wheat remove 52 pounds of potash from the soil, how much will 40 bushels to the acre remove per acre?

10. If the straw weighs $1\frac{1}{2}$ times as much as the grain, and 1000 lb. of straw remove 6.3 lb., how much will the straw of 40 bu. remove?

11. The corresponding amount of phosphoric acid is 7.9 for the grain to 2.2 for the straw. How much phosphoric acid will be removed by the 40 bu. of wheat and the straw?

12. The corresponding amount of nitrogen is 20.8 lb. for 1000 lb. of grain and 4.8 lb. for 1000 lb. of straw. How much nitrogen will be removed by 40 bu. of grain and its straw, on the basis of the tenth problem?

13. 1000 lb. of tobacco take from the soil 42 lb. of nitrogen, 5 lb. of phosphoric acid, and 57 lb. of potash. The nitrogen is worth 15¢ a pound, and each of the others 5¢ a pound. What is the whole worth?

14. 1000 lb. of corn take from the soil 18 lb. of nitrogen, 17 lb. of phosphoric acid, and 4 lb. of potash. How much is it all worth?

15. If barnyard manure contains 10 lb. of nitrogen, 6 lb. of phosphoric acid, and 9 lb. of potash, how much manure would it be necessary to apply to replace the nitrogen used by the corn in problem 14? Is this sufficient to replace the phosphoric acid? How much more is necessary?

16. How many tons of barnyard manure will be required to supply the nitrogen removed from an acre by 25 bu. of wheat? By 75 bu. of corn?

MISCELLANEOUS PROBLEMS OF INTEREST TO THE FARMER

WRITTEN

248. 1. The annual wheat crop of the United States is about 500,000,000 bushels. If this could be increased 2% by seed selection, how much would the addition be worth at 80¢ per bushel?

2. How many agricultural experiment stations would this increase support at $500,000 a year for each station?

3. If grass seed costs 10¢ a pound and is 20% weed seed, what is the real cost of clean grass seed? Would it be cheaper to buy a clean seed at 12¢?

4. If half a day extra were spent each year for 5 years in selecting and caring for seed corn, and the yield were thereby increased 100 bushels, would it pay?

5. If there are 40 birds per square mile, how many birds are there in your state?

6. Allowing each bird 50 insects a day for the 120 days of summer, how many insects would there be destroyed during the year?

7. Three golden-winged woodpeckers were found by King to have in their stomachs, respectively, 255, 220, and 200 ants. Suppose this to represent 1 day's ration, how many ants would these 3 birds eat in 4 months?

8. The answer to the seventh question may be multiplied by 3 to find the number for a year.

9. During an outbreak of Rocky Mountain locusts, a marsh wren was seen to carry 30 locusts to her young in 1 hour. At this rate, how many locusts would she carry in 7 hours?

10. If there are 20 broods to the square mile, how many insects would be destroyed in 78,000 square miles of Nebraska's area in 7 hours?

11. If each locust were to consume its own weight of standing crops each day, how much would be saved by the birds, allowing $10 per ton for the crops? Each locust weighs 15 grains, and there are 7000 grains in a pound.

12. If soil is cultivated 5 inches deep, how many cubic feet to the acre are cultivated? If 7 inches deep, how many cubic feet? If 9 inches deep, how many cubic feet?

13. How many acres are there in a piece of land 32 rods long and 20 rods wide?

14. Supposing it could be plowed back and forth with furrows 9 in. wide, how far would a man travel to plow it?

15. How much more available plant food is there if it is plowed 7 in. than if it is plowed 5 in.?

16. If by extra cultivation a man can raise 100 bu. of potatoes more an acre, how many days' work would thus be paid for, allowing $3 per day for man and team, if the potatoes bring $.25 per bushel?

SPECIFIC GRAVITY

249. The specific gravity of a body is the weight of that body compared with the weight of an equal volume of water. If a stone has a specific gravity of 3, it means that a cubic foot of it weighs three times as much as a cubic foot of water.

The weight of a cubic foot of water is 62.5 lb.

The following table shows the approximate specific gravity of several substances:

Alcohol	.84	Gravel	2
Applewood, dry	.79	Hickory, dry	.80
Brass	8.4	Hickory, green	.95
Cedar	.61	Iron	7.2
Coal	1.3	Maple, hard, dry	.72
Copper	8.9	Mercury	13.5
Cork	.24	Oak, dry	.73
Earth, loose	1.5	Pine, dry	.55
Elm, dry	.67	Sugar	1.5
Glass	2.9	Tin	7.3
Gold	19.3	Zinc	7

WRITTEN

1. What is the weight of a cord of dry oak?

2. What is the weight of a cubic yard of gravel?

3. Find the weight of 20 cu. ft. of cork.

4. A cubic foot of lead weighs 706$\frac{1}{4}$ lb. Find from this the specific gravity of lead.

5. Find the specific gravity of silver, one cubic foot of which weighs 656$\frac{3}{4}$ lb.

6. A basement is to be excavated 24 ft. wide, 80 ft. long, and 9 ft. deep. Supposing that the soil in breaking up increases one third of its original volume, what is the weight of the soil to be excavated?

7. If a cubic foot of a substance weighs 1080 oz., what is its specific gravity?

8. What is the loss of weight in the drying of a cord of green hickory?

9. How many cubic inches of glass will weigh as much as a cubic inch of gold?

10. Formulate a problem in specific gravity, and solve it.

HOUSEHOLD PROBLEMS

Written

250. 1. If the cost of milk is 3.9¢ per pound, and the cost of eggs is 13.3¢ per pound, and the fuel value is 290 and 590 calories per pound respectively, what would be the comparative fuel value in 10¢ worth of each?

Solution

10¢ ÷ 3.9 = 2.6, or number of pounds of milk to be obtained for 10¢.
10¢ ÷ 13.3 = .75, or number of pounds of eggs to be obtained for 10¢.
290 × 2.6 = 754, fuel value of 10¢ worth of milk.
590 × .75 = 443, fuel value of 10¢ worth of eggs.
754 ÷ 443 = 1.7 times as much fuel value in 10¢ worth of milk as in 10¢ worth of eggs.

2. Mary wished to make gingerbread. Her mother told her to take one half the following recipe:

½ cup butter	1 level teaspoonful cinnamon
½ cup sugar	1 level teaspoonful ginger
½ cup New Orleans molasses	1 level teaspoonful cloves
2½ cups flour	1 cup boiling water
2½ level teaspoonfuls soda	2 eggs

½ teaspoonful salt

How much of each ingredient did Mary use? If she took one fourth of the recipe, how much of each would she use?

3. What would be the cost of covering a floor 8 ft. × 12 ft. with linoleum 2 yd. wide at $1.50 per yard?

4. A loaf of bread weighs 1 lb. If 4 oz. of this weight is what the sugar, salt, butter, etc., weigh, and the rest is the weight of the flour in it, how many cups of flour were used in the loaf?

16 oz. = 1 lb.
4 cups of flour = 1 lb.

5. If butter is \$.30 per pound; sugar, \$1 per 20 lb.; flour, \$1.50 per 50 lb.; eggs, \$.25 per dozen; 2 teaspoonfuls baking powder, \$.00⅔; and 1 teaspoonful spices, \$.00⅓, determine the cost of the cookies in the following recipe:

1 cup butter	4 eggs
2 cups sugar	2 teaspoonfuls baking powder
3 cups flour	1 teaspoonful spice

2 cups butter = 1 lb.
2 cups sugar = 1 lb.
4 cups flour = 1 lb.

6. How much cream of tartar, soda, and cornstarch must be substituted for 5 teaspoonfuls baking powder, if the composition of baking powder is ¼ soda, ¼ cornstarch, and ½ cream of tartar?

7. Reduce the quarts, pints, and pounds in the following recipe for reception chocolate to cups. Compute one fourth of the recipe.

2 qt. milk	1 pt. cream
1 lb. cocoa	2 eggs
3 tablespoonfuls white sugar	3 teaspoonfuls vanilla extract

2 cups = 1 pt.
2 pints = 1 qt.
2 cups cocoa = 1 lb.

8. Find the cost of making 6 times the quantity of apple jam called for in the following recipe, if apples are \$.12 a quart (1½ lb. to a quart), sugar is \$.05 a pound, lemons are 3 for \$.05, and cinnamon enough for the entire quantity is \$.01.

4 lb. sour cooking apples	Grated rind and juice of 2 lemons
3 lb. sugar	¼ teaspoonful ground cinnamon
¼ pt. cold water	

USE OF SYMBOLS IN ARITHMETICAL ANALYSIS

251. In the analysis of problems it is frequently necessary to perform long operations in addition, subtraction, multiplication, or division in order to find the several results that must be obtained to secure the final result.

The analysis of the following problems illustrates the necessity of such operations in order to find the answer.

1. If 125 acres of land are bought for $62.50 an acre and sold for $9375, what is the gain per acre?

ANALYSIS: If land costs $62.50 an acre, 125 acres cost 125 × $62.50, or $7812 50. If land costing $7812.50 is sold for $9375, the gain equals $9375 − $7812.50, or $1562.50. If $1562.50 is gained by the sale of 125 acres of land, the gain on 1 acre is $1562.50 ÷ 125, or $12.50.

In this analysis three operations are required and three results are found. $62.50 must be multiplied by 125; $7812.50 must be subtracted from $9375; $1562.50 must be divided by 125.

When pupils have been trained to add, subtract, multiply, and divide accurately and rapidly, it is entirely unnecessary to have *all* the problems in the arithmetic worked out completely, as the analysis demands all the reasoning involved in the solution. Much time usually given to figuring in arithmetic can thus be saved for work in other subjects.

In the solutions of problems, any letter may be substituted for, or used instead of, any number which might be the result of any operation, provided the same letter is not used instead of any other number in the solution of the problem. Numbers resulting from other operations in the same solution may be represented by other letters.

The use of letters for numerical results of indicated operations is illustrated in the following analysis of problem 1.

ANALYSIS: If land costs $62.50 an acre, 125 acres cost 125 times $62.50, or $$x$. If land costing $$x$ is sold, at a gain, for $9375, the gain

equals $\$9375 - \$x$, or $\$y$. If $\$y$ are gained by the sale of 125 acres of land, the gain on 1 acre is $\frac{1}{125}$ of $\$y$, or $\$y \div 125$, or $\frac{\$y}{125}$. The expression $\frac{\$y}{125}$ represents the answer. If desired, $\$z$ may be substituted for $\frac{\$y}{125}$.

In an analysis of this kind, wherever we indicate an operation, we at once assume it to have been performed and substitute a letter for the result. As we proceed with the analysis, whenever it is necessary to use this result, we use instead the substituted letter, and substitute another letter for the result of the next operation, and so on until the expression, or letter, representing the final result is found. We can thus proceed with the analysis without the delay or interruption of our course of reasoning, resulting from the performance of each operation.

This analysis might also be indicated as follows:

$$\frac{\$9375 - (125 \times \$62.50)}{125} = \$x \text{ gain, } \textit{Ans.}$$

A number of illustrations of the first form of analysis are here given. The same method may be used at the teacher's discretion with other problems throughout the book.

2. What change should be received from $50, in payment for 16 doz. buttons at $.36 a dozen, 6 yards of velvet at $4.75 a yard, and 15 yards of lining at $.18 a yard?

ANALYSIS: 16 doz. buttons at $.36 a dozen cost 16 times $.36, or x dollars. 6 yards of velvet at $4.75 a yard cost 6 times $4.75, or y dollars. 15 yards of lining at $.18 a yard cost 15 times $.18, or z dollars. The cost of the buttons, velvet, and lining equals $x + y + z$, or p, dollars. The amount of change to be received is $\$50 - \$p$, or $\$r$.

Or, $\$50 - (16 \times \$.36 + 6 \times \$4.75 + 15 \times \$.18) = \$x$. *Ans.*

3. At $1.50 a day, how much does a man earn in 3 weeks, if he works $5\frac{3}{4}$ days the first week, $4\frac{3}{5}$ days the second week, and $5\frac{2}{3}$ days the third week?

ANALYSIS: During the 3 weeks he works $5\frac{3}{4}$ days + $4\frac{3}{5}$ days + $5\frac{2}{3}$ days, or x days. If he earns $1.50 a day, in x days he earns x times $1.50, or $\$y$.

Indicate this analysis in another way.

4. A drover with $2144 bought as many horses as possible for $165 each, and spent the remainder for sheep at $4 a head. How many of each did he buy?

ANALYSIS: If horses cost $165 each, for $2144 a drover can buy $\frac{\$2144}{\$165}$, or x horses, and have $\$y$ left. At $4 each, for $\$y$ he can buy $\frac{\$y}{\$4}$, or z sheep. He bought x horses and z sheep.

5. Corn weighs 56 lb. and wheat 60 lb. a bushel. If a farmer sells 6640 lb. corn at $.35 a bushel, and 4290 lb. wheat at $.60 a bushel, how much should he receive?

ANALYSIS: 6640 lb. corn $= \frac{6640}{56}$ bu., or x bu. x bu. corn at $.35 a bushel are worth $x \times \$.35$, or $\$y$. 4290 lb. wheat $= \frac{4290}{60}$ bu., or z bu. z bu. wheat at $.60 a bushel are worth $z \times \$.60$, or $\$r$. The farmer should receive $\$y + \$r$, or $\$s$.

Or, $\frac{6640}{56} \times \$.35 + \frac{4290}{.60} \times \$.60 = \$x$. *Ans.*

6. A bin 8 ft. square and 6 ft. deep is $\frac{2}{3}$ full of wheat. What is the value of the wheat at $.70 a bushel?

ANALYSIS: If the bin is 6 ft. in depth and is $\frac{2}{3}$ full, the depth of the wheat in the bin is $\frac{2}{3}$ of 6 ft., or 4 ft. If the mass of wheat in the bin is 8 ft. wide, 8 ft. long, and 4 ft. deep, its volume is $8 \times 8 \times 4$ cu. ft, or 256 cu. ft. 256 cu. ft. $= 256 \times 1728$ cu. in., or x cu. in. One bushel contains 2150.4 cu. in. x cu. in. $= \frac{x}{2150.4}$ bushels, or y bushels. The value of y bushels of wheat at $.70 a bushel $= y \times \$.70$, or $\$z$.

As in the first part of the above analysis, whenever the results of the indicated operations can be found mentally without trouble, letters should not be substituted for such results.

7. A lot 120 ft. long and 80 ft. wide is surrounded by a sidewalk 8 ft. wide. What will be the cost of the sidewalk at $1.50 per square yard?

ANALYSIS: Since the sidewalk is 8 ft. wide, the length of each end walk, including the corners, is 80 ft. + 8 ft. + 8 ft., or 96 ft. The length of both end walks is 2 × 96 ft., or 192 ft. The length of each side walk is 120 ft.; of both side walks is 2 × 120 ft., or 240 ft. The entire length of the walk is 192 ft. + 240 ft., or x ft. The area of the walk is $x \times 8$, or y sq. ft. As there are 9 sq. ft. in a square yard, the number of square yards in the walk is $\frac{y}{9}$. At \$1.50 per square yard, the cost of $\frac{y}{9}$ sq. yd. is $\frac{y}{9} \times \$1.50$, or $\$z$.

The above problem may be analyzed correctly in a number of ways, in any one of which letters may be used for the results that cannot be easily found mentally.

8. How much will a planter receive who sends to market 127 bales of cotton, averaging 500 pounds, if the cotton is sold at \$.09¾ a pound, the freight being \$.57 a hundredweight, drayage \$60, and commission 3⅛ %?

ANALYSIS: If the bales average 500 pounds, 127 bales weigh 127 × 500 pounds, or x pounds. x pounds, at \$.09¾ per pound, sell for $x \times \$.09\frac{3}{4}$, or $\$y$. The commission for selling is .03⅛ of $\$y$, or $\$z$. As the freight rate is \$.57 a hundred pounds, the expense for freight on x pounds is $\frac{x}{100} \times \$.57$, or $\$r$. The entire expense of selling is, — commission, $\$z$, + freight, $\$r$, + drayage, \$60, or $\$z + \$r + \$60$, or $\$s$. Since the cotton sold for $\$y$, and the expenses for selling were $\$s$, the planter will receive $\$y - \$s$, or $\$t$.

9. 36 men are engaged to do a piece of work in 48 days. After 12 days 12 men stop. In how many days can the rest finish the work?

ANALYSIS: If 36 men can do a piece of work in 48 days, one man can do it in 36 × 48 days, or x days. If 36 men work 12 days, they do the work of 1 man for 36 × 12 days, or y days. Then there remains work for one man for $x - y$, or z days. If 12 men stop at the end of 12 days, there are 24 men left to do the work of 1 man for z days. It will take 24 men $\frac{1}{24}$ of z days, or r days, to complete the work.

EXPRESSING EQUATIONS

252. 1. If x represents the number of dollars A earns in 1 day, 3 times x, which may be written $3x$, will represent the number of dollars he can earn in 3 days at the same rate. $\frac{x}{2}$ will represent the number of dollars he can earn in half a day, and $\frac{5}{2}x$ or $\frac{5x}{2}$ will represent the number of dollars he can earn in $2\frac{1}{2}$ days, or $\frac{5}{2}$ days. What, then, will represent the number of dollars he can earn in 2 days? In 5 days? In 10 days? In $\frac{1}{3}$ of a day? In $2\frac{1}{3}$ days? In $4\frac{1}{4}$ days?

2. How many are $1\not{x}+5\not{x}+6\not{x}$? $1x+5x+6x$? ($1x$ or once x is written simply x.)

3. If John has x dollars and Henry has 4 times as many dollars, how many dollars has Henry? How many dollars have John and Henry? (*Ans.* Once x plus 4 times $x = 5$ times x, or $x + 4x = 5x$.) How many more dollars has Henry than John? (*Ans.* $4x - x = 3x$.) The number of dollars John has is what part of the number Henry has? $\left(\textit{Ans. } x \text{ is } \frac{1}{4} \text{ of } 4x, \text{ or } \frac{x}{4x} = \frac{1}{4}.\right)$

4. John has two more apples than James. If we let x equal the number James has, what is the expression for the number John has? (*Ans.* $x + 2$.) If we let x equal the number John has, how many has James? (*Ans.* $x-2$.) If Henry has $\frac{1}{2}$ as many apples as John has, and John's are represented by x, what is the expression for the number Henry has? $\left(\textit{Ans. } \frac{x}{2}.\right)$ If John gives Henry 2 apples, what is the expression for the number John then has? (*Ans.* $x - 2$.) For the number Henry then has? $\left(\textit{Ans. } \frac{x}{2} + 2.\right)$

5. If A has x dollars and B has \$8 more than A, how many dollars has B? (*Ans.* $x + 8$.) If C's money is \$8 less than A's, how many dollars has C? (*Ans.* $x - 8$.) If D's money is 8 times A's, how many dollars has D? (*Ans.* $8x$.) If E's money is $\frac{1}{8}$ of A's, how many dollars has E? $\left(\textit{Ans.}\ \frac{x}{8}.\right)$ If F's money is \$2 more than 8 times A's, how many dollars has F? (*Ans.* $8x + 2$.) If G's money is \$2 less than 8 times A's, how many dollars has G? (*Ans.* $8x - 2$.)

6. If James has 8 apples and William has 6 apples, they both have $(8 + 6)$ apples. If Henry has $\frac{1}{2}$ as many apples as both James and William, he has $\frac{8+6}{2} = \frac{8}{2} + \frac{6}{2} = \frac{14}{2}, = 7$ apples. Substitute x for the number of apples James has, and y for the number William has, and find the expression for the number Henry has.

7. What number added to 8 will equal 10? This may be stated $? + 8 = 10$. The **known numbers** here are 8 and 10 and the **unknown number** represented by ? is the answer 2. If we represent the unknown number by x, the statement is $x + 8 = 10$. This is an **equation**, since what precedes the equality sign equals what follows it. The part on the left, $x + 8$, is called the **first member**, and the part on the right, 10, the **second member**.

Make equations expressing the following, representing the unknown number by x:

8. If \$4 is added to Frank's money, he will have \$6.

9. James, who has \$3 less than John, has \$6.

10. Five times Alice's age equals her mother's age, 25 years.

11. One fourth of Anna's age equals 4 years.

12. What number increased by 7 equals 25?

13. The sum of two numbers is 30 and one of the numbers is 10.

14. The difference of two numbers is 5 and the larger number is 17.

15. Twice a number increased by 4 equals 50.

16. One fourth a number diminished by 3 equals 10.

17. One half of a number plus one fourth of it equals 8.

18. Two pieces of lead weigh 35 pounds and one piece weighs 4 pounds more than the other.

19. If a piece of work can be done in 6 days, what part of it can be done in 1 day? In 3 days? In y days? $\left(Ans.\ \frac{1}{6};\ \frac{3}{6};\ \frac{y}{6}.\right)$ If a piece of work can be done in x days, what part of it can be done in 1 day? In 3 days? In y days? $\left(Ans.\ \frac{1}{x};\ \frac{3}{x};\ \frac{y}{x}.\right)$

20. If A does a piece of work in 4 days, and B can do the same piece of work in 6 days, what part can each do in 1 day? What part can both do in 1 day? If x equals the number of days it takes A and B to do the work together, what is the expression for the part they can both do in 1 day? From these expressions form an equation.

21. If 6 bushels of potatoes cost \$3, the expression for the part of a dollar 1 bushel costs is $\frac{3}{6}$. The expression for the number of bushels \$1 will buy is $\frac{6}{3}$. If c bushels of potatoes cost d dollars, what is the cost of 1 bushel?

How many bushels will \$1 buy? How many bushels will \$2 buy?

22. The sum of three numbers is 21. The second is twice the first and the third is twice the second. What is the expression for the second? For the third? For the sum of the first and second? For the sum of the three? For the difference between the first and second?

23. The sum of two numbers is 25. If the first number is 8, what is the expression for the second? If the first number is b, what is the second?

24. The sum of two numbers is a. The first number is 6. What is the second?

25. A father's age is three times that of his son. Make an expression for the age of each. What is the expression for the age of each 10 years later? The son is then one half as old as his father. Using the son's age, make an expression for the father's age. What two expressions are equal? Put them in the form of an equation. State the problem which the solution of the equation will answer.

SOLVING EQUATIONS

253. To **solve an equation** is to find the value of the unknown number.

1. Solve the equation $x + 8 = 10$.

In solving equations it is best, by transposing, to get the unknown numbers represented by letters alone in the first member and the known numbers in the second member.

SOLUTION: Since 10 is 8 more than x, x must be 8 less than 10; $x = 10 - 8$, or 2. PROOF: $2 + 8 = 10$.

2. Solve the equation $x - 8 = 10$.

SOLUTION: Since 8 less than x equals 10, x must equal 8 more than 10; $x = 10 + 8 = 18$. PROOF: $18 - 8 = 10$.

Notice in each of the preceding solutions that the known number is transposed from the left to the right member of the equation by changing the sign preceding it. In Ex. 1, $+8$ is changed to -8 on being transposed. In Ex. 2, -8 is changed to $+8$ on being transposed.

Any term may be transposed from one member to another of an equation, if the sign $+$ preceding the term is changed to $-$ or the sign $-$ to $+$.

Find the value of x in the following:

3. $x+3=5$ **5.** $3+x=5$ **7.** $x+7=15$

4. $x-8=12$ **6.** $20+x=24$ **8.** $x-10=20$

9. Add 2 to both members of the equation $x+3=5$. (*Ans.* $x+3+2=5+2$, or $x+5=7$.) Solve $x+5=7$ and compare the result with the solution of $x+3=5$.

10. Subtract 2 from both members of the equation $x+3=5$. (*Ans.* $x+3-2=5-2$, or $x+1=3$.) Solve $x+1=3$ and compare with the solution of $x+3=5$.

The same number may be added to or subtracted from both members of an equation without changing the value of the unknown number.

If $x=4$, $4\times x$, or $4x$, $=4\times 4=16$. If $4x=16$, $2\times 4x=2\times 16$, or $8x=32$.

If $4x=16$, $\frac{1}{4}$ of $4x$, or x, $=\frac{1}{4}$ of 16, or 4. If $8x=32$, $4x=\frac{1}{2}$ of 32, or 16.

Both members of an equation may be multiplied or divided by the same number without changing the value of the unknown number.

Find the value of x in the following equations:

11. $5x=25$ **13.** $4x+x=45$ **15.** $2x+3x=35$

12. $6x=36$ **14.** $5x-x=24$ **16.** $9x-5x=12$

17. If $\frac{1}{3}$ of x, or $\frac{x}{3}$, equals 12, how much does x equal? (*Ans.* 3×12, or 36.) If $\frac{2}{3}$ of x, or $\frac{2x}{3}$, equals 12, how much does x equal? (*Ans.* $\frac{1}{2}$ of 3×12, or $\frac{3}{2}$ of 12, or 18.)

18. If $\frac{2x}{3} = 3$, find the value of x.

Since 3 times $\frac{2x}{3}$ equals $2x$, multiply both members by 3 to make the first member an integer, $2x = 9$. Then divide both members by 2. *Ans.* $x = 4\frac{1}{2}$.

Solve the following equations:

19. $\frac{x}{2} = 10$ **21.** $\frac{3x}{5} = 15$ **23.** $\frac{3x}{4} = 9$

20. $\frac{2x}{3} = 8$ **22.** $\frac{4x}{9} = 16$ **24.** $\frac{5x}{7} = 25$

PROBLEMS

WRITTEN

254. In the statement of a problem certain statements, or suppositions, are given, and a question asks for a number whose value is unknown. The finding of this unknown number requires the making and solving of an equation in which the unknown number is represented by a letter.

The first step is to let x, or some other letter, represent the unknown number. This letter may then be used whenever it is necessary to employ the unknown number in the solution of the problem.

1. If a number is increased by 250, the result is 3296. What is the number?

SOLUTION

Let x = the number.
Then $x + 250 = 3296$.
$x = 3296 - 250$.
$x = 3046$.

PROOF: $3046 + 250 = 3296$.

2. If 295 is subtracted from a number, the result is 488. What is the number?

3. Ella's age is 12 years less than Frank's, who is 25 years old. How old is Ella?

4. Eva and Ruth together have $54. Eva has twice as much as Ruth. How much has each?

SOLUTION

Let x = the number of dollars Ruth has.
Then $2x$ = the number of dollars Eva has,
and $x + 2x = 54$.
Hence, $3x = 54$.
$x = 18$, the number of dollars Ruth has.
$2x = 2 \times 18 = 36$, the number of dollars Eva has.

PROOF: 36 = 2 times 18; 36 + 18 = 54.

5. Separate the number 600 into two parts, one of which shall be 5 times the other.

6. A horse and carriage cost $274. Find the cost of each if twice the cost of the carriage is $34 less than the cost of the horse.

7. If $\frac{3}{5}$ of a number is 72, what is the number?

8. What number added to 3 times itself equals 240?

9. If 6 times a number plus 15 equals 315, what is the number?

10. If 5 times a number less 48 equals 302, what is the number?

11. If a number is increased by 3 times itself, the result will be 72. What is the number?

12. A school in which there are 500 pupils has 56 more girls than boys. How many of each are there?

13. Three men, A, B, and C, formed a partnership with \$30,000 capital. B's capital was 3 times A's, and C's as much as the difference between B's and A's. How much capital did each furnish?

14. A girl made three purchases which amounted to \$27. For shoes she paid \$2 more than for gloves, and for a dress \$17 more than for shoes. How much did each purchase amount to?

15. The sum of three numbers is 24. The second is 5 less than the first, and the third is 10 more than the second. What are the numbers?

16. Divide the number 33 into three parts such that the second shall be 3 less than the first, and the third 4 times the first.

17. What must be added to 15 to make 42? What must be added to x to make 42?

18. Divide 18 into two parts, the smaller of which is x. What is the other part? The smaller part equals $\frac{1}{2}$ the larger part. Find each part.

19. A pipe will fill a tank in 8 hours. What part of the tank will it fill in 1 hour? In 3 hours? A pipe will fill a cistern in b hours, what part of the tank will it fill in 1 hour? In c hours?

20. 60 is 75 % of what number?

21. Divide the number 57 into three parts such that the second shall be 4 more than the first and the third 8 less than the second.

22. If a number is increased by 7 times itself and 40, the result is 360. What is the number?

23. Separate the number 240 into three parts the second of which shall be 3 times the first and the third 4 times the first.

24. What number diminished by 50 equals 712?

25. What number divided by 17 equals 20?

26. What number multiplied by 25 equals 625?

27. What number increased by 83 equals 214?

28. What number decreased by 20 % of itself equals 40?

29. What number increased by 20 % of itself equals 30?

30. Separate 100 into two parts, the smaller of which, x, equals $\frac{1}{3}$ of the larger part.

31. A mother is 4 times as old as her daughter, and the sum of their ages is 40 years. How old is the daughter? The mother?

32. The sum of the three sides of a triangle is 24 inches. If the second side is three times the first, and the third side twice the first, how long is each side?

33. Two poles are together 55 feet long. If one is 4 times the length of the other, how long is each?

34. Construct and solve six problems, one each like problems 24–29 on this page.

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ANSWERS

Page 21. — **1.** 282. **2.** 793. **3.** 720. **4.** 1105. **5.** 17,762. **6.** 101,421.

Page 22. — **7.** 3876. **8.** 2670. **9.** 2726. **10.** 3424. **11.** 3392. **12.** 38,340. **13.** 43,394. **14.** 30,341. **15.** 33,581. **16.** 22,785. **17.** 29,713. **18.** 32,736. **19.** 32,218. **20.** 212,645. **21.** 123,244. **22.** 3208. **23.** 3037.

Page 23. — **24.** 1968. **25.** 2575. **26.** 2386. **27.** 15,133. **28.** 24,183.

Page 24. — **2.** $102.81. **3.** $123.77. **4.** $59.05. **5.** $257.88. **6.** $1785.47. **7.** $31.07. **8.** $12,645.31. **9.** $4398.21.

Page 27. — **17.** $6.02.

Page 28. — **18.** $302.25. **19.** 479 bu. **20.** $241.23. **21.** 2d half, 184 da. 1st half, 181 da. **22.** $8054.50; $8650.50. **23.** 156. **24.** $29.90.

Page 29. — **25.** $3,077,406,908.

Page 31. — **4.** 928. **5.** 1918. **6.** 2994. **7.** 5087. **8.** 4198. **9.** 9172. **10.** 33,629. **11.** 10,504. **12.** 29,704. **13.** 39,106. **14.** 2369. **15.** 4720. **16.** 962,845. **17.** 14,869. **18.** 24,502. **19.** 42,089. **20.** 69,923.

Page 32. — **2.** $389.37. **3.** $28. **4.** $200.98. **5.** $187.95. **6.** $9.45. **7.** $9610.39. **8.** $36,196.78. **9.** $1488.69. **10.** $8361.05. **11.** $2458.78.

Page 34. — **13.** 38 cords. **14.** 97 cords. **15.** 145 cords.

Page 35. — **16.** $8468; $9062. **17.** 15¢. **18.** $2707.50. **19.** 95¢. **20.** $2400. **23.** $2. **24.** $265. **25.** $18 loss.

Page 36. — **26.** $1850. **28.** $4.83 grocer owes. **29.** 85¢. **30.** $1.11.

Page 37. — **32.** $550.

Page 42. — **3.** 273. **4.** 24,276. **5.** 20,861. **6.** 28,092. **7.** 8932. **8.** 40,732. **9.** 510,321. **10.** 127,710. **11.** 77,142. **12.** 85,084. **13.** 289,391. **14.** 8610.

Page 43. — **15.** 51,948. **16.** 478,695. **17.** 310,128. **18.** 336,288. **19.** 1,736,304. **20.** 538,424.

§ 25. **3.** 11,340. **4.** 4260. **5.** 15,720. **6.** 37,800. **7.** 14,820. **8.** 269,100. **9.** 42,650. **10.** 108,000. **11.** 370,200. **12.** 3,600,000. **13.** 3600. **14.** 82,000.

Page 44. — **1.** $57. **2.** $40.80. **3.** $1.12. **4.** $5222.75. **5.** $24,048. **6.** $104.91. **7.** $391,322.75. **8.** $7830. **9.** $750.

Page 45. — **7.** $28.75. **8.** $72. **9.** 60 min.; 1440 min.; 10,080 min.

Page 46. — **10.** 3360 mi. **11.** $11.40. **12.** 229,921. **13.** $626 gain. **14.** $825. **15.** $28.50. **16.** $13.04. **17.** 23,813,040 yd. **18.** 1140 mi. **19.** $510.

Page 48. — **2.** $92.76. **3.** $79.04. **4.** $115.90.

Page 61. — 7. $8\frac{17}{75}$. 8. $8\frac{21}{77}$. 9. $4\frac{89}{91}$. 10. $8\frac{215}{378}$. 11. $8\frac{65}{2112}$. 12. $4\frac{1884}{3773}$. 13. $3\frac{1778}{4140}$. 14. $4\frac{1799}{4356}$. 15. $8\frac{419}{432}$. 16. $8\frac{77}{141}$. 17. $6\frac{8}{45}$. 18. $8\frac{62}{77}$. 19. $5\frac{52}{135}$. 20. $3\frac{28}{41}$. 21. $6\frac{1}{417}$. 22. $2\frac{4}{49}$. 23. $5\frac{73}{497}$. 24. $7\frac{761}{91}$.

Page 62. — 2. $1029\frac{24}{25}$. 3. $162\frac{1011}{1400}$. 4. $197\frac{346}{420}$. 5. $56\frac{4946}{5000}$.
§ 43. 2. $9.81. 3. 91¢. 4. $8.79. 5. $93.65. 6. $8.03. 7. $75.18.

Page 64. — 14. 175 cows. 15. 4360 bu. 16. $2.75. 17. 51, and 705 sq. mi. remaining. 18. 244. 19. 12 horses; 41 sheep. 20. Ans. to last 3600 rev. 21. $113.60.

Page 65. — 22. $67.50. 23. 18¢. 24. $1110.49. 25. $4.80 gain. 26. $13.14. 27. 12 yr. 28. $9\frac{43}{49}$ av. age. 29. 75° nearly. 30. $21\frac{23}{37}$ da.

Page 66. — 31. 8 yr. 32. 35. 33. 94,621.

Page 67. — 1. 12. 2. 31. 3. 27. 4. 18. 5. 44. 6. 64. 7. 84. 8. 4. 9. 45. 10. 17.

Page 68. — 1. 1354. 2. 11,583. 3. 12,512. 4. 120,530. 5. 117,176. 6. 187,569.

Page 69. — 7. 187,176. 8. $1394.92 bal. 9. $593.78. 10. $284.

Page 70. — 11. 11,460. 12. $2.44. 13. 674,214. 14. 5 yr. 15. $7.50. 16. $135.48. 17. 61¢. 18. $32.85 for year; $32.94 if leap year; $16.29 for first six months, $16.38 if leap year; $8.10 for winter months, $8.19 if leap year; $8.28 for spring, and for summer months; $8.19 for autumn months.

Page 71. — 19. $244 at end of year; 20\frac{1}{3}$ av. per month. 20. 152 hr. and 5 min. in one common year. 21. $55. 22. 816. 23. $1700. 24. 10 yd.

Page 78. — 10. 15. 11. 3. 12. 15. 13. 6. 14. 7. 15. 144. 16. 2.

Page 80. — 28. 252. 29. 120. 30. 360. 31. 450. 32. 300. 33. 1575. 34. 720. 35. 1728. 36. 660. 37. 450. 38. 840. 39. 336. 40. 120. 41. 600. 42. 1000. 43. 420. 44. Tank contains 84 gal.

Page 87. — 28. $\frac{1}{4}$; $\frac{1}{5}$. 29. $\frac{5}{24}$; $\frac{5}{24}$. 30. $\frac{17}{10}$; $\frac{7}{8}$. 31. $\frac{1}{4}$; $\frac{3}{5}$. 32. $\frac{3}{8}$; $\frac{1}{2}$. 33. $\frac{5}{8}$; $\frac{9}{8}$. 34. $\frac{3}{4}$; $\frac{9}{10}$. 35. $\frac{5}{8}$; $\frac{1}{4}$. 36. $\frac{10}{11}$; $\frac{1}{2}$. 37. $\frac{7}{5}$; $\frac{11}{15}$.

Page 90. — 20. $2\frac{9}{140}$. 21. $1\frac{173}{180}$. 22. $2\frac{17}{72}$. 23. $1\frac{147}{440}$. 24. $2\frac{59}{252}$. 25. $1\frac{9}{110}$. 26. $1\frac{173}{990}$. 27. $1\frac{119}{300}$. 28. $2\frac{231}{990}$. 29. $1\frac{55}{144}$. 30. $2\frac{178}{1320}$. 31. $1\frac{209}{300}$. 32. $2\frac{229}{1320}$. 33. $2\frac{233}{495}$. 34. $1\frac{991}{2000}$. 35. $\frac{187}{1760}$.

Page 91. — 14. $10\frac{5}{9}$. 15. $8\frac{28}{45}$. 16. $14\frac{13}{24}$. 17. $6\frac{3}{4}$. 18. 23. 19. 8.
§ 63. 12. $\frac{49}{72}$. 13. $\frac{93}{11}$. 14. $\frac{109}{11}$. 15. $\frac{188}{15}$. 16. $\frac{2202}{8}$. 17. $\frac{277}{3}$. 18. $\frac{513}{8}$. 19. $\frac{1148}{9}$. 20. $\frac{605}{8}$. 21. $\frac{404}{3}$.

Page 93. — 3. $2\frac{13}{10}$. 4. $2\frac{19}{180}$. 5. $1\frac{26}{45}$. 6. $1\frac{57}{104}$. 7. $1\frac{139}{144}$. 8. $1\frac{1}{4}$. 9. $\frac{125}{264}$. 10. $\frac{1}{4}$. 11. $\frac{31}{60}$. 12. $\frac{9}{16}$. 13. $\frac{5}{40}$.

Page 94. — 17. $5\frac{43}{56}$. 18. $71\frac{1}{12}$. 19. $91\frac{41}{48}$. 20. $129\frac{41}{70}$. 21. $26\frac{17}{18}$. 22. $95\frac{13}{16}$. 23. $67\frac{85}{96}$. 24. $84\frac{9}{44}$.

Page 95. — 1. $2\frac{2}{15}$. 2. $3\frac{5}{24}$. 3. $2\frac{233}{360}$. 4. $4\frac{79}{360}$. 5. $37\frac{127}{140}$. 6. $50\frac{9}{40}$. 7. $1\frac{83}{140}$. 8. $\frac{23}{84}$. 9. $\frac{1}{36}$. 10. $\frac{56}{325}$. 11. $11\frac{23}{25}$. 12. $2\frac{13}{18}$. 13. $4\frac{11}{12}$.

Page 96. — 14. $3\frac{5}{8}$. 15. $77\frac{121}{440}$. 16. $427\frac{1}{90}$. 17. $432\frac{101}{175}$. 18. $61\frac{21}{70}$. 19. $362\frac{45}{231}$. 20. $4\frac{8}{17}$. 21. $14\frac{33}{78}$. 22. $887\frac{81}{800}$. 23. $231\frac{121}{144}$. 24. $461\frac{91}{231}$.

§ 69. 1. $158\frac{35}{120}$ A. 2. $33\frac{1}{4}$ cd. 3. $218\frac{1}{4}$ rd. 4. $\frac{13}{60}$. 5. $1814\frac{71}{120}$ mi. 6. $99\frac{33}{64}$ yd.

Page 97. — 7. $9\frac{7}{8}$ yd. 8. $32\frac{5}{21}$; $19\frac{23}{44}$.

Page 98. — 5. $4\frac{1}{2}$. 6. $67\frac{1}{2}$.

Page 99. — 7. 96. 8. $\frac{7}{12}$. 9. $9\frac{3}{5}$. 10. 108. 11. $742\frac{1}{2}$. 12. $\frac{11}{16}$.

Page 101. — 31. $\frac{1}{3}$. 32. $\frac{3}{25}$. 33. $1\frac{1}{3}$. 34. $12\frac{1}{2}$. 35. $205\frac{5}{64}$. 36. $26\frac{1}{4}$. 37. 100. 38. 12. 39. $211\frac{1}{3}$. 40. $47\frac{3}{11}$.

Page 102. — 42. $12{,}588\frac{1}{3}$. 43. $218{,}056\frac{1}{4}$. 44. $95{,}144\frac{7}{8}$.

Page 103. — 18. $5.63. 19. $146.69.

Page 104. — 21. $48.58. 22. $62.50. 23. $90.25. 24. $95.13. 25. $100.50. 26. $16.70. 27. $24.03. 28. $1.68. 29. $205 for entire lot.

Page 106. — 39. 1. 40. $1\frac{1}{2}$. 41. $1\frac{1}{2}$. 42. $\frac{2}{3}$. 43. $3\frac{3}{4}$. 44. $\frac{4}{15}$. 45. $6\frac{17}{18}$. 46. $\frac{14}{125}$. 47. $2\frac{1}{3}$. 48. 56. 49. 25. 50. 50.

Page 107. — 52. $112\frac{8}{35}$. 53. $26\frac{13}{35}$. 54. $48\frac{21}{50}$. 59. $\frac{5}{16}$. 60. $\frac{5}{6}$. 61. $1\frac{58}{117}$. 62. $\frac{50}{161}$.

Page 108. — 8. 36 rd. 9. 120 bbl. 10. $281\frac{1}{4}$ lb.

Page 109. — 11. 8 times. 13. 640 rev. 14. 10 strips. 15. 16 bows. 16. $2. 17. 10 yd.

Page 110. — 18. $10\frac{1}{4}$ da. 19. $127\frac{93}{141}$ mi. 20. 24 ft. 21. 48 ft. 2 in. 22. 20 rd. 23. 1 mi. in 2 min. 48 sec.; $4\frac{3}{4}$ mi. in 13 min. 18 sec. 24. $8\frac{1}{3}$ min. 25. $2\frac{3}{4}$ mi. 26. 85 tickets. 27. 40 lb. 28. $459\frac{3}{23}$ times. 29. 11 da.

Page 111. — 31. $\frac{48}{125}$. 32. $\frac{8}{9}$. 33. Answer to last, $\frac{1}{12}$. 34. Answer to last, $\frac{3}{4}$. 35. Answer to last, $\frac{5}{24}$. 37. $2\frac{1}{2}$ da. 38. $1\frac{1}{7}$ da. 39. 17 da. 40. $4\frac{1}{4}$ da.

Page 114. — 28. 30. 29. 104. 30. 168. 31. 96. 32. 105. 33. 207. 34. 164. 35. 21. 36. 27. 37. 546. 39. $25\frac{1}{3}$. 40. $114\frac{1}{3}$. 41. $8\frac{8}{17}$. 42. 9. 43. $14\frac{1}{4}$.

Page 115. — 6. $39. 7. 9 books. 8. $6\frac{3}{4}$ lb. 9. $1.00.

Page 119. — 17. $\frac{6}{11}$. 18. Gain $\frac{3}{10}$ of cost; $300, gain. 19. $\frac{13}{30}$. 20. $\frac{21}{71}$. 21. $\frac{24}{43}$.

Page 122. — 17. 12 girls. 18. 189 girls. 19. $60,000. 20. 264 sheep. 21. 70 ft. 22. $308,000.

Page 123. — 23. 768. 24. 120. 25. Answer to last, $5\frac{3}{7}$ da. 26. $15. 27. $1.24. 28. $\frac{8}{9}$. 29. $2\frac{2}{9}$ hr. 30. 20 hr. 31. Answer to last, $\frac{4}{21}$. 32. $\frac{17}{60}$; $3\frac{9}{17}$ da. 33. $4\frac{13}{16}$ da.

Page 124. — 34. Cost, $300; $\frac{1}{4}$ the cost. 35. Answer to last, $5.33. 36. $3.20 loss. 37. $212.10. 38. $153.60.

Page 141. — 3. 138.1737. 4. 107.01056. 5. 98.8705. 6. 2.0662. 7. 136.0886. 8. 53.000671. 9. 3.736.

Page 142. — 21. 3.66366. 22. .13077. 23. 217.55075. 24. 61,140.3436. 25. 68.0967. 26. 5408.1. 27. 1542.38. 28. .07279725. 29. $153.79. 30. $10.58. 31. 77.63. 32. 35.76. 33. 67. 34. .763. 35. .0906. 36. 78,300.

Page 146. — **1.** 1.18. **2.** .112$\frac{2}{3}$. **3.** .02507. **4.** .00044. **5.** 12.73$\frac{1}{3}$. **6.** .091. **7.** .059$\frac{1}{4}$. **8.** 64.86$\frac{2}{3}$. **9.** 246.14$\frac{6}{11}$. **10.** .00005. **11.** 128. **12.** .00005. **13.** 1.955. **14.** .273. **15.** .36$\frac{6}{11}$. **16.** 15,314.28+. **17.** 24. **18.** .875. **19.** .082+. **20.** .04. **21.** .097+. **22.** $115.50. **23.** $11.17. **24.** $51. **25.** $.077+. **26.** .375. **27.** .57$\frac{1}{7}$. **28.** .77$\frac{7}{9}$. **29.** .625. **30.** .08$\frac{1}{3}$. **31.** .11$\frac{1}{9}$. **32.** .178+. **33.** .0625.

Page 147. — **6.** $\frac{1}{5}$; $\frac{1}{6}$. **7.** $\frac{5}{8}$; $\frac{1}{12}$. **8.** $\frac{1}{3}$; $\frac{1}{5}$. **9.** $\frac{1}{4}$; $\frac{2}{5}$. **10.** $\frac{3}{40}$; $\frac{4}{7}$. **11.** $\frac{1}{1000}$; $\frac{5}{8}$. **12.** $\frac{1}{2}$; $\frac{3}{10}$. **13.** $\frac{4}{5}$; $\frac{888}{1000}$. **14.** $\frac{1}{3}$; $\frac{2}{3}$. **15.** $\frac{4}{5}$; $\frac{1}{2}$. **16.** $\frac{1}{100}$; $\frac{1}{15}$. **17.** $\frac{2}{7}$; $\frac{1}{15}$. **18.** $\frac{7}{15}$; $\frac{1}{11}$. **19.** $\frac{5}{16}$; $\frac{1}{100}$. **20.** $\frac{1}{16}$; $\frac{1}{110}$. **21.** $\frac{1}{12}$; $\frac{9}{100}$. **22.** $\frac{7}{16}$; $\frac{7}{12}$. **23.** $\frac{9}{16}$; $\frac{11}{12}$.

Page 149. — **12.** .777+; .416+; .857+. **13.** .529+; .555+; .106+. **14.** .771+; .188+; .950$\frac{40}{81}$. **15.** 1.8; 1.857+; 2.657+.

§ 98. 2. $42.

Page 150. — **2.** $223.78.

Page 151. — **2.** $30.94.

§ 101. 1. 250.275 A. **2.** 16,975 T. **3.** 89.725 T. **4.** $30.47. **5.** 244.712 T. **6.** $18.21. **7.** 142.3125 mi. **8.** $10.01. **9.** $21.25.

Page 152. — **10.** $2812.50. **12.** $2.75. **13.** $14.67; $13.75; $5. **15.** 50¢. **16.** $28.44.

Page 153. — **17.** 962.25 bbl. **20.** $1560.60. **21.** $577.50. **22.** $16.99. **23.** $13.60. **24.** $66.17. **25.** $738.37.

Page 154. — **26.** Coal dealer owes $42.83. **27.** 25$\frac{2}{3}$ bu. **28.** 500.

Page 156. — **6.** 9.72. **7.** 16.64. **8.** 13.44. **9.** 64.5. **10.** 7.5. **11.** 15.75. **12.** 25.625. **13.** 1.155. **14.** 1.92. **15.** .456. **16.** 5.85. **17.** 68.75. **18.** 3.75. **19.** 140.

Page 158. — **5.** .0375. **6.** 2$\frac{1}{2}$%. **7.** 4$\frac{1}{8}$%. **8.** 66$\frac{2}{3}$%. **9.** 50%.

Page 159. — **10.** 60%. **11.** .375; 37$\frac{1}{2}$%. **12.** Answer to last, 88$\frac{8}{9}$%.

Page 160. — **3.** 40 pupils.

Page 177. — **3.** 10$\frac{2}{3}$ bd. ft. **4.** 320 bd. ft. **5.** 280 ft. of lumber. **6.** $6.24. **7.** $33.60. **8.** $31.68. **9.** $12.80.

Page 184. — **7.** 2 hr. 40 min. **8.** 59 min. 20 sec. **9.** 5 hr. 8 min. 12 sec. ahead. **10.** 10 hr. 40 min. 55 sec. A.M. **11.** 10 hr. 26 min. 56 sec. P.M. April 30. **13.** 13° 15′. **14.** 10° 23′.

Page 185. — **15.** Traveled east; 68° 10′ W. Long. **16.** 11 hr. 47 min. 4 sec. A.M. **17.** 10 hr. 44 min. 16 sec. A.M.

Page 192. — **15.** 98 pk. **16.** 824 qt. **17.** 59 pt. **18.** 215 hr. **19.** 10,380 min. **20.** 81 sq. ft. **21.** 4392 sq. in. **22.** 396 oz. **23.** 7500 lb. **24.** 475 lb. **25.** 624 cu. ft. **26.** 41,582 cu. in. **27.** 58 yd.

Page 193. — **1.** 7 gal. **2.** 6 yd. 9 in. **3.** 1 da. 17 hr. 15 min. **4.** 17 cu. yd. 16 cu. ft. **5.** 7 bu. 2 pk. 7 qt. **6.** 9 rd. .5 yd. **7.** 3 mi. 283$\frac{1}{11}$ rd. **8.** 25 cwt. 60 lb. **9.** 4 lb. 11 oz. **10.** 12 bu. 5 qt. **11.** 2 T. 57 lb. 10 oz. **12.** 40 A.

Page 194. — **8.** 3 gal. 3 qt. 1 pt. 2 gi. **9.** 2 ft. 7.5 in.

Page 195. — **12.** 1.68; 3.6. **13.** 6; 3.6. **16.** .03. **17.** .09625. **20.** $\frac{25}{48}$ or .5208+.

Page 196. — **21.** $\frac{9}{13}$ or .281+. **22.** $\frac{14}{29}$ or .484+. **23.** $\frac{9}{10}$ or .9.

Page 197. — **4.** 6 mi. 68 rd. 3 yd. 1 ft. 6 in. **5.** 7 mi. 222 rd. 4 ft. **6.** 51 bu. 2 pk. 2 qt.

Page 198. — **8.** 5 bu. 4 qt. **9.** 5 gal. 3 qt. **10.** 6 yd. 2 ft. 10 in.

§ **143.** **2.** 55 yr. 2 mo. 15 da. **5.** 1 yr. 9 mo. 27 da.

Page 199. — **6.** 8 mo. 27 da. **8.** 1 mo. 9 da.; 9 mo. 26 da.; 1 yr. 6 mo. 6 da.; 6 mo. 19 da.

Page 200. — **3.** 115 rd. 7 ft. 6 in. **4.** 360 bu. 2 pk. **6.** 7 rd. 11 ft. 6 in. **7.** 26 A. 123$\frac{1}{8}$ sq. rd.

Page 201. — **8.** 1 lb. 9 oz.

§ **145.** **1.** $28.88. **2.** 21 cwt. 53 lb.; $118.42. **3.** 180 lb. **4.** $5202.34. **5.** $59.42. **6.** 126 T. 1495 lb. **7.** $60.30. **8.** $178.20.

Page 202. — **9.** 328$\frac{7}{8}$ T. **10.** 166 rd. 3 yd. 2 ft. **11.** 24 steps. **12.** 6204 steps. **13.** 720 times. **14.** 8910 times. **15.** 8 strips, lengthwise; 10 strips, crosswise. **16.** 8 strips, lengthwise; 9 strips, crosswise. **18.** 288 sq. rd. **19.** 1$\frac{1}{8}$ A.

Page 203. — **20.** 43$\frac{81}{32}$ A. **21.** 6$\frac{1}{4}$ A. **22.** 32 sq. ft. in walk. **23.** $576. **24.** 1296 sq. in.; 9 bd. ft. **25.** 224 sq. ft.; 104 sq. ft. **26.** $1975. **27.** 64 sq. in., sides; 48 sq. in., ends; 96 sq. in., top and bottom; 64 sq. in. more than one sq. ft.

Page 204. — **28.** 126 sq. yd. **29.** $9000; $2250. **30.** 864 sq. ft. **31.** 30 sq. yd. in. ceiling; 40 yd. of carpet. **32.** $2.40; $12.91. **33.** 288 tiles. **34.** 296 sq. ft. **35.** 70 ft. **36.** 216 ft. **37.** $13.71.

Page 205. — **38.** $8.40. **39.** 22.5 M; $90. **42.** $82.63. **43.** 281$\frac{17}{18}$ sq. ft. **44.** $20. **45.** 10 ft. by 10 ft.; 100 sq. ft.; 50 ft.

Page 206. — **46.** 15,697$\frac{11}{17}$ times. **47.** 14 gal. 1 qt. **48.** 16 baskets. **49.** 13 ties on 24 ft. of road; $1056.40 cost of ties for 1 mi. **50.** 37 bbl. **51.** 9 da. **52.** 34 mi. **53.** $9.19. **54.** $18.74. **55.** 35 bu. 3 pk. 4 qt. **56.** 1498 bu. 3 pk. 4 qt.

Page 207. — **57.** .719−. **58.** 40¢ per bu. **59.** 6 gross. **60.** $3.70 gain per gross. **61.** 359.06+ gal.; 38.57+ bu. **62.** 96 sq. in. **63.** 48 cu. yd. **64.** 192.85+ bu. **65.** 5984.41+ gal. **66.** 216 cu. ft. **67.** 144 bd. ft. **68.** 260 bottles.

Page 208. — **69.** $\frac{2}{33}$ mi. **70.** 14,400 shingles.

Page 215. — **51.** 371$\frac{7}{8}$ A. remained. **52.** 66$\frac{2}{3}$ lb. **53.** 320. **54.** 1305. **55.** $150. **56.** $\frac{11}{48}$; $17,966.67. **57.** $16, value of watch; $15, value of chain.

Page 216. — **58.** 115$\frac{1}{2}$ mi. **59.** $40,000. **60.** 131$\frac{1}{2}$%; 76$\frac{4}{21}$%. **61.** 91$\frac{2}{3}$%. **62.** $754.29. **63.** 4$\frac{1}{16}$%; 3$\frac{3}{4}$%; $\frac{1}{4}$%. **64.** $16,000. **65.** A's money 80% of B's. **66.** $45 lost. **67.** $570, Smith receives. **68.** B's is 83$\frac{1}{3}$% of A's; A's is 12% of B's.

Page 219. — **14.** $3.30. **15.** $31.

Page 220. — **16.** $18.70. **17.** 6% gain. **18.** 66$\frac{2}{3}$% gain. **19.** $231.25. **20.** 36% gain. **21.** 17+% gain. **23.** $7.50 loss. **24.** 50%.

Page 221. — **25.** 66$\frac{2}{3}$%. **26.** 33$\frac{1}{3}$% gain; 20% gain. **27.** 6$\frac{2}{3}$%. **28.** 100%. **29.** 99% loss; 9900% gain. **30.** $485 gain; 4$\frac{1}{2}$%. **31.** $57.60.

Page 222. — **32.** $25. **33.** $1.00. **34.** 5000 bu. **35.** 20% gain; $40 cost; $53.33 list price. **36.** 80¢ gain; 32% gain. **37.** 38⅛% gain. **38.** $4. **39.** 50%.

Page 224. — **1.** $16.80 discounts; $43.20 net price. **2.** $54 selling price. **3.** $152 received. **4.** Single discount is 1% better. **5.** 46%. **6.** 80%.

Page 225. — **2.** 2½%. **3.** $28.25 commission; $536.75 remitted. **4.** $300. **5.** $300 commission; $500 returned. **6.** $300 selling price; $294 received.

Page 226. — **7.** $25928 selling price. **9.** $5000 amount of debt. **10.** $315.78. **12.** $800 value of goods. **13.** $3756.25.

Page 227. — **2.** $8.75. **3.** $8531.25. **4.** $50.40.

Page 228. — **5.** $.80; ⅛%. **6.** $34,775.00.

Page 229. — **2.** $147.19. **3.** .02486; $149.16 tax. **4.** $102.70.

Page 230. — **3.** $5300. **4.** $135.60. **5.** $243.75. **6.** $2160. **7.** $648. **8.** $310. **9.** $1485. **10.** $6550.58.

Page 235. — **3.** $22.83. **4.** $24.06. **5.** $14.63. **6.** $80.00. **7.** $52.40. **8.** $22.16. **9.** $930.00. **10.** $4252.50. **11.** $83,093.75. **12.** $20.70. **13.** $44.38. **14.** 371.58.

Page 237. — **22.** $53.25. **23.** $29.58. **24.** $62.75. **25.** $48.98. **26.** $26.32. **27.** $33.48. **28.** $245.33. **29.** $40.73.

Page 240. — **1.** $.44. **2.** $1.36. **3.** $1.32. **4.** $2.06. **5.** $1.26. **6.** $9.86. **7.** $8.63.

Page 241. — **8.** $5.50. **9.** $5.72. **10.** $60.59.

Page 244. — **5.** 4 mo., drew interest 6%, $153.00.

Page 245. — **11.** $2687.50. **12.** $559.17. **13.** $1015.00. **14.** $888.13 simple interest; $887.95 exact interest. **15.** June 1, 1908, date of maturity, $1512.50.

Page 248. — **2.** 6%. **3.** 5%. **4.** 8%. **5.** 5%. **6.** $\frac{1}{10}$, ½. **11.** 10 mo.

Page 249. — **12.** 2 yr. 10 mo. 13 da. **13.** 3 mo. 24 da. **14.** 5 yr. **15.** 14 yr. 3 mo. 13 da. **16.** $365.00. **19.** $297.20. **20.** $5000. **21.** March 8, 1910. **22.** $1000.

Page 250. — **23.** $300. **24.** 300.

Page 253. — **2.** $4400.70. **3.** $485.05. **4.** $4485.61. **5.** $474.72.

Page 255. — **7.** $856.34. **8.** $874.02.

Page 257. — **2.** $75.91. **3.** $843.73.

Page 259. — **3.** $198.00 proceeds. **4.** $122.81 proceeds. **5.** $173.83 proceeds. **6.** $244.37 proceeds. **7.** $148.87 proceeds.

Page 260. — **9.** $4.88 discount; $451.87 proceeds. **10.** $4.15 discount; $753.25 proceeds. **11.** $.62 discount; $177.91 proceeds. **12.** $1.28 discount; $126.22 proceeds. **13.** $3.44 discount; $226.06 proceeds. **14.** $2.43 discount; $500.90 proceeds. **15.** $.985. **16.** $10,000. **17.** $.99. **18.** $252.52. **19.** $5000.

Page 262. — **23.** $44 discount; $1056 proceeds. **24.** $1066.67. **25.** 12%.

Page 263. — **26.** $.90. **27.** $90. **28.** $8000. **29.** $887.50. **30.** 218 da. **31.** $43.88. **32.** $114.25. **33.** $3\frac{4}{13}$% gain. **34.** 15% gain. **35.** $1045.50 premium; $4100 loss on goods; $5145.50 loss, including premium. **36.** $1.51.

Page 264. — **37.** $825 commission; $3675 remitted. **38** 9500%; 95 pk. **39.** $1\frac{2}{15}$% loss. **40.** $.13$\frac{1}{3}$ cost first grocer; $.15 cost second grocer. **41.** $2.70 asking price; $1.62 selling price; 8% gain. **42.** $76.25. **44.** 5%. **45.** $22\frac{2}{9}$%.

Page 265. — **46.** $20,000. **47.** $25 loss. **48.** 100%. **49.** 17%. **50.** $\frac{1}{8}$%. **51.** $2000. **52.** $446.06. **53.** $75. **54.** 20%. **55.** $\frac{3}{4}$% rate; $30 premium; $3970 company's loss, counting premium received.

Page 266. — **56.** $1924.28. **57.** $1370. **58.** 2% rate; $394 Giles pays. **59.** $320. **60.** $14.58. **61.** $412.31. **63.** $300.81. **64.** $1370.27. **65.** $4.70 first option.

Page 273. — **13.** $3\frac{1}{7}$. **14.** 15. **15.** $2\frac{1}{2}$. **16.** $12\frac{1}{2}$. **17.** $\frac{8}{21}$. **18.** $\frac{11}{14}$.

Page 277. — **1.** 92 bu. **2.** 160 men. **3.** 5 da. **4.** 3 men. **5.** 34 lb. **6.** 187.5 bu. **7.** 100 da. **8.** 7 hr.

Page 278. — **9.** 8 hr. **10.** 40 men. **11.** 5¢. **12.** 6 da.

Page 281. — **4.** 9; 6. **8.** 8 mi. first man travels, 12 mi. second man travels; 4 hr. **9.** 4 ft.; 6 ft.; 8 ft. **10.** 150 A., first child; 120 A., second child; 90 A., third child.

Page 282. — **11.** $2.50; $4.00. **12.** $1250 first man's gain; $2000 second man's gain.

Page 288. — **3.** 48. **4.** 45. **5.** 16. **6.** 65. **7.** 85. **8.** 64. **9.** 27. **10.** 34. **11.** 72.

Page 289. — **13.** 365. **14.** 324. **15.** 723. **16.** 314. **17.** 625. **19.** 702. **20.** 506. **21.** 250. **22.** 700. **23.** 6.72. **24.** 54.8. **25.** 5.25. **26.** 6.31. **27.** 7.13. **28.** 8.14.

Page 290. — **31.** 8.66+. **32.** $\frac{1}{4}$. **33.** .612+. **34.** .534+.

§ **196.** 1. 480 rd. **2.** 64 in. square. **3.** $2560. **4.** $492, $246 additional.

Page 296. — **2.** 6. **3.** 1. **4.** 15. **5.** 24.

Page 297. — **2.** 48. **3.** 630. **4.** 6048. **5.** 2850.

Page 298. — **2.** $1963.20. **3.** $1620.40. **4.** $1260.76. **5.** $257.23. **6.** $238.30. **7.** $746.75.

Page 299. — **2.** $378.74; $78.74. **3.** $57.58.

Page 300. — **2.** $1609.76. **3.** $5518.06. **4.** $1713.90. **5.** $456.96. **6.** $9948.95. **7.** $193.56.

Page 302. — **2.** $90. **3.** $9000. **4.** $690.30. **5.** $2100.

Page 303. — **6.** $80. **7.** $4558.22. **8.** $339.29. **9.** $186.24. **10.** $15.86 in favor of first offer.

Page 307. — **1.** $2.38; $2.60; $8.08; $55.93; $98.90. **2.** $2369.95. **4.** $476.19. **5.** $354.71.

Page 308. — **7.** $20,737.50. **8.** $1990.

Page 309. — **4.** $974. **5.** $963.30. **6.** $2378.13. **7.** $2442.93. **8.** 1766.35 taels.

Page 313. — **2.** $4462 market value; $138 discount. **3.** $4062.50. **4.** $7790.63. **5.** $1662.50. **6.** $11,875 loss. **7.** 24. **8.** $636. **9.** 45. **10.** 634.

Page 314. — **11.** $250,000. **12.** $6\frac{1}{4}\%$. **13.** 1000 shares. **14.** 42 shares; $651 gain. **15.** $15\frac{1}{2}\%$. **16.** $9000. **17.** $43.20. **18.** $210,210. **19.** $40 gain. **21.** $9000.

Page 315. — **22.** $11,250. **23.** $4100, saving in buying the latter. **24.** $380 less. **26.** $5\frac{4}{13}\%$. **27.** $6\frac{12}{13}\%$. **28.** 4's at 112. **29.** 7's at 90. **30.** $4\frac{12}{17}\%$. **32.** $75. **33.** 20%. **34.** $120.

Page 318. — **6.** $826\frac{2}{3}$ ft.

Page 319. — **7.** 20 da. **8.** $24\frac{8}{10}$ da. **9.** $53\frac{1}{2}\frac{1}{2}$ da.

Page 320. — **2.** 32. **3.** 43. **4.** 85. **5.** 7.2. **6.** .17.

Page 327. — **1.** 1240 gr. **3.** 2 lb. 5 3 13 19. **4.** 4 lb. 13 13 29 5 gr. **8.** 5 3 13 5 gr. **9.** $\frac{14}{175}$. **10.** $\frac{175}{192}$.

Page 328. — **11.** 5 oz. 2 pwt. 17 gr. **12.** 6 lb. 5 oz. 6 pwt. 21 gr. **13.** 9 pwt. 16.2 gr. **14.** $9\frac{1}{7}$ lb. Av. **15.** $2480.62.

Page 329. — **5.** 7680 lk. 307.2 rd. **6.** 58 ch. **7.** 34.385 mi. **8.** 240 ch.

Page 330. — **5.** $4000. **6.** $36,000.

Page 335. — **4.** 142.5 sq. in. **5.** $56\frac{7}{8}$ A. **6.** 100 sq. rd. **7.** $16\frac{1}{4}$ ft. **8.** 16.2 ft.

Page 336. — **2.** 150 sq. in. **3.** 5304 sq. in. **4.** 2424 sq. in. **5.** $35\frac{1}{16}$ A.

Page 337. — **4.** 69 in. **5.** 25 ft. **6.** 240 mi.

Page 338. — **7.** 3037.5 sq. rd.

Page 339. — **8.** 175 sq. in. **9.** 810 sq. in. **10.** $15\frac{1}{8}$ in.

§ **230.** **6.** 67,260 sq. rd. **7.** 5144 A. **8.** 1848 sq. in.

Page 340. — **9.** 1 sq. mi. **10.** 6 sq. yd. 4 sq. ft. 72 sq. in. **11.** $3.20. **13.** $495.93, including ceiling. **14.** $16.19, including ceiling. **15.** $6.69, no allowance for doors or windows. **16.** $205.11.

Page 341. — **17.** $27 lengthwise; $24 crosswise. **18.** $49.79 lengthwise; $51.94 crosswise. **19.** $1285.88. **20.** $1650. **21.** 1,474,560 A. **22.** 2.5 mi. **23.** $117,612. **24.** $195.56 cheaper to pave.

Page 343. — **4.** 452.39 sq. ft. **5.** 1964.5 sq. ft. **6.** 1964.5 sq. ft. **7.** 36 in. radius. **8.** 180 rd. circumference.

Page 346. — **2.** 240 sq. in. lateral surface; 252 sq. in. entire surface. **3.** 144 sq. in. lateral surface; 176 sq. in. entire surface. **4.** 603.19 sq. in. lateral surface; 829.37 sq. in. entire surface. **5.** 1005.31 sq. in. lateral surface; 1407.44 sq. in. entire surface.

Page 347. — **6.** 480 sq. in. lateral surface; 624 sq. in. entire surface. **7.** 324 sq. in. **8.** 1256.64 sq. in. lateral surface; 1570.8 sq. in. entire sur-

face. **9.** 4476.78 sq. ft. **10.** 314.16 sq. in. **11.** 314.16 sq. in. area of circle; 1256.64 sq. in. area of sphere; 2400 sq. in. area of cube.

§ **235.** **1.** 2340 sq. ft. **2.** 19½ A. **3.** 2062.5 sq. rd. **4.** 1287 ft. **5.** 737$\frac{1}{11}$ in.

Page 348. — **6.** 636.5 sq. ft. **7.** 18$\frac{3}{5}$ A. **8.** $4000. **9.** 266$\frac{2}{3}$ rd. **10.** 314.16 sq. ft., answer to last. **11.** 54.9+ sq. in. **12.** 150.79+ sq. in. **13.** 12,063.74+ lb. **14.** 73.83 sq. in. **15.** 1.12+ in. **16.** 81.2+ ft. **17.** 216 sq. in. first cube; 864 sq. in. second cube; 10.392 in. diagonal of first cube; 20.784 in. diagonal of second cube.

Page 349. — **18.** 376 sq. in. **19.** 420 sq. in. **20.** 32$\frac{5}{14}$ sq. yd. **21.** 240 sq. in. **22.** 471.24 sq. in. lateral surface; 628.32 sq. in. entire surface. **23.** 27.93− sq. yd. **24.** 30. **25.** 172,800 sq. ft. **26.** 432 sq. in. **27.** 201.06+ sq. in. lateral surface; 251.33− sq. in. total surface. **28.** 508.93+ sq. in.

Page 350. — **29.** 3.1416 sq. in. first sphere; 314.16 sq. in. second sphere. **30.** $4.71. **31.** $523.60. **32.** No difference.

Page 352. — **16.** 108 cu. ft. **18.** $333.33. **19.** 4693½ cu. yd. **20.** 41$\frac{1}{54}$ cu. yd. **22.** 5266$\frac{2}{7}$ bbl.

Page 353. — **23.** 26$\frac{2}{11}$ bbl. **24.** 771.4+ bu. **25.** 603.19− cu. in. **26.** 791.68 cu. in. volume of cone; 904.78 cu. in. volume of sphere. **27.** 100.22+ lb. **28.** 69.81+ lb. **29.** 201.06+ cu. in. **30.** 386.1+ cu. in. **31.** 52.5+ bu. **32.** 381.18 cu. in.

Page 355. — **2.** 19.635 sq. in. first circle; 1256.64 sq. in. second circle; 78.54 sq. in. third circle.

Page 356. — **3.** 16 in. **4.** 12 ft. **5.** 26.12+ in. diameter when B receives it; 18.47+ in. diameter when C receives it. **7.** 6$\frac{3}{4}$ T. **8.** 4.7+ ft. **9.** 163$\frac{3}{11}$ bbl. **10.** 6.2+ in. when B receives it; 7.8+ in. when C receives it.

Page 357. — **1.** 57$\frac{3}{5}$ T. **2.** 78 T. **3.** $17.10. **4.** 65.35 T. **5.** $215.99. **6.** 88 T. **7.** $135.28. **8.** 29 ft. 2 in.

Page 358. — **9.** 56 ft. 4 in. **10.** $234.33. **11.** 62 ft. 6 in. **12.** $423.92 value of wild hay greater than timothy.

§ **241.** **1.** $36.48. **2.** $116.40. **3.** $349.20. **4.** $83.20. **5.** $12.00.

Page 359. — **6.** $47.60 cheaper to fence square.

§ **242.** **1.** 72 T. **2.** 20.11 A. **3.** 36 cows. **4.** 31 ft. 8+ in. **5.** 20.11 da. **6.** 42 cows.

Page 360. — **1.** Doe 144.92+ lb.; Dean 258.99+ lb.; King 124.09+ lb.; Rule 241.18+ lb.; Riley 291.48 lb.; Gimlin 194.23+ lb.; Johns 98.20+ lb.; Lund 206.01+ lb.; Bell 114.24+ lb.

Page 361. — **2.** $714.16. **3.** $67.13. **4.** $77.71.

§ **244.** **1.** 192 lb. **2.** $12.35.

Page 362. — **3.** $97.92. **4.** 5046 lb. **5.** 56¢ per bu. **6.** $504.

§ **245.** **1.** 14,400 lb. **2.** 3$\frac{21}{121}$ in.

Page 363. — **3.** 51,000 cu. ft. **4.** 40,474.5 T. **5.** 4.46 in.; 20,237.25 T. **6.** 7,440,000 lb. more to mature corn. **8.** 7875 lb.; 29.92+ bbl. **9.** 21+%.

Page 365. —**4.** $984. **5.** 20,240 lb.; $809.60. **6.** 11,867.7 lb.; $74.17. **7.** $137.50.

Page 366. —**9.** 124.8 lb. **10.** 22.68 lb. **11.** 26.88 lb. **12.** 67.20 lb. **13.** $9.40. **14.** $3.75. **15.** 1.8 T. to replace nitrogen; $1\frac{1}{10}$ T. more to replace phosphoric acid. **16.** 3.12 T. for wheat; 7.56 T. for corn.

Page 367. — **1.** $8,000,000. **2.** 16. **3.** $12\frac{1}{2}$ ¢ cost of clean seed. **7.** 81,000 in four months of 30 days each.

Page 368. —**10.** 327,600,000. **11.** $3510 in one day of 7 hours. **12.** 18,150 cu. ft.; 25,410 cu. ft.; 32,670 cu. ft. **13.** 4 A. **14.** 44 mi. **15.** 40% more. **16.** $8\frac{1}{3}$ da.

Page 369. —**1.** 2.92 T. **2.** $1\frac{11}{18}$ T. **3.** 300 lb. **4.** 11.3. **5.** 10.5. **6.** 1080 T. **7.** 1.8. **8.** 1200 lb. **9.** 6.65 cu. in. glass.

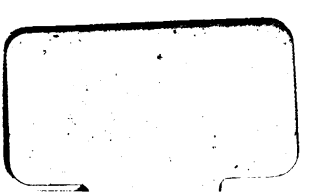
Page 370. —**3.** $8. **4.** 3 cups.

Page 371. —**5.** $.32. **8.** $3.03.

Page 380. —**3.** $x = 2$. **4.** $x = 20$. **5.** $x = 2$. **6.** $x = 4$. **7.** $x = 8$. **8.** $x = 30$. **11.** $x = 5$. **12.** $x = 6$. **13.** $x = 9$. **14.** $x = 6$. **15.** $x = 7$. **16.** $x = 3$.

Page 381. —**19.** $x = 20$. **20.** $x = 12$. **21.** $x = 25$. **22.** $x = 36$. **23.** $x = 12$. **24.** $x = 35$.

Page 382. —**2.** 783. **3.** 13. **5.** 100; 500. **6.** $80 carriage; $194 horse. **7.** 120. **8.** 60. **9.** 50. **10.** 70. **11.** 18. **12.** 222 boys; 278 girls.

Page 383. —**13.** A's $5000; B's $15,000; C's $10,000. **14.** Gloves $2; shoes $4; dress $21. **15.** First 8; second 3; third 13. **16.** First 6; second 3; third, 24. **17.** 27; $42 - x$. **18.** 6; 12. **20.** 80. **21.** 19; 23; 15. **22.** 40.

Page 384. —**23.** 30; 90; 120. **24.** 762. **25.** 340. **26.** 25. **27.** 131. **28.** 50. **29.** 25. **30.** 25; 75. **31.** Daughter 8; mother 32. **32.** First 4 in.; second 12 in.; third 8 in. **33.** 11; 44.